

THE
ELEMENTS
OF
NAVIGATION;

CONTAINING THE
THEORY and PRACTICE.

With the necessary TABLES.

To which is added,

A TREATISE
OF
MARINE FORTIFICATION.

Composed for the Use of

The ROYAL MATHEMATICAL School at CHRIST'S HOSPITAL,
The ROYAL ACADEMY at PORTSMOUTH,
And the GENTLEMEN of the NAVY.

VOLUME THE SECOND.

The THIRD EDITION, with ADDITIONS,

And COMPENDIUMS for finding
The LATITUDE and LONGITUDE at Sea.

By J. ROBERTSON,

LIBRARIAN to the Royal Society,

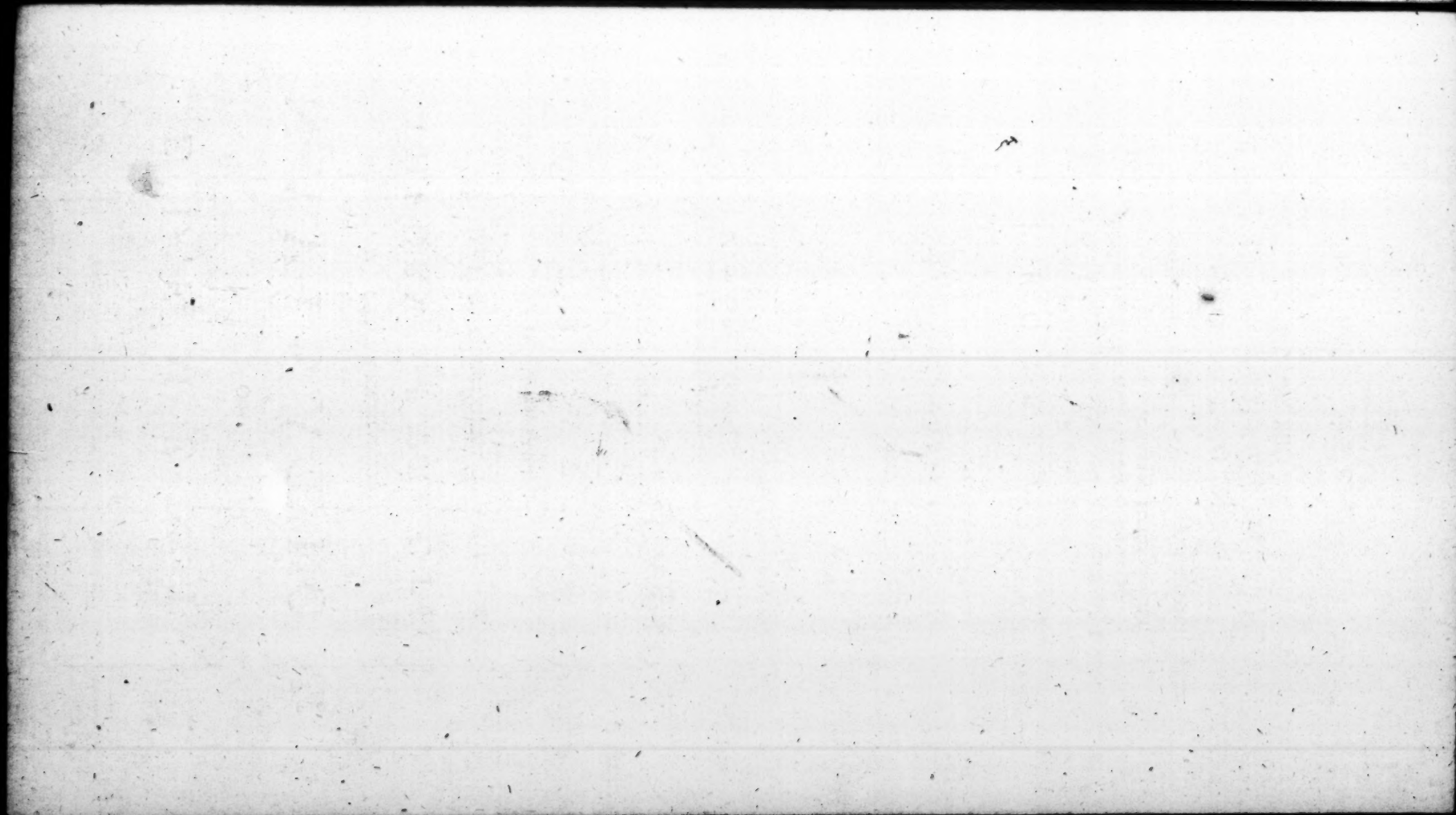
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THE
ELEMENTS
OF
NAVIGATION.

BOOK VII.
OF PLANE SAILING.

SECTION I.

Definitions and Principles.

1. **N**AVIGATION is the art of conducting a ship from one port to another.
This art may be divided into two parts; namely, Mechanical and Theoretical.

MECHANICAL, or **WORKING NAVIGATION**, comprehends the art of working a ship; that is, of causing it to observe such motions and directions as are assigned by the Navigator.

This art must be learned on shipboard, and in the practice of sailing.

THEORETICAL NAVIGATION, or the art of Piloting, concerns those methods whereby the Navigator, or Pilot, discovers in what track the ship must steer, so as to arrive safely at the intended port.

The art of Piloting is either common or proper.

COMMON PILOTING is that knowledge which teaches how to coast along shore, or to sail within sight of land.

PROPER PILOTING is the art of sailing to distant places through the ocean, and out of sight of land.

The art of proper piloting, (which partly includes that of common piloting) chiefly depends on a right application of the principles contained in the preceding books.

Vol. II.

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2. The

2. The navigating or conducting a ship through the Ocean to distant countries, depends on the Navigator's being beforehand furnished with the following elements.

1st. A Table of the latitudes and longitudes of the most remarkable parts of the Sea-coasts, Islands, Rocks, Shoals, &c. in the frequented parts of the world.

2d. Maps or Charts of the Seas and Lands; together with the depths of water, and the times and setting of the tides, upon the coasts he may have occasion to approach near.

3. The use and application of several Instruments necessary to point out the way the ship is to steer, to measure the rate she runs at, and to find the place she is in at any time.

4th. A sufficient stock of Mathematical Learning to enable him rightly to use the observations that may be deduced from the preceding elements.

The latitudes and longitudes of places cannot be otherwise obtained, than from proper observations made by persons who have been in those places: consequently, the collecting a sufficient number of these must be the work of many ages. And hence arises the great uncertainty of the positions of many places, and even of the little knowledge we have of many parts on the surface of the Terraqueous globe. See the Geographical Table, at the end of Book VI.

3. The Maps of Countries, and the Charts of the Oceans and their boundaries, are made chiefly from the catalogue of the latitudes and longitudes of places: But where these are partly wanting, and a good collection of the positions and distances of places can be obtained, those places may be inserted in the map as truly as the others, whose latitudes and longitudes are known.

The most natural way of delineating the surface of the Earth, is to do it on a sphere; for then every place on the Globe, laid down by its latitude and longitude, will have the same position with respect to other places, they really have on the surface of the Earth. But as a Globe is a bulky thing, and not so easily managed, or worked upon, as a plane; therefore has the representation of the surface of the Earth been made on planes, and these are called *Maps* or *Charts*.

In the constructing of Maps and Charts, great care should be taken, that the several parts in them preserve their position one to the other in the same order as they do on the Earth; and it is likely, that the finding out of proper methods to do this, gave rise to the several methods of projection.

4. There are many ways of constructing Maps and Charts; but they chiefly depend on two principles.

First,

First, From considering the Earth as a large extended flat surface: The Charts made on this supposition, are usually called PLANE CHARTS.

Secondly, From considering the Earth as a sphere: The Charts made on this principle are sometimes called GLOBULAR CHARTS, or MERCATOR'S CHARTS, or REDUCED CHARTS, or PROJECTED CHARTS.

Plane Charts, when made for any large extent of several degrees in high latitudes, are very erroneous; for in them the difference of longitude between any two Meridians, taken at the Equator, is used as the distance of those meridians in all latitudes; and consequently the distances of places in high latitudes will be too great, and thereby their due positions will not be preserved: But Charts made for a small part, as a degree or two in length and breadth, may be reckoned as tolerably exact, especially if those places be within the *Torrid Zone*; nay, a Plane Chart made for all this Zone will differ but little from the truth.

SECTION II.

5. *The Construction and Use of the Plane Charts.*

CONSTRUCTION.

1st. Having determined the limits of the Chart, that is, how many degrees of latitude and longitude, or meridional distance (they being in this Chart the same), it is to contain: Suppose from the lat. of 20° N. to the lat. of 71° N; and from the longitude of London in 0 deg. to the lon. of 50° W; then fix on some scale of equal parts, such whereby the chart may be contained within the size of the sheet of paper it is intended to be drawn on. In the chart annexed, see Pl. VII. the scale is such, that each degree of latitude and longitude is $\frac{1}{3}$ part of an inch.

2d. Make a parallelogram (II. 69) ABCD, whose length AB from north to south shall contain 51 degrees, the difference of latitude between the limits of 20° and 71° ; and the breadth AD from east to west shall contain the proposed 50 degrees of longitude, the degrees being taken from the said scale of 8 degrees to an inch; and this parallelogram will be the boundaries of the chart.

3d. About the boundaries of the chart, make (II. 81) scales containing the degrees, halves and quarters of degrees (if the scale is large enough); drawing lines across the chart through every 5 or 10 degrees; let the degrees of latitude and longitude have their respective numbers annexed, and the sheet is then fitted to receive the places intended to be delineated thereon.

4th. On a strait slip of pasteboard, or stiff paper, let the scale of the degrees and parts of degrees of longitude, in the line AD, be laid close to the edge; and the divisions numbered from the right hand towards the left, being all west longitude.

B 2

5th. Seek

5th. Seek in the geographical table (at the end of Book VI.) for the latitudes and longitudes of the places contained within the proposed limits; and let them be wrote out in the order in which they increase in latitude.

6th. Then, to lay down any place, lay the edge of the pasteboard scale to the divisions on each side the chart, shewing the latitude of the place; so that the beginning of its divisions fall on the right hand border A B; and against the division shewing the longitude of the given place make a point, and this gives the position of the place proposed; and in like manner are all the other places to be laid down.

7th. Draw waving lines from one point to the other, where the coast is contiguous, and thus the representation of the lands within the proposed limits will be delineated.

8th. Write the names to the respective parts, and in some convenient place insert a compass, and the chart will be completed.

The Use of the Plane Chart.

6. To find the lat. and lon. of any proposed place in the Chart.

Take, with a pair of compasses, the least distance between the given place and some parallel of latitude; this distance applied (the same way) to the graduated meridian on the border of the chart, keeping one point on the same parallel of latitude, the other point will shew the degrees of latitude the proposed place is in.

And in like manner, the distance between the given place and some meridian, applied (the same way) to the graduated parallel on the top or bottom of the chart, will shew its degrees of longitude.

Thus the distance between Cape Finesterre and the parallel of latitude of 40° , laid on the border from 40° upwards, gives $43^{\circ} 15'$ N. for Cape Finesterre's latitude: And the distance of the Cape from the meridian passing through 10° of longitude, being laid on the bottom border from 10° toward the right, gives $9^{\circ} 20'$ W. for the longitude.

7. Two places being given, suppose Cape Clear and the Island of St. Michael's, one of the Azores, to find the bearing and distance between them.

Lay a ruler by the two places, C. Clear and St. Michael's, take the nearest distance between the center of the compass and the edge of the ruler; in this position slide one point along the ruler, and the other point will run along that point of the compass shewing the bearing, which in this case is S. W.; that is, St. Michael's lies to the S. W. of C. Clear, or C. Clear to the N. E. of St. Michael's.

The distance between the two places applied to the graduated parallel from 0 degrees, will give the number of degrees, which converted into miles shews their distance.

Thus the distance from Cape Clear to St. Michael's will be found to be about $15^{\circ} 25'$, which multiplied by 60 gives 925 miles.

8. *Th*

8. *The distance sailed on a given course, and from a given place, being known, to find the place the ship is in. Thus suppose a ship sails from Cape Clear S. W. for 925 miles: Required her present place?*

Reduce the given miles (925) into degrees and minutes, and take them ($15^{\circ} 25'$) with one pair of compasses from the graduated scales.

Lay a ruler by the centre and the given point of the compass (S. W.); take the nearest distance between (Cape Clear) the place sailed from and the edge of the ruler with another pair of compasses; in this position, slide one point along the ruler until the other point is got as far distant from the given place as is the given distance, which is found by applying the opening of the other pair of compasses from (Cape Clear) the given place, till the points of both pair of compasses meet; and that will give the present place of the ship, which will be found in the latitude and longitude of St Michael's; and these are known by Art. 6.

The Mariner working by the preceding precepts, will preserve his chart free from a multitude of lines: But when this is not regarded (as indeed it is scarcely worth while), the operations will become somewhat shorter.

9. *The longitude of a place (A), and its bearing from a given place (B) being known; to find that place (A) on the chart, and its distance from B.*

Through the given place B draw a line parallel to the given point of the compass or bearing; then an east and west line drawn through the latitude of A, its intersection with the former line will give the place sought. The distance may be measured, as in Art. 7.

10. *The latitude of a place (A), and its distance from a given place (B) being known; to find the place of A on the chart.*

Draw an east and west line through the latitude of A; take the given distance in degrees from the graduated scale; then this distance being applied from the given place B will cut the said east and west line in the required place of A.

11. *The longitude of a place A, and its bearing from a given place B being known; to find the place of A on the chart.*

Draw a meridian through the longitude of A; then the given distance taken as before, and applied from B, will cut the said meridian in the required place of A.

12. *The longitude of a place A, and its bearing from a given place B being known; to find the place of A on the chart.*

Through the given place B draw the given bearing. (7)
Draw a meridian through the longitude of A, and its intersection with the said bearing will be the place of A.

And in this manner may every possible case of sailing by the plane chart be readily solved: But as the accuracy of these solutions cannot be depended upon nearer than five or six minutes more or less than the truth; therefore the more careful Navigators depend on computation, as will be shewn in the following sections.

SECTION III.

Of Plane Sailing.

13. **PLANE SAILING** is the art of navigating a ship upon principles deduced from the notion of the Earth's being an extended plane.

On this supposition, the meridians are esteemed as parallel right lines, the parallels of latitude are at right angles to the meridians, the lengths of the degrees on the meridians, equator and parallels of latitude, are every where equal; and the degrees of longitude are reckoned on the parallels of latitude as well as on the equator.

14. The **PLANE CHART** is a map, or representation, of the several parts of the Earth, laid down from these principles, according to their latitudes and longitudes, on a sheet of paper, or other flat surface. (5)

When the parts of the Earth are thus delineated on a plane, it is easy to see the track by which the ship may go from one place to another; and also what angle this track makes with the meridian: And ships at sea are kept in this track, or path, by the help of an instrument called the Mariner's Compass.

15. The **MARINER'S COMPASS** is an artificial representation of the horizon of every place, by the means of a circular piece of paper called a Card, divided like the horizon into degrees and points, which are called Rhumbs: Now the card being properly fixed to a piece of steel called a Needle, that has been touched with a loadstone (whose property is such as to cause one end of the needle, so touched, to point towards the north, when turning freely on something supporting it), all the points of the card will be directed towards their corresponding points of the horizon.

From this construction of the Mariner's Compass, it follows, that in every place the north point of the card shews the position of the meridian in that place; and some one rhumb, or point of the card, will coincide with, or be directed along, the track that makes any given angle with the meridian: Consequently, by the help of the card, or compass, a ship may be kept on any proposed track, or course.

16. A **RHUMB LINE** is a right line drawn from the center of the compass to the horizon; and is named from that point of the horizon it falls in with.

17. The **COURSE** is an angle which any Rhumb line makes with the meridian, and is sometimes reckoned in degrees, and sometimes in points of the compass.

18. **DISTANCE** is the number of leagues, miles, &c. intercepted between two places reckoned on a Rhumb; or it is the way or length a ship has gone on a direct course in a given time.

19. **DEPARTURE** is the east or west distance of a ship from the meridian of some noted place, reckoned on its parallel of latitude in the Plane Chart; and therein it is the same as the difference of longitude.

20. If a ship sails either due north or south, she sails on a meridian, makes no departure, and her distance and difference of latitude are the same.

If a ship sails either due east or west, she runs on a parallel of latitude, makes no difference of latitude, and her departure and distance are the same.

21. The difference of latitude and departure always make the legs of a right angled triangle, whose hypotenuse is the distance the ship has sailed.

When the course is 45 degrees, or 4 points, the difference of latitude and departure are equal.

When the course is less than 45 degrees, or 4 points, the difference of latitude is greater than the departure.

But when the course is greater than 4 points, or 45 degrees, the departure is greater than the difference of latitude.

22. When a figure relating to a ship's course is to be constructed, it must first be considered whether the ship is sailing northward or southward; and whether she goes to the eastward or westward of the place she departed from; for the lines in the figure must be drawn accordingly.

Thus. Let the upper part of the paper, or what the figure is drawn on, always represent the north, then the lower part will be south, the right hand side east, and the left hand side the west.

Draw a north and south line to express the meridian of the place the ship sails from; then if the course is to the southward, mark the upper end of the line for the place sailed from: But if the course is northward, mark the lower end for that place.

Through the point sailed from, draw a line perpendicular to the meridian to represent the parallel of latitude of that place; and from the same point, with the chord of 60 degrees, describe a quadrantal arc on the east side of the meridian, if the course is eastward, or on the west side, if the course is to the westward.

When the course is given, the chord of the degrees, or points expressing it, is to be taken with a pair of compasses, either from the scale of chords, or of rhumbs*; and is always to be laid off on the quadrantal arc, beginning at the meridian.

The line expressing the distance the ship has run, is always to be drawn from the center of the quadrant, or point sailed from, through that point of its arc which limits the course.

The difference of latitude, in leagues or miles, taken with the compasses from a scale of equal parts, is ever to be laid on the meridian, reckoning from the center of the quadrant.

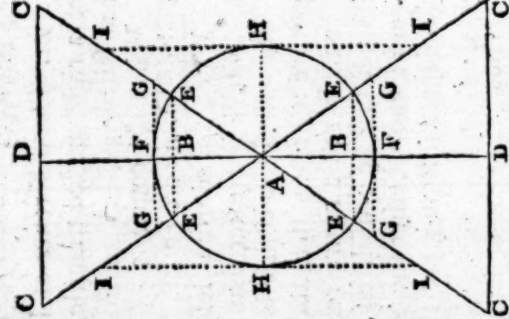
* A quadrant, or arc of 90° , being divided into eight equal parts, and each of these into quarters; then if from one end of the arc the several divisions be transferred to the chord of 90° , as in Book III. Art. 18. a line of rhumbs is thereby constructed, whose use is the same as the chords, namely, to make an angle whose measure is given in points instead of degrees.

The departure is to be a right line parallel to the east and west line before drawn, and joins the ends of the lines expressing the distance and difference of latitude.

23. In the annexed figure, suppose the circle $FHFH$ to represent the horizon of the place A , from whence the ship sails; AC the rhumb she sails on, and c the place come to; then HH represents the parallel of latitude she sailed from, cc the parallel of the latitude arrived in: So that AD becomes the difference of latitude, DC the departure, AC the distance sailed, the angle DAC is the course, and the angle DCA is the complement of the course.

These particulars will be alike represented, whether the ships sails in the N. E. or N. W. S. E. or S. W. quarters of the horizon.

Hence it is evident, that the difference of latitude, departure and distance, form the sides of a right angled triangle, whose angles are the course, its complement and the right angle, whose sine is radius; therefore among these four things, *course, distance, diff. lat.* and *departure*, any two of them being given, the rest are to be found by plane trigonometry.



24.

Of the Traverse Table.

For the ready working of any single course, there is a table, called a *Traverse Table*, usually annexed to Treatises of this kind; which is so contrived, that by finding therein the given course and a distance not exceeding 100 miles, the *diff. lat.* and departure is had by inspection: The course is to be found at the top of the table, when under 4 points, or 45 deg.; but at the bottom of the table when it exceeds 45 deg. Each column signed with a course consists of two parts, one for the *diff. lat.*, marked *Lat.*; the other for the departure, marked *Dep.*; which names are both at the top and bottom of these columns. The distance is to be found in the column, marked *Dist.* next the margin of the page.

This table will serve for greater distances, by taking their halves, thirds, fourths, &c. and doubling, tripling, quadrupling, &c. the *diff. lat.* and departure found to those parts of the distance.

The traverse table contained in this Treatise, at the end of Book VII, has the distances continued to 120, which is a number more capable of subdivisions than 100: Besides, the table is divided into two parts; the first contains the whole points and quarter points in a quadrant; and the second part is fitted to every degree and quarter of a degree in the quadrant; whereby the course and distance, resulting from a day's work, is more accurately obtained than by the common traverse tables.

25. All the proportions that can arise in plane sailing are exhibited in the following table; they refer to the preceding figure, where AE , AF , or AH represent the radius of the tables, EB the sine of the course, GF its tangent, and AG its secant; AB stands for the sine of the complement of the course, HI its tangent, and AI its secant.

These

These proportions are here inserted, to exercise the learner in reasoning on the relations subsisting between these lines and those representing the distance, departure and difference of latitude: But in what follows, only the analogies in the first column, and one against the last case in the second column, will be used; the others being rather matters of curiosity than use.

Case	Given	Req.	Solution by fines	Solution by tangents and secants
—	—	—	by $\text{fin. } \Delta \text{ADC, ABE}$	by $\text{fin. } \Delta \text{ADC, AFG}$ by $\text{fin. } \Delta \text{ADC, AHI}$
1	$\angle A, AC$	AD DC	AE:AB::AC:AD AE:EB::AC:DC	AG:AF::AC:AD AG:FG::AC:DC
2	$\angle A, AD$	AC DC	AB:AE::AD:AC AB:BE::AD:DC	AF:AG::AD:AC AF:FG::AD:DC
3	$\angle A, DC$	AC AD	BE:AE::DC:AC BE:AB::DC:AD	FG:AG::DC:AC FG:AF::DC:AD
4	AD, AC	$\angle A$ DC	AC:AD::AE:AB AE:EB::AC:DC	AD:AC::AF:AG AF:FG::AD:DC
5	AC, DC	$\angle A$ AD	AC:DC::AE:EB AE:AB::AC:AD	
6	AD, DC	$\angle A$ AC		AD:DC::AF:FG AF:AG::AD:AC
				DC:AC::AH:AI AH:HI::DC:AD
				DC:AD::AH:HI AH:AI::DC:AC

26. To assist the learner, the proportions hereafter used are given in their nautical terms.

		SOLUTIONS.	
Case	Given	Required	
1	Course & Distance	Diff. lat. Depart.	Rad : Dist :: Cof. Cou : Diff. lat. N ^o I. Rad : Dist :: sin. Cou : Depart N ^o II.
2	Course Diff. lat.	Distance Depart.	Cof. Cou: Diff. lat.: Rad : Distance. N ^o III. Cof. Cou: Diff. lat.: sin. Cou: Depart or by N ^o II.
3	Course Depart.	Distance Diff. lat.	Sin. Cou: Depart.: Rad : Dist. N ^o V. Sin. Cou: Depart.: Cof. Cou: Diff. lat. or by N ^o I.
4	Distance Diff. lat.	Course Depart.	Dist : Rad :: Diff. lat : Cof. Cou. N ^o V. Rad : Dist :: sin. Cou : Depart. by N ^o II.
5	Distance Depart.	Course Diff. lat.	Dist : Rad :: Depart. : sin. Cou. N ^o X. Rad : Dist :: Cof. Cou : Diff. lat by N ^o I.
6	Dist. lat. Depart.	Course Distance	Diff. lat : Depart.: Rad : tan Cou. N ^o XI. Sin. Cou: Depart.: Rad : Dist. by N ^o IV.

These proportions (or Canons as they are usually called by Mariners,) are applied to the numeral solutions of all the varieties of single courses in the following section.

The Nos I, II, &c. placed at the end of these Canons, shew which proportion they refer to in the upper table; reckoning two in each case.

27. A TABLE of the angles, which every rhumb, or point of the compass, makes with the meridian.

NORTH.	SOUTH.	Points.	D. M.	NORTH.	SOUTH.
N. b. E.	S. b. E.	0 $\frac{1}{4}$ 0 $\frac{1}{2}$ 0 $\frac{3}{4}$ 1 0	2 49 5 37 $\frac{1}{2}$ 8 26 11 15	N. b. W.	S. by W.
N. N. E.	S. S. E.	1 $\frac{1}{4}$ 1 $\frac{1}{2}$ 1 $\frac{3}{4}$ 2 0	14 04 16 52 $\frac{1}{2}$ 19 41 22 30	N. N. W.	S. S. W.
N. E. by N.	S. E. b. S.	2 $\frac{1}{4}$ 2 $\frac{1}{2}$ 2 $\frac{3}{4}$ 3 0	25 19 28 07 $\frac{1}{2}$ 30 56 33 45	N. W. by N.	S. W. b. S.
N. E.	S. E.	3 $\frac{1}{4}$ 3 $\frac{1}{2}$ 3 $\frac{3}{4}$ 4 0	36 34 39 22 $\frac{1}{2}$ 42 11 45 00	N. W.	S. W.
N. E. b. E.	S. E. by E.	4 $\frac{1}{4}$ 4 $\frac{1}{2}$ 4 $\frac{3}{4}$ 5 0	47 49 50 37 $\frac{1}{2}$ 53 26 56 15	N. W. b. W.	S. W. b. W.
E. N. E.	E. S. E.	5 $\frac{1}{4}$ 5 $\frac{1}{2}$ 5 $\frac{3}{4}$ 6 0	59 04 61 52 $\frac{1}{2}$ 64 41 67 30	W. N. W.	W. S. W.
E. b. N.	E. by S.	6 $\frac{1}{4}$ 6 $\frac{1}{2}$ 6 $\frac{3}{4}$ 7 0	70 19 73 07 $\frac{1}{2}$ 75 56 78 45	W. b. N.	W. b. S.
E. A. S. T.	E. A. S. T.	7 $\frac{1}{4}$ 7 $\frac{1}{2}$ 7 $\frac{3}{4}$ 8 0	81 34 84 22 $\frac{1}{2}$ 87 11 90 00	W. E. S. T.	W. E. S. T.

Before the learner begins to work the cases of plane sailing, he should be so well acquainted with the compass, or this table, as to be readily able to tell how many points any of these courses or rhumbs are distant from the meridian, or from the parallel.

SECTION

SECTION IV.

Of Single Courses.

28. CASE I. Given the course steered, and the distance run : Required the difference of latitude and departure?

EXAM. *A ship from the latitude $47^{\circ} 30' N.$ has sailed S. W. b. S. 98 miles : What lat. is she in, and what dep. has she made?*

By the TRAVERSE TABLE.

Find the given course at the top or bottom of the table among the points or degrees ; and in that column, right against the distance taken in its column, stand the diff. lat. and departure in their columns.

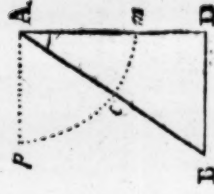
Thus the course is S. W. b. S. or 3 points, in which column, and against the distance 98 in the column of distances, stand 81,5 miles for the diff. lat. ; and 54,45 miles for the departure.

By CONSTRUCTION. (22)

1. Draw the meridian AD, and make the quadrant Amcp.

2. Make mc = rhumb of 3 points, and through c draw AB equal to 98 miles.

3. Draw BD parallel to pA, meeting AD in D. Then AD the diff. lat. measures about $81\frac{1}{2}$ miles. And DB the departure measures about 54,4 miles.



By COMPUTATION.

The course being 3 points = $33^{\circ} 45'$. The comp. course is 5pts. = $56^{\circ} 15'$. To find the diff. latitude.

As Rad	= $90^{\circ} 00'$	10,00000	As Rad	= $90^{\circ} 00'$	10,00000
To diff.	= 38 m.	1,99123	To diff.	= 98 m.	1,99123
So cof. Co.	= $33^{\circ} 45'$	9,91985	So sin Co.	= $33^{\circ} 45'$	9,74474
To diff. lat.	= 81,48	1,91108	To dep.	= 54,45	1,73597

By GUNTER'S SCALE.

On the line of fine Rhumbs*, the extent from Radius, or 8 points, to 3 points, applied to the line of numbers, will reach from 98, in either interval, to 54,4.

And the extent from 8 points, to 5 points among the Rhumbs, reaches from 98 to 81,5 on the line of numbers.

Or thus. The extent from Radius, or 8 points, in the S. R. to 98 in the Nos. will reach from 3 points in the Rhumbs to 54,4 in the Nos. and the same extent will reach from 5 points in the Rh. to 81,5 in the Nos.

The lat. from $47^{\circ} 30' N.$
diff. lat. 1 21 S.
 $\frac{81}{60} = 1,21.$

Lat. in 46 9 N. Dep. 54,4 miles West.

* The lines on the Gunter's scale marked S. R. and T. R. Signifying *Sine Rhumbs* and *Tangent Rhumbs*, are the logarithmic lines and tangents answering to the degrees and minutes in each point and quarter point of the Compass, transferred from the scales of log. sines and log. tangents.

29. CASE II. Given the course steered, and difference of latitude :
Required the distance run, and the departure?

EXAM. *A ship has sailed S. E. b. S. from the lat. $47^{\circ} 30' N.$ to the lat. $46^{\circ} 08' N.$ What distance has she run, and what departure has she made?*

By the TRAVERSE TABLE.

Find the course among the degrees, and the diff. lat. in its column, right against which stand the departure and distance in their columns.

Now taking $46^{\circ} 08'$ from $47^{\circ} 30'$, remains $1^{\circ} 22' = 82$ m. the diff. lat.

Then the course S. E. b. S. is 3 points, under which in the column of lat. the nearest number to 82 is 82,3. the corresponding departure is 55, and the distance is 99 miles.

By CONSTRUCTION.

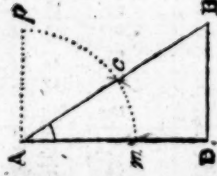
1st. Describe the quadrant Amp .

2d. Make mc = rhumb of 3 points, and continue A till $AD = 82$ miles.

3d. Through D draw DB parallel to Ap , meeting the line AB drawn through c , in the point B .

Then the distance AB measures about 99 miles.

And the departure DB measures about 55 miles.



By COMPUTATION.

The course 3 points = $33^{\circ} 45'$. The com. course is 5 pts = $56^{\circ} 15'$.
To find the distance.

As cos. Co.	= $33^{\circ} 45'$	0,08015
To diff. lat.	= 82 m.	1,91381
So Rad	= $90^{\circ} 00'$	10,00000
To dist.	= 98,62	1,99396

To find the departure.

As cos. Co.	= $33^{\circ} 45'$	0,08015
To diff. lat.	= 82 m.	1,91381
So sin Co.	= $33^{\circ} 45'$	9,74474
To dep.	= 54,79	1,73870

By GUNTER'S SCALE.

On the line of fine rhumbs, the extent from 5 points to 8 points, or radius, will, on the line of numbers, reach from 82 to 99.

And the extent from 5 points to 3 points on the S. R. will reach from 82 to 55 on the line of numbers.

Or thus. The extent from 5 points in the fine rhumbs, to 82 miles in the line of numbers, will reach from 8 points in those rhumbs to 99 in the numbers; and the same extent will reach from 3 points in the rhumbs to 55 in the numbers.

Note. In all these cases, wherever the cos. Co. is used, the deg. put down is the course itself, yet the log. belongs to the comp. of that course.

30. CASE III. Given the course steered and the departure:
Required the dist. run, and diff. latitude?

EXAM. *A ship from lat. $47^{\circ} 30' N.$ sailing $N. W. b. W.$ finds she has made 82 miles of departure: What is her distance run, and what lat. is she now in?*

By the TRAVERSE TABLE.

Find the course among the degrees, and the departure in its column; right against which stand the difference of latitude, and the distance in their respective columns.

Now the course $N. W. b. W.$ is 5 points, over which, in the column of departure, the nearest number to 82, is 82.3; and the corresponding diff. latitude is 55 miles, and the distance is 99 miles.

By CONSTRUCTION.

1st. Describe the quadrantal arc Amp .

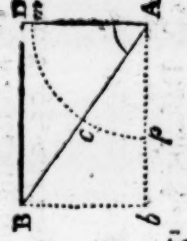
2d. Make mc = rhumb of 5 points, and continue Ap till Ab = 82 miles.

3d. Through b draw bB parallel to Am , meeting the line AB , drawn through c , in B .

4th. Through B draw BD parallel to bA , meeting Am continued, in D .

Then the distance AB measures about 99 miles.

And the diff. lat. AD measures about 55 miles.



By COMPUTATION.

The course 5 points is $56^{\circ} 15'$. The comp. course is 3pts. = $33^{\circ} 45'$.
To find the distance.

As fin. Co. = $56^{\circ} 15'$ 0,08015
To dep. = 82 m. 1,91381
So Rad. = $90^{\circ} 00'$ 10,00000

As fin. Co. = $56^{\circ} 15'$ 0,08015
To dep. = 82 m. 1,91381
So cos. Co. = $56^{\circ} 15'$ 9,74474

To dist. = 98,62

To diff. lat. = 54,79

By GUNTER'S SCALE.

On the line of fine rhumbs, the extent from 5 points to 8 points, or radius, will, on the line of numbers, reach from 82 to 99.

And the extent from 5 points to 3 points on the line of S. R. will reach from 82 to 55 on the line of numbers.

Or thus. The extent from 5 points in the fine rhumbs to 82 miles in the line of numbers, will reach from 8 points in the rhumbs to 99 in the numbers: And the same extent will reach from 3 points in the rhumbs to 55 in the numbers.

Lat. from $47^{\circ} 30' N.$
Diff. lat. 0 55 N.

Lat. in 48 25 N.

31. CASE IV. Given the distance and difference of latitude:
Required the course and departure?

EXAM. *A ship from lat. $47^{\circ} 20' N.$ sails between the north and east 98 miles, and is arrived in lat. $48^{\circ} 42' N.$ What course did she steer, and what departure has she made?*

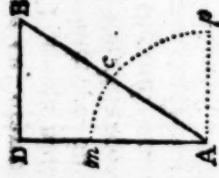
By the TRAVERSE TABLE.

Seek in the table, till against the distance taken in its column be found the given diff. lat. in one of the following columns; and adjoining to it stands the departure, which, if less than the diff. lat. the course is found at the top of the column; but if greater, the course is found at bottom.

Now taking $47^{\circ} 20'$ from $48^{\circ} 42'$, leaves $1^{\circ} 22' = 82$ m. the diff. lat. Then seeking till against dist. 98 stands lat. 82 , the nearest will be lat. $81,95$, and here the departure is $53,73$ m. and the course is $33^{\circ} 15'$.

By CONSTRUCTION.

- 1st. Describe the quadrant Amc .
 - 2d. Continue $A m$, till $AD = 82$ m.
 - 3d. Through D draw DB parallel to $A p$.
 - 4th. On A with the distance 98 cut DB in B .
- Then the departure DB measures about 53 m.
And the course DAB is measured by the arc mc of about 33° .



By COMPUTATION.

To find the course.		To find the departure.	
As dist.	$= 98$ m.	As Rad	$= 90^{\circ} 00'$
To Rad	$= 90^{\circ} 00'$	To dist.	$= 98$ m.
So diff. lat.	$= 82$ m.	So sin Co.	$= 33^{\circ} 13'$
To col. Co.	$= 33^{\circ} 13'$	To dep.	$= 53,69$
			1,72986

By GUNTER'S SCALE.

On the line of numbers, the extent from 98 m. to 82 m. will, on the line of fines, reach from 90, or radius, to about 57 degrees.

And the extent on the line of fines, from 90° to 33° , will reach on the line of numbers from 98 m. to about 53 miles.

Or thus. The extent from 98 m. in the line of numbers, to 90° in the line of fines, will reach from 82 m. in the numbers to about 57° in the fines: And the same extent will reach from about 33° in the fines, to about 53 in the numbers.

So that the course is $N. 33^{\circ} 15' E.$ or $N. E. b. N.$ nearly.
And the departure is 53,69 miles to the Eastward.

32. CASE

32. CASE V. Given the distance and departure:
Required the course, and difference of latitude?

EXAM. From the lat. $50^{\circ} 13' N.$ a ship, in sailing between the South and east 98 miles, makes her departure 82 miles: What course did she keep, and what lat. is she arrived at?

By the TRAVERSE TABLE.

Seek in the table, till against the distance, taken in its column, be found the given departure in one of the following columns, and adjoining to it stands the diff. lat. which if greater than the departure, the course is found at the top of the column; but if less, the course is found at the bottom.

Now seeking till against the dist. 98 stands 82, the nearest will be 81,95, signed Dep. at bottom; and here the diff. lat. is $53,73$ m. and the course is $56^{\circ} 45'$.

By CONSTRUCTION.

1st. Describe the quadrant Amp .
2d. Continue Ap , till $A b = 82$ m.; and through b draw $b B$ parallel to Am .
3d. From A with the dist. 98 m. cut $b B$ in B .
4th. Through B draw BD parallel to Ab , meeting Am continued, in D .
Then the diff. lat. AD measures about 53 miles.
And the course DAB is measured by the arc mc of about 57° .



By COMPUTATION.

To find the course.		To find the diff. lat.	
As dist.	= 98 m.	As Rad	= $90^{\circ} 00'$
To Rad.	= $90^{\circ} 00'$	To dist.	= 98 m.
So dep.	= 82 m.	So cos. Co.	= $56^{\circ} 47'$
To sin Co.		To diff. lat.	
= $56^{\circ} 47'$		= 53,69	
		1,72986	

Then $50^{\circ} 13' N. - 0^{\circ} 54' N. = 49^{\circ} 19' N.$ For 53,69 m. is nearest 54.

By GÜNTER'S SCALE.

On the line of numbers, the extent from 98 m. to 82 m. will, on the line of sines, reach from 90° , or radius, to about 57 degrees.

And the extent on the line of sines, from 90° to 33° , will reach from 98 m. to about 53 m. on the line of numbers.

Or thus. The extent from 98 m. in the line of numbers to 90° in the line of sines, will reach from 82 m. in the numbers to about 57° in the sines: And the same extent will reach from about 33° in the sines, to about 53 m. in the numbers.

So that the course is S. $56^{\circ} 47'$ E. or S. E. b. E. nearly.

And the latitude arrived in is $49^{\circ} 19' N.$

33. CASE

33. CASE VI. Given the difference of latitude and departure:
Required the course and distance?

EXAM. A ship from lat. $48^{\circ} 32' N.$ is arrived in lat. $40^{\circ} 54' N.$ and is got 54 miles to the westward of her departed meridian: What course did she steer, and what is the direct distance she has run?

By the TRAVERSE TABLE.

Seek in the table till the given diff. lat. and departure are found together in their respective columns; then right against them will be found the distance in its column, and the course stands among the degrees at the top or bottom of the column, where the diff. lat. and depart. were found.

Now taking $48^{\circ} 32'$ from $49^{\circ} 54'$, leaves $1^{\circ} 22' = 82m.$ the diff. lat. Then in the table it will be found, that the numbers standing together the nearest to 82 and 54, are 81, 95 and 53, 73; the distance answering to these is 98 m. and the course is $33^{\circ} 15'$.

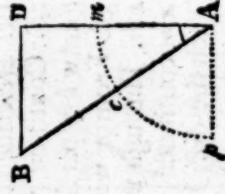
By CONSTRUCTION.

1st. Describe the quadrant Amp , and continue Am till $AD = 82m.$

2d. Through D draw DB parallel to Ap , and equal to 54 miles, and draw $AB.$

Then the distance AB measures about 98 miles.

And the course DAB is measured by the arc mc of about $33^{\circ}.$



By COMPUTATION.

To find the course.		To find the distance.	
As diff. lat.	= 82 m.	As fin. Co.	= $33^{\circ} 22'$ 0,25964
To depart.	= 54 m.	To depart.	= 54 m. 1,73239
So Rad	= $45^{\circ} 00'$ 10,00000	So Rad	= $90^{\circ} 00'$ 10,00000
To tan. Co. = 33 22 9,81858		To dist. = 98,18 1,99203	

By GUNTER'S SCALE.

On the line of numbers, the extent from 82 m. to 54 m. will reach on the line of tangents from 45° to about $33^{\circ}.$

And the extent on the line of fines, from about 33° to 90° , will reach on the line of numbers from 54 m. to about 98 m.

Or thus. The extent from 82 in the line of numbers to 45° in the line of tangents, will reach from 54 in the line of numbers, to about 33° in the line of tangents.

Also, the extent from about 33° in the fines to 54 in the numbers, will reach from 90° in the fines to about 98 m. in the numbers.

So the course is $N. 33^{\circ} 22'. W$ or $N. W. b. N.$ nearly.

And the distance is 98,18 miles.

QUEST. III. *A ship in lat. $3^{\circ} 52' S.$ is bound to a port bearing NW. by W. $\frac{1}{4} W.$ in lat. $4^{\circ} 30' N.$: How far does that port lie to the westward, and what is the ship's distance from it?*

Lat. from	$3^{\circ} 52' S.$
Lat. to	$4^{\circ} 30' N.$
Diff. lat.	$8^{\circ} 22' = 502 \text{ miles.}$



CONST. Describe the quadrant Amp : Draw the meridian $AD = 502$, and through D draw DB parallel to Ap : Make $m = 5\frac{1}{2}$ points: Through c draw AB meeting DB in B .

COMPUTATION.

As col. Co.	$= 61^{\circ} 52'$	$0,32650$	As col. Co.	$= 61^{\circ} 52'$	$0,32650$
To diff. lat.	$= 502 \text{ m.}$	$2,70070$	To diff. lat.	$= 502 \text{ m.}$	$2,70070$
So Rad	$= 90^{\circ} 00'$	$10,00000$	So sin. Co.	$= 61^{\circ} 52'$	$9,94540$
To dist.	$= 1065$	$3,02720$	To dep.	$= 938,9$	$2,97260$

Answer { The port lies $938,9 \text{ m.}$ to the westward.
The direct distance is 1065 .

QUEST. IV. *A ship from lat. $30^{\circ} 14' N.$ sails SE. by E. $\frac{1}{2} E.$; and four days after meets a sloop who had sailed directly east at the rate of 6 miles an hour, she having been four days before under the meridian the ship departed from: Required the difference of their rate of sailing, and their present latitude?*

As the ship and sloop sailed from the same meridian, and the sloop runs directly east; therefore her distance will be the ship's departure.

Now in 4 days are 96 hours.

And $96 \times 6 = 576$ miles of departure.



CONST. Describe the quadrant Amp .

In A continued take $Ab = 576$, and draw bB parallel to Am . Make $m = 5\frac{1}{2}$ points, and through c draw Ac meeting bB in B . Through B draw BD parallel to bA meeting Am continued in D .

COMPUTATION.

As sin. Co.	$= 61^{\circ} 52'$	$0,05460$	As sin. Co.	$= 61^{\circ} 52'$	$0,05460$
To dep.	$= 576 \text{ m.}$	$2,76042$	To dep.	$= 576 \text{ m.}$	$2,76042$
So Rad.	$= 90^{\circ} 00'$	$10,00000$	So col. Co.	$= 61^{\circ} 52'$	$9,67350$
To dist.	$= 653,2 \text{ m.}$	$2,81502$	To diff. lat.	$= 308 \text{ m.}$	$2,48852$
Now $96(653,2(6,8 \text{ m. per hour the$			And $6,0(30,8(Then lat. fr. 30^{\circ} 14' N.$		
[ship's rate,			diff. lat. $5^{\circ} 08' S.$		
772			$5^{\circ} 08'$		
			Lat. in $25^{\circ} 06' N.$		

So the difference of their rate of sailing is $\frac{8}{10}$ or $\frac{4}{5}$ of a mile per hour.

QUEST.

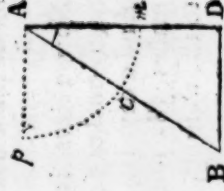
QUEST. V. *A ship from the lat. $48^{\circ} 17' N.$ sails SW. by S. until she has depressed the north pole two degrees: What direct distance has she run, and how many miles is she got to the westward?*

To depress the pole two degrees, the ship must sail southward until her diff. lat. is 2 degrees, or 120 miles.

CONST. 1st. Draw $AD = \text{diff. lat. } 120 \text{ m.}$

2d. Describe the quadrant AmP , and make $mC = 3$ points.

3d. Through c draw AB , meeting DB drawn through D parallel to AP .



COMPUTATION.

Asc. Co.	$= 33^{\circ} 45'$	Asc. Co.	$= 33^{\circ} 45'$	0,08015
To diff. lat.	$= 120 \text{ m.}$	To diff. lat.	$= 120 \text{ m.}$	2,07918
So Rad	$= 90^{\circ} 00'$	So sin. Co.	$= 33^{\circ} 45'$	9,74474
To dist.	$= 144.3$	To dep.	$= 80.18$	1,90407

Answer { Ship has run 144.3 miles.

And has got to the westward 80.18 miles.

QUEST. VI. *Two ports lie under the same meridian, one in latitude $52^{\circ} 30' N.$ and the other in latitude $47^{\circ} 10' N.$ A ship from the southern port sails due east 9 knots an hour, and two days after meets a sloop that had sailed from the northernmost port: Required the sloop's direct course and distance run?*

Here the distance the ship sails east is the sloop's departure.

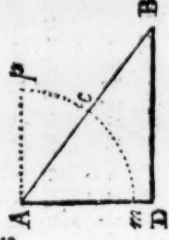
Now 2 days $= 48$ hours.

Then $48 \times 9 = 432 \text{ m.} = \text{departure.}$

Sloop's lat. $52^{\circ} 30' N.$

Ship's lat. $47^{\circ} 10' N.$

Diff. lat. $5^{\circ} 20' = 320 \text{ m.}$



CONST. In the meridian, take AD for the diff. lat. 320 m. and perpendicular to AD draw the departure $DB = 432 \text{ m.}$ and draw the distance AB .

COMPUTATION.

As diff. lat.	$= 320 \text{ m.}$	As sin. Co.	$= 53^{\circ} 28'$	0,09501
To depart.	$= 432 \text{ m.}$	To depart.	$= 432 \text{ m.}$	2,63548
So Rad	$= 45^{\circ} 00'$	So Rad	$= 90^{\circ} 00'$	10,00000
To tan. Co.	$= 53^{\circ} 28'$	To dist.	$= 537.6$	2,73049

Answer { The sloop has sailed S. $53^{\circ} 28' E.$ or SE. $\frac{1}{4} E.$
And has run 537.6 miles.

SECTION V.

Of Compound Courses.

35. A COMPOUND COURSE is an irregular track which a ship makes in a known time, by sailing on several different courses: Or where two or more cases of Plane sailing are concerned.

A TRAVERSE is a compound course, wherein several different successive courses and distances are known.

36.

TO WORK A TRAVERSE,

Or, to reduce a compound course to a single one.

1st. Make a table of six columns, title them Course, Dist. N. S. E. W. beginning at the left side, and write the given courses and distances in their respective columns.

2d. In the traverse table seek the given courses and distances, and let the corresponding differences of latitude and departures be wrote in their proper columns in the table made for the question.

3d. Add up the columns of northing, southing, easting, and westing; then the difference between the sums of the northing and southing gives the whole difference of latitude of the same name with the greater; and the difference between the sums of the easting and westing will be the whole departure, of the same name with the greater.

4th. The whole diff. lat. and departure to the compound course being found, the direct course and distance will be found as in Case VI. of single courses.

37.

TO CONSTRUCT A TRAVERSE.

1st. Describe a circle with the chord of 60 degrees, and divide it into 4 quadrants; marking the north point with N.

2d. Lay off each course on the circumference, reckoned from its proper meridian; and from the center to each point draw so many blank radii; mark these with the proper number of the course.

3d. On the first radius, lay the first distance, mark its extremity; through this extremity, and parallel to the second radius, draw the second distance of its proper length; through the extremity of the second distance, and parallel to the third radius, draw the third distance of its proper length: and thus proceed until all the distances are drawn.

4th. A line drawn from the extremity of the last distance, to the center of the circle, will represent the distance made good; and a line drawn from the same point, perpendicular to the meridian (lengthened if necessary) will represent the departure.

38. When a ship from a given place is bound to a known port; and on some occasion she is obliged to run on one or more different courses, not in the direct one to the port; and then wants to find the direct course and distance she must now run to gain her port, observe the following precepts.

1st. Find the diff. lat. and dep. between the ship's first place and the port bound to.

2d. Find the diff. lat. and dep. between the ship's first place and the place she is come to.

3d. If these differences of latitude are both of the same name, take their difference for a new diff. lat.: But if of contrary names, take their sum.

4th. If the departures are both of the same name, take their difference for a new departure: But take their sum, if of contrary names.

5th. To this new difference of latitude and departure, find the course and distance (33).

6th. If between the place failed from, and the place come to, there has been more than one course: To each course let the diff. lat. and departure be found; and let their sum or difference be taken according as they are of the same, or of contrary, names; and this will give the diff. lat. and departure between the place failed from and the place come to: Then proceed by the 3d, 4th, and 5th precepts.

39. TO CONSTRUCT A COMPOUND COURSE.

See either of the figures after Question VII.

1st. Describe a circle, in which draw the meridian and parallel of the place failed from.

2d. Construct the triangle between the place failed from, marked A, and the place bound to, marked B; and let the intersection of the diff. lat. and departure be marked D.

3d. Construct the figure between the place failed from, or A, and the place come to, marked C; and let the intersection of this diff. lat. and departure be marked E.

4th. Through C draw CF parallel to AD, meeting BD, continued, if necessary, in F; and draw CB.

5th. Then in the triangle CFB, the lines CF, FB, CB, will represent the diff. lat. departure and distance between the place come to, and the place bound to.

6th. From the center A draw a line parallel to CB, in the direction from C to B, and this shews the course wanted.

40. Examples to exercise Traverse Sailing.

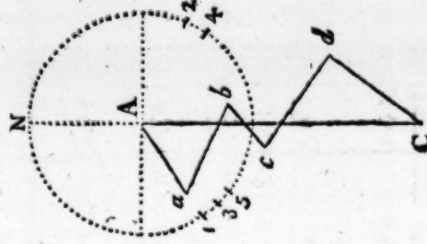
QUESTION. I. A ship sails from a place in latitude $24^{\circ} 32' N.$ and has run the following courses and distances viz.

1st. SW. b. W. dist. 45 m. 2d. ESE. dist. 50 m. 3d. SW. dist. 30 m.
4th. SE. b. E. dist. 60 m. 5th. SW. b. S. $\frac{1}{4}$ W. dist. 63 m.

Required her present latitude, with the direct course and distance between the place sailed from, and the place come to?

Construct the traverse table by art. 36, and the figure by art. 37.

TRAVERSE TABLE.				
Courses.	D.	N.	S.	E.
SW. b. W.	45		25,0	
ESE.	50		19,1	46,2
SW.	30		21,2	
SE. b. E.	60		33,3	49,9
SW. b. S. $\frac{1}{4}$ W.	63		50,6	
		149,2	96,1	96,1



The traverse table being completed; the sum of the southings is 149,2 miles, and so far the ship has got to the southward, or altered her latitude.

The lat. from $24^{\circ} 32' N.$

$$\text{Diff. lat. } \frac{149}{60} = 2 \text{ } 29 \text{ S.}$$

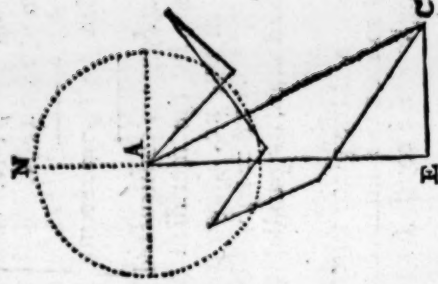
Present lat. $22 \text{ } 03 \text{ N.}$

The miles of departure in the east column are 96,1; and those in the west column are also 96,1: But as the east and west departures are directly opposite, they destroy one another, and the ship in her present station has made no departure; therefore she is under the same meridian sailed from: Consequently her course made good is directly south; and her distance is the same as the difference of latitude, which is 149,2.

In this figure the circle, which is described with the chord of 60° , represents the horizon, the center whereof is the place the ship sailed from; through which are drawn the two diameters at right angles to one another, the one representing the meridian, the other the parallel of latitude the ship departed from: On the circumference is laid the several courses, and numbered 1, 2, 3, 4, 5. On the first rhumb A 1, is laid the first distance 45 ; through a , and parallel to the rhumb A 2, is drawn the second distance 50 ; through b , and parallel to the rhumb A 3, is drawn the third distance 30 ; through c , and parallel to the rhumb A 4, is drawn the fourth distance 60 ; through d , and parallel to the rhumb A 5, is drawn the fifth distance 63 .

QUEST. VI. The last 24 hours we have run the following courses, viz.
 1st. SE. 40m. 2d. NE. 28 m. 3d. SW. b. W. 52. m.
 4th. NW. b. W. 30m. 5th. SSE. 36 m. 6th. SE. b. E. 58 m.
 Required the diff. latitude, departure, direct course and distance?

TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
SE.	40	—	28,3	28,3	—
NE.	28	19,8	—	19,8	—
SW. b. W.	52	—	28,9	—	43,2
NW. b. W.	30	16,7	—	—	24,9
SSE.	36	—	33,3	13,8	—
SE. b. E.	58	—	32,2	48,2	—
		36,5	122,7	110,1	68,1
			36,5	68,1	—
		D. lat.	86,2	42,0	Dep.

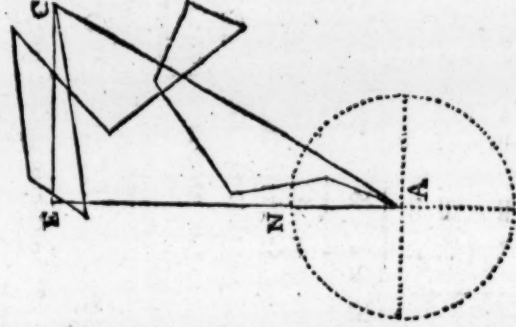


For the course. As 86,2m. : 42m. :: Rad : t. $25^{\circ} 59'$. Or SSE. $\frac{1}{4}$ E. nearly.
 For the distance. As f. $25^{\circ} 59'$: 42 m. :: Rad : 95,87 m.

QUEST. VII. Yesterday noon we were in lat. $3^{\circ} 18' S$. and since then we have plied on the following courses, viz.

1st. NNE. 22m. 2d. N. b. W. 30m. 3d. NE. b. E. 40m.
 4th. ESE. 25m. 5th. SSW. 18m. 6th. NW. b. N. $\frac{1}{4}$ W. 50m.
 7th. NE. $\frac{1}{4}$ E. 42m. 8th. W. b. S. $\frac{1}{4}$ W. 45m. 9th. SW. b. S. 20m.
 10th. E. b. N. $\frac{1}{4}$ E. 62m. Required our present lat. and departure, with the course and distance made good?

TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
NNE.	22	20,3	—	—	—
N. b. W.	30	29,4	—	8,4	5,8
NE. b. E.	40	22,2	—	23,3	—
ESE.	25	—	9,6	23,1	—
SSW.	18	—	16,6	—	6,8
NW. b. N. $\frac{1}{4}$ W.	50	38,6	—	—	31,7
NE. $\frac{1}{4}$ E.	42	26,6	—	32,5	—
W. b. S. $\frac{1}{4}$ W.	45	—	4,4	—	44,8
SW. b. S.	20	—	16,6	—	11,1
E. b. N. $\frac{1}{4}$ E.	62	9,1	—	61,3	—
		146,2	47,2	158,6	100,2
		47,2	—	100,2	—
		99,0	D. lat.	58,4	Dep.



For the course. As 99m. : 58,4 m. :: Rad : t. $30^{\circ} 32'$. Or NNE. $\frac{1}{4}$ E.
 For the dist. As f. $30^{\circ} 32'$: 58,4 m. :: Rad : 115. the distance.

QUEST.

QUEST. VIII. Yesterday noon we were in lat. $33^{\circ} 15' N.$ and bound to a port in lat. $28^{\circ} 35' N.$ lying 196 miles to the west; and this day at noon we were in lat. $30^{\circ} 20' N.$ having made 168 miles of westing: Required the direct course and distance to our intended port?

Lat. from $33^{\circ} 15' N.$	Lat. from $33^{\circ} 15' N.$
Lat. to $28^{\circ} 35' N.$	Lat. in $30^{\circ} 20' N.$

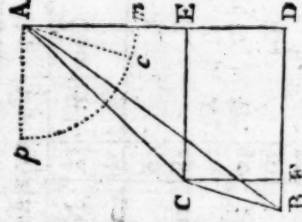
Diff. lat. 4	40 = 280 m.	Diff. lat. 2	55 = 175.
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The figure is constructed by art. 39.

Thus: In the meridian take $AD = 280$, $AE = 175$; in DB , EC , drawn parallel to the parallel of lat. A , take $BD = 196$, $EC = 168$; draw AB , AC , CB ; and draw GF parallel to AD , and AC parallel to CB .

Then A is the place failed from; c the place come to; B the place bound to; CB the distance the ship has to sail; and the arc mc measures the angle of the course she must sail on.

From AD	= 280	From DB	= 196
Take AE	= 175	Take $CE = FD$	= 168
Leaves $EB = CF$	= 105	Leaves BF	= 28



In the triangle CFB .

To find the Course.

As CF , diff. lat. = 105	7,97881	As $\sin. Co.$	= $14^{\circ} 55'$	0,58937
To BF , dep. = 28	1,44715	To dep.	= 28 m.	1,44716
So Rad	= $45^{\circ} 00'$	So Rad	= 90 00	10,00000

To find the distance.

To $t. \angle BCF$, $t. Co.$ = 14 55	9,42596	To Dist.	= 108,8	2,03653
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Answer: Course is $S. 14^{\circ} 55' W.$ or $S. b. W. \frac{1}{2} W.$ Dist. 108,8 miles.

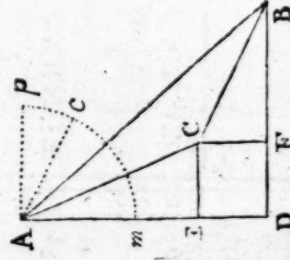
QUEST. IX. A ship from latitude $14^{\circ} 25' N.$ and bound to a port in latitude $5^{\circ} 18' N.$ lying 476 miles to the eastward; finds she has made 400 miles of southing, and 150 of easting: Required the course and distance she has to sail, to reach her intended port?

The figure is constructed by art. 39.

Lat. from $14^{\circ} 25' N.$
Lat. to $5^{\circ} 18' N.$

Diff. lat. 9	7 = 547 miles.
--------------	----------------

AD	= 547	DB	= 476
AE	= 400	$EC = DF$	= 150
$ED = CF$	= 147	FB	= 326



Then $CF : FB :: \text{Rad} : t. \angle BCF$. Or $147 : 326 :: t. 45^{\circ} : t. 65^{\circ} 44'$.
And $f. \angle FCB : FB :: \text{Rad} : CB$. Or $f. 65^{\circ} 44' : 326 :: f. 90^{\circ} : 357,6$.
Answer. Course is $S. 65^{\circ} 44' E.$; or $SE. b. E. \frac{1}{2} E.$ Distance 357,6 m.

QUEST

QUEST. X. Four days ago we were in lat. $4^{\circ} 39' S.$ long. $83^{\circ} 16' E.$ of London; and now we are in lat. $0^{\circ} 35' N.$ having made 200 miles of westing: Required our present course and distance to Achén in the Island of Sumatra?

The fig. is constructed by art. 39.

Lat. from 4° 39' S.
Lat. to 5 15 N.

Diff. lat. $9^{\circ} 54' = 594$ miles.

Lat. from 4° 39' S.
Lat. in 0 35 N.

Long. from $83^{\circ} 16' E.$
Long. to $95^{\circ} 55' E.$

Diff. lat. 5 14=314 miles.

Diff. long. 12 39=759 miles.

AD = 594

DB = 759

$$AE = 314$$
$$FD = 200 = CR.$$
$$ED = 280 = CF.$$
$$FB = 959$$

In the triangle $C'B'$, to find the course and distance.

ASCf : FB :: Rad : t. $\angle$ FCB. Or 280 : 959 :: t. 45° 00' : t. 73° 43'.

As f. $\angle_{FCB} : FB :: Rad : CB$. Or f. $73^{\circ}43' : 959 :: f. 90^{\circ}00' : 999$.

Answer { The course is N. $73^{\circ} 43'$ E. or ENE. $\frac{1}{2}$ E.

Answer { The distance is 999 miles.

QUEST. XI. A ship from the lat. $10^{\circ} 38' N.$ and bound to a port lying 282 miles to SW. b. S. after three successive days of bad weather, finds herself in lat. $3^{\circ} 58' N.$ and 120 miles to the eastward: Required her present course and distance to her intended port?

The figure is constructed by art. 39.

In the triangle ABD, find AD and BD.

$$\text{As Rad} : \text{AB} :: f. \angle B : \text{AD}.$$

Or Rad: 282::f. 56^o 15': 234,5.

And Rad: $AB :: f. \angle BAD : BD.$

Or Rad: 282::f. 33^o45': 156,6.

Lat. from $10^{\circ} 38' N.$

A B

Lat. in	3	58 N.
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9
10	10	10
11	11	11
12	12	12
13	13	13
14	14	14
15	15	15
16	16	16
17	17	17
18	18	18
19	19	19
20	20	20
21	21	21
22	22	22
23	23	23
24	24	24
25	25	25
26	26	26
27	27	27
28	28	28
29	29	29
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31	31	31
32	32	32
33	33	33
34	34	34
35	35	35
36	36	36
37	37	37
38	38	38
39	39	39
40	40	40
41	41	41
42	42	42
43	43	43
44	44	44
45	45	45
46	46	46
47	47	47
48	48	48
49	49	49
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51	51	51
52	52	52
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54	54	54
55	55	55
56	56	56
57	57	57
58	58	58
59	59	59
60	60	60
61	61	61
62	62	62
63	63	63
64	64	64
65	65	65
66	66	66
67	67	67
68	68	68
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72	72	72
73	73	73
74	74	74
75	75	75
76	76	76
77	77	77
78	78	78
79	79	79
80	80	80
81	81	81
82	82	82
83	83	83
84	84	84
85	85	85
86	86	86
87	87	87
88	88	88
89	89	89
90	90	90

AD

Diff. lat.

$$DE = CF$$

३४

$$= 156,6$$

120

$$= 276.6$$

In the triangle CFB find the course and distance.

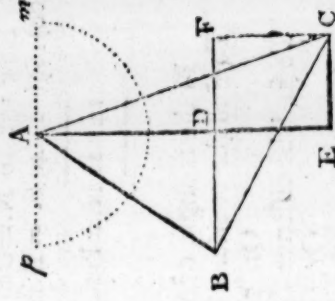
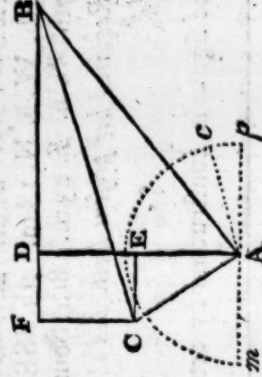
AscF:FB::R:t.∠FCB. Or 165,5:276,6::Rad:t.59°6'.

As f. $\angle_{FCB} : FB :: R : CB$. Orf. $59^{\circ}6' : 276,6 :: Rad : 322,4$.

Answer } Course is N. $59^{\circ} 6' W.$, Or NW. b. W. $\frac{1}{4} W.$

Answer { Distance is 322,4 miles.

QUEST.



QUEST. XII. *A ship in the lat. $17^{\circ} 10' N.$ is bound to a port in the lat. of $13^{\circ} 10' N.$ the departure being 180 miles westward, sails these courses:*

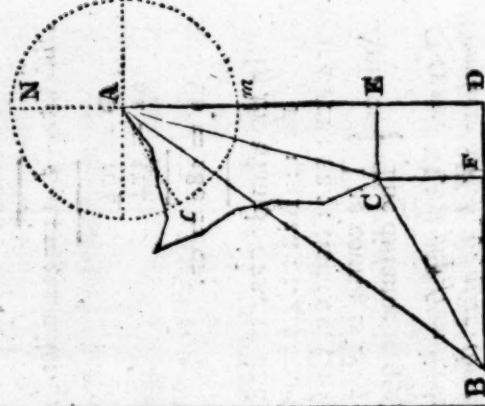
1st SW. b. W. $27 m.$ 2d WSW. $\frac{1}{2} W.$ $30 m.$ 3d W. b. S. $25 m.$
 4th W. b. N. $18 m.$ 5th SSE. $32 m.$ 6th SSE. $\frac{3}{4} E.$ $27 m.$
 7th S. b. E. $25 m.$ 8th South $31 m.$ 9th SSE. $39 m.$

Required her present latit. and the direct course and distance to the desired port?

The figure is constructed by art. 39. Depart. lat. $17^{\circ} 10' N.$
 Lat. bound to $13^{\circ} 10' N.$

TRAVERSE TABLE.				
Courses.	D.	N.	S.	E. W.
SW. b. W.	27	—	—	—
WSW. $\frac{1}{2} W.$	30	—	15,0	22,4
W. b. S.	25	—	8,7	28,7
W. b. N.	18	3,6	4,9	24,5
SSE.	32	—	29,6	12,2
SSE. $\frac{1}{2} E.$	27	—	23,1	13,9
S. b. E.	25	—	24,5	4,9
South	31	—	31,0	—
SSE.	39	—	36,0	14,9
		D. lat. $169,2$	Dep. $47,4$	

Diff. lat. $4^{\circ} 00' = 240 m.$



Now $17^{\circ} 10' N. = \text{Dep. lat.}$ And $AD = 240,0$ Also $BD = 180,0$
 $60) 169,2 \quad 2 \quad 49 S. = \text{Diff. lat.}$ $AE = 169,2$ $CE = 47,4 = FD$
 $14 \quad 21 N. = \text{Pres. lat.}$ $CF = ED = 70,8$ $BF = 132,6$

In the triangle CFB;

To find the course. $CF : FB :: \text{Rad} : t. \angle FCB.$

Or, $70,8 : 132,6 :: \text{Rad} : t. 61^{\circ} 54'.$

To find the distance. $f. \angle FCB : FB :: \text{Rad} : CB.$

Or, $f. 61^{\circ} 54' : 132,6 :: \text{Rad} : 150,3.$

The present latitude is $14^{\circ} 21' N.$

Answer { Course is S. $61^{\circ} 54' W.$ or SW. b. W. $\frac{1}{2} W.$
 Distance is 150,3 miles.

In this question A is the place the ship sails from.

B is the place she is bound to.

C is the place come to by the traverse.

And CB is the rhumb she must sail on to come to B.

QUEST.

QUEST. XV. A ship in the lat. $1^{\circ} 00' S$. is bound to a port in lat. $1^{\circ} 10' N$. distance 220 miles in the NW. quarter; and meeting with contrary winds, runs the following courses, viz.

NE. b. N. 63 m. NW. $\frac{1}{4}$ W. 85 m. North 96 m. NNW. 87 m.

Required the ship's place, and the course and distance to her port?

In the triangle ADB, the course DAB will be found $= 43^{\circ} 47'$; and the departure DB $= 177,5$ miles.

In the traverse, the difference of lat. AE $= 285,9$.

The depart. EC $= 61,3$.

In the triangle CEB.

Where $CF = (AE - AD) = 155,9$.

$BF = (DB - CE) = 116,2$.

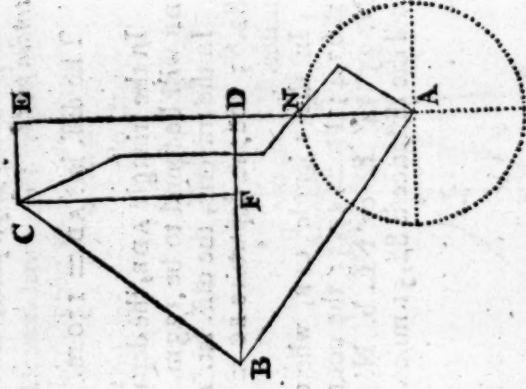
The course FCB $= 36^{\circ} 42'$.

The dist. CB $= 194,4$.

The pref. lat. $3^{\circ} 46' N$.

Answer { The course SW. b. S. $\frac{1}{4}$ W.

Distance 194,4 m.



QUEST. XVI. A ship in the lat. $1^{\circ} 36' N$. is bound to a port lying to the S. eastward, distant 420 miles, and 180 to the eastward; and having plied on the following courses, viz.

SE. b. E. 59 m. SSE. $\frac{1}{2}$ E. 84 m. S. $\frac{1}{4}$ W. 92 m. WSW. 43 m.

W. b. N. 25 m. S. b. E. $\frac{1}{2}$ E. 62 m. ESE, 89 m. E. b. N. 37 m.

ESE. $\frac{1}{2}$ E. 95 m.

Desires to know her place, and the direct course and distance to her port?

In the triangle ADB.

The course is S. $25^{\circ} 23' E$.

The diff. lat. AD $= 379,5$.

In the traverse ABC.

The diff. lat. AE $= 324,2$.

The depart. EC $= 247,4$.

The pref. lat. is $3^{\circ} 48' S$.

In the triangle CFB.

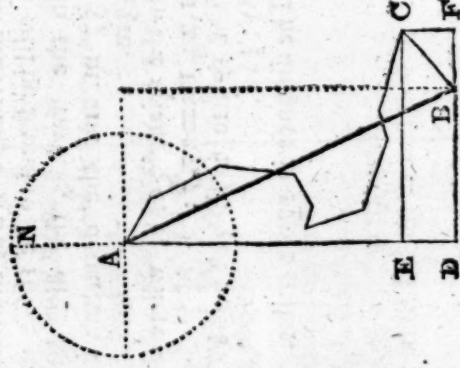
The diff. lat. CF $= 55,3$.

Departure FB $= 67,4$.

Course is S. $50^{\circ} 38' W$.

Or SW. $\frac{1}{4}$ W.

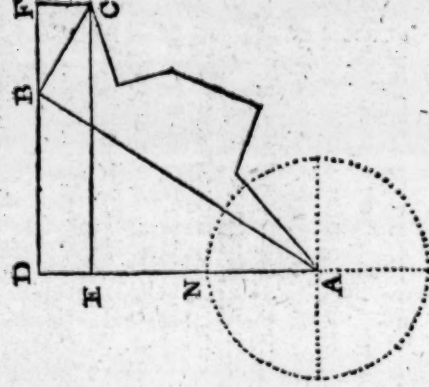
The distance CB $= 87,18$ miles.



QUEST.

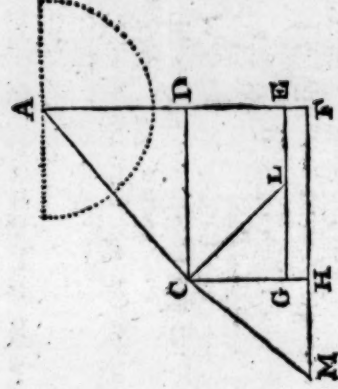
QUEST. XVII. *A ship from the latitude of $4^{\circ} 10' N$. being to make an Island that bears NE b. N. distant 196 miles; having run these courses, viz. NE. $\frac{1}{2}$ E. 75 m. ESE. $\frac{1}{4}$ E. 42 m. NNE. $\frac{1}{4}$ E. 52 m. N. b. W. 34 m. ENE. $\frac{1}{2}$ E. 49 m. Desires to know her present latitude, with her direct course and distance to the Island?*

In the triangle ADB.
The diff. lat. $AD=163$.
The depart. $DB=108,9$.
In the traverse AEC.
The diff. lat. $AE=128$.
The depart. $EC=160$.
And the present lat. $6^{\circ} 18' N$.
In the triangle CFB.
The diff. lat. $CF=35$.
The depart. $FB=51,1$.
The course is $N. 55^{\circ} 36' W$.
Or $NW. b. W$. nearly.
The distance $CB=61,93$ miles.

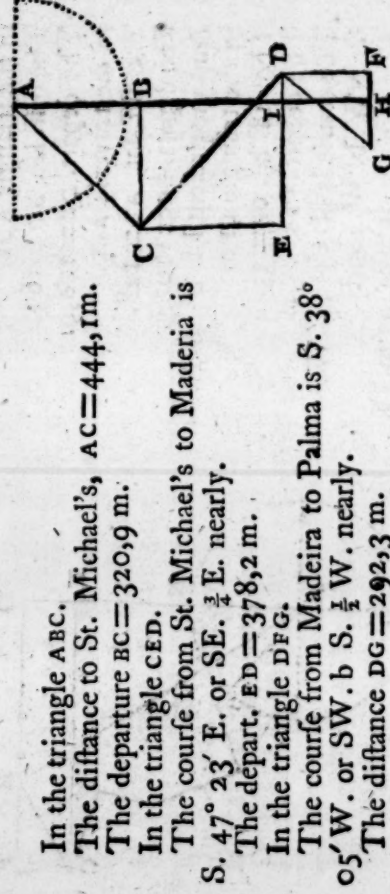


QUEST. XVIII. *Two ships take their departure from the Lizard in latitude $49^{\circ} 57' N$.; one bound to St. Michael's, which lies 715 m. to the S. and 745 m. to the west; the other bound to Lisbon, lying 661 m. to the S. and 215 m. west, reckoning from the Lizard: They sail in company SW. $\frac{1}{2}$ W. 610 miles, and then part: What is the direct course and distance of each ship to her port?*

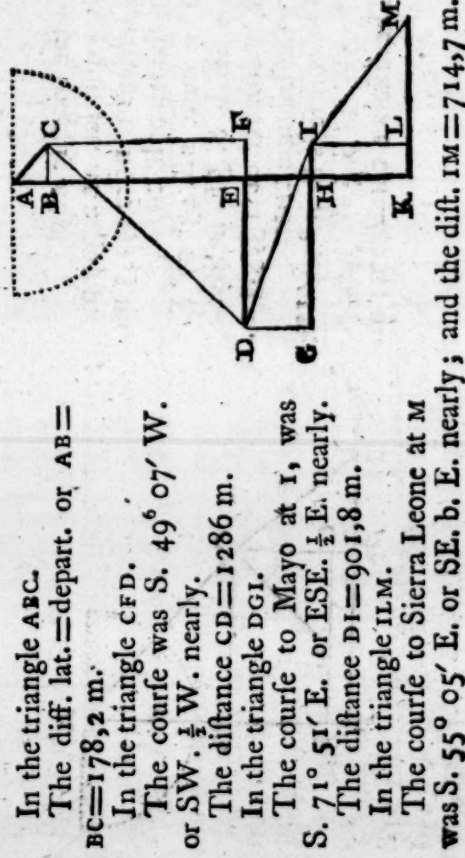
In the triangle ADC.
The diff. lat. $AD=387$ m.
The depart. $DC=471,5$ m.
In the triangle CGL.
The course is $S. 43^{\circ} 07' E$.
Or SE. b. S. $\frac{1}{4}$ E. nearly.
The distance $CL=375,3$ m.
In the triangle CHM.
The course is $S. 39^{\circ} 49' W$.
Or SW. b. S. $\frac{1}{2}$ W. nearly.
The distance $CM=427,1$ m.

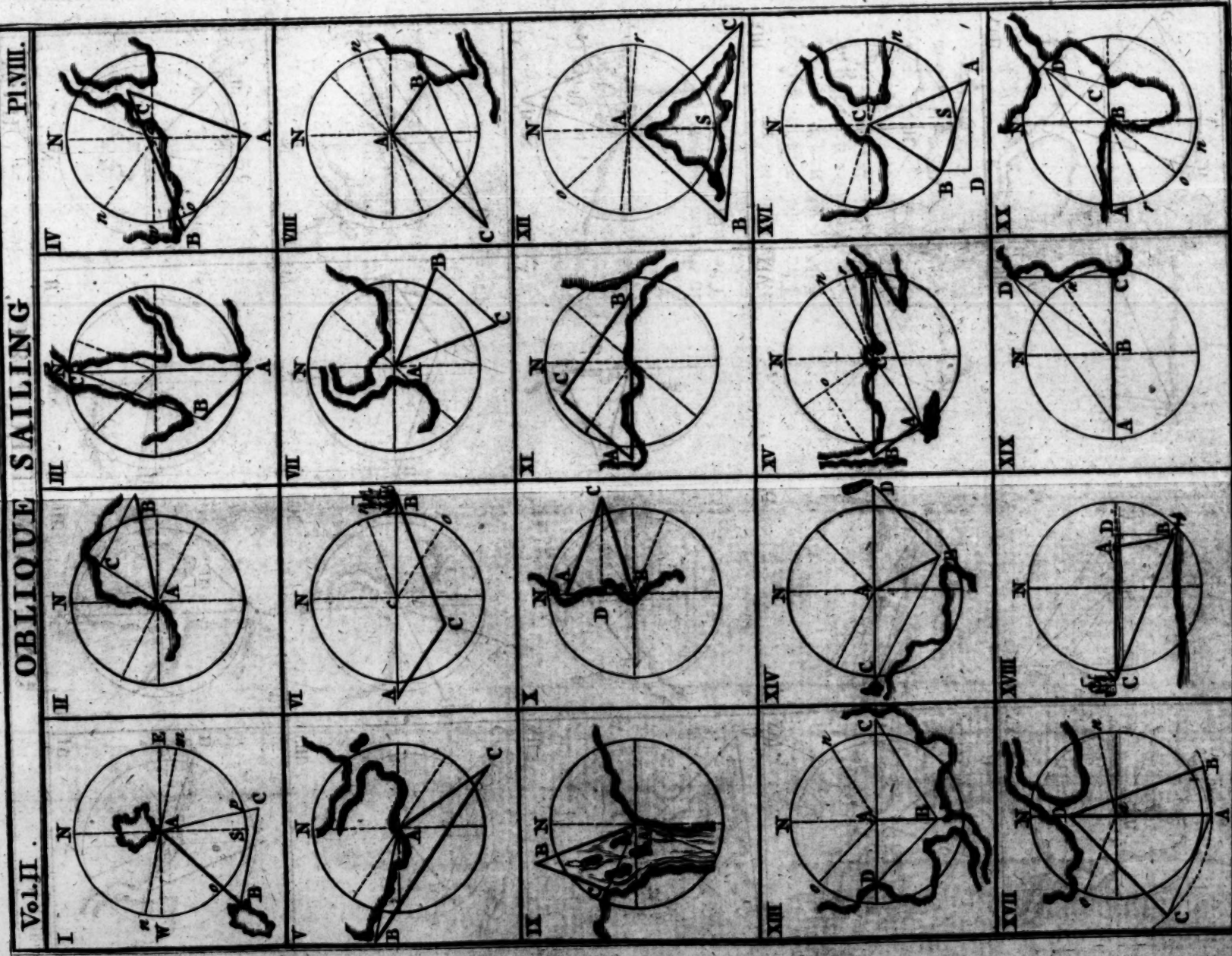


QUEST. XIX. A ship in lat. $43^{\circ} 20' N$. long. $15^{\circ} 10' W$. is bound to St. Michael's, bearing S. $46^{\circ} 16' W$. in the lat. $38^{\circ} 13' N$.; and from thence she is bound to Madeira in lat. $32^{\circ} 25' N$. and distant from St. Michael's 514 m. in the SE. quarter: Required the distance to St. Michael's, the course from thence to Madeira, together with the course and distance from Madeira to the Island of Palma in lat. $28^{\circ} 35' N$. long. $17^{\circ} 13' W$.?



QUEST. XX. Departing from St. Mary's, one of the Azores, and bound to the river Sierra Leone, after running SE. 36 hours at 7 knots an hour, my sea compass was spoiled by a flash of lightning; then after 8 days direct run between the S. and W. I met a ship in lat. $20^{\circ} 00' N$. long. $36^{\circ} 10' W$. and bound to Mayo, one of the Cape Verd Isles, to which I sailed in her company: Required my course and distance in the 8 days run; also the same, while I sailed in company; and the same, from Mayo to Sierra Leone?





SECTION VI.

41. *Of oblique Sailing.*

In the preceding sections, the bearing and distances of places were determined by the solution of right angled triangles; But at sea there being many cases wherein only oblique positions can be observed, therefore it is necessary to shew how such cases are to be solved; which will be amply illustrated in this, and some of the following sections.

To set an object, is to observe what rhumb or point of the nautical compass is directed to it.

The bearing of an object, is the rhumb on which it is seen: And the bearing of one place from another is reckoned by the name of the rhumb passing through those two places.

In every figure relating to any case of plane sailing, the bearing of a line, not running from the center of the circle or horizon, is found by drawing a line parallel thereto, from the center, and towards the same quarter.

The figures belonging to the questions in this section are in Plate VIII. at the end, wherein the figures are referred to by the number of the question.

QUEST. I. Two ships, A and B, sail from two Islands bearing the one from the other NE. and SW. their distance being 76 miles; A sails S. b. E. and B runs E. b. S. at last they meet: How far has each sailed?

CONSTRUCTION. With the chord of 60° describe a circle representing the horizon of one of the places sailed from, suppose of A, the northern; draw the meridian NS, and at right angles to it draw the parallel of latitude EW.

In this circle lay off the SW. rhumb so, the S. b. E. rhumb sp, and the E. b. S. sm: Through o draw AB equal to 76 miles.

Through B draw BC parallel to Am, meeting AC drawn through p.

Then A and B represent the places sailed from; AC and BC the distances and courses (when properly compared with the meridian) the ships had sailed when they met at C.

The distances AC and BC may be measured from the scale AB was taken from.

COMPUTATION. In the triangle ABC there is given one side AB, and all the angles, to find the other sides.

Thus, $\angle A = 5$ pts. $= 56^\circ 15'$. For so and sp make 5 pts. meas. $\angle A$.

$\angle D = 5$ pts. $= 56^\circ 15'$. For BC is paral. Am; and $\angle B = \angle BAN$

(II. 95) $= 5$ pts.

$\angle C = 6$ pts. $= 67^\circ 30'$. For BC is paral. Am; and $\angle C = \angle CAM$

(II. 95) $= 6$ pts.

Therefore the solution belongs to Problem I Book III. 45.

And f. $\angle C : AB :: f. \angle A : BC$. Or, As f. $67^\circ 30' : 76$ m. :: f. $56^\circ 15' : 68,4$ m.

Now as $\angle B = \angle A$: Therefore AC is equal to BC.

(II. 104)

Answer $\left\{ \begin{array}{l} A \text{ sails S. b. E. } 68,4 \text{ miles.} \\ B \text{ sails E. b. S. } 68,4 \text{ miles.} \end{array} \right.$

D 2

QUESTION

QUESTION II. *Coasting along shore, a headland bore NE. b. N. ; then having run E. b. N. 15 miles, the headland bore WNW. : Required the distance from the headland at each time of setting ?*

The construction and computation of this question is made as in Quest. I. ; c being the headland, and AB the ship's run.

Answer, the distance is $\begin{cases} 8,496 \text{ m. at first setting.} \\ 10,811 \text{ m. at second} \end{cases}$

QUESTION III. *In doubling a cape, I observed a castle up a river bore due N. ; then running NW. 6 miles, the castle bore NNE. : Required my distance from the castle at each time of setting ?*

Here c is the castle, and AB the ship's run.

Answer, the castle's distance at $\begin{cases} \text{first is } AC = 14,48 \text{ m.} \\ \text{last is } BC = 11,08 \text{ m.} \end{cases}$

QUESTION IV. *At noon a headland bore NW. b. W. ; then sailing NNE. 5 miles an hour, we anchored at 9 in the evening in a creek 18 leagues distant from the headland : Required its distance at noon, and bearing from the creek ?*

CONSTRUCTION. Having described the horizon, and drawn the meridian and parallel, in the meridian take any point A for the ship's place at first : Draw the NW. b. W. and NNE. rhumbs, and parallel to them draw AB, AC.

Make $AC = 45 \text{ m. } (= 9 \text{ h} \times 5 \text{ m.})$: And from c with 54 m. ($= 18 \text{ l.} \times 3$) cut AB in B. Then B the headland, and c the creek.

COMPUTATION. Here is givn the side $AC = 45 \text{ m.}$; the side $CB = 54 \text{ m.}$ and the $\angle A = 78^\circ 45' = 5 \text{ pts. for NW. b. W.} + 2 \text{ pts. for NNE.}$

Therefore the solution falls under Problem I. Book III. 45.

And $\angle B = 54^\circ 49'$; therefore $\angle C = 46^\circ 26'$; and $AB = 39,9 \text{ miles.}$

Draw ae parallel to CB , then $\angle CBA = \angle ean = 54^\circ 49'$.

Therefore $\angle ean - \angle naw = \angle oaw = 21^\circ 04'$; which taken from $90^\circ 00'$ leaves $68^\circ 56'$.

Answer, the headland's $\begin{cases} \text{distance at noon is } 39,9 \text{ miles.} \\ \text{bearing from the creek is } S. 68^\circ 56' W. \text{ or} \\ WSW. \frac{1}{2} W. \text{ nearly.} \end{cases}$

QUESTION V. *A ship after sailing SE. by S. 62 leagues from her port in lat. $29^\circ 16' N.$ meets a ship that had run 124 leagues from a place bearing W. b. N. of the port the ship departed from : What was the ship's course, and the distance of the two ports ?*

The construction and computation being made as in the last, A and B represent the ports sailed from ; and c the place where the vessels met.

Answer $\begin{cases} \text{The distance of the ports is } 72,18 \text{ leagues.} \\ \text{The sloop's course along CB is } S. 58^\circ 03' E. \text{ or } SE. b. E. \text{ nearly.} \end{cases}$

QUESTION

QUESTION VI. A merchant ship meeting with a ship of war who had sailed SE. b. E. 39 leagues from the latitude of $45^{\circ} 30' N.$ informed them that she had been plundered the day before in the same latitude; and had since run 67 leagues on a direct course between the south and west; and that the pirates were cruising about the place where they robbed her: What course must the man of war steer to come up with them.

Suppose A the place the man of war failed from, B the place where the merchant man was robbed, C the place where they met; then the man of war must sail along CB to meet the pirates; which course will be shewn by cn drawn parallel to CB.

Here the given distance AC is not drawn from the center of the circle representing the horizon, but parallel to co the given course; this was done, in order to prevent the figure from being so small as it must otherwise have been.

Here are two given sides and an angle opposite to one of them, to find the angle B opposite to the other side AC.

Then $CB : f. \angle A :: AC : f. \angle B$. That is, $As 67 l. : 33^{\circ} 45' :: 39 l. : 18^{\circ} 52'$.

ANSWER. The course the man of war must steer is $N. 71^{\circ} 08' E.$

For $\angle nCB = \angle CBc$ (II. 95); and $90^{\circ} - \angle nCB = \angle Ncn$.

QUESTION VII. Two ships sail at the same time from one port; one sails ESE. and the other SSE.: Required their bearing and distance when each has run $37\frac{1}{2}$ miles?

Here $AB = AC$; therefore $\angle B = \angle C$. Now $\angle A = 4$ points; then $\angle C = \angle B = 6$ points.

ANSWER. B bears from C, $N. 45^{\circ} E.$ distant 28,7 miles.

QUESTION VIII. Sailing on a SW. course at the rate of $4\frac{1}{2}$ miles an hour, at 9 o'clock in the forenoon a cape bore SE. b. E. distant 15 miles: Required its bearing and distance at 4 o'clock in the afternoon?

ANSWER. The cape B bears $N. 68,08 E.$ distant 37,44 miles.

Here the ship is at A at 9 o'clock, and at C at 4 o'clock: The two sides AC, AB, and the contained angle $A = 9$ points, are given, to find the rest.

QUESTION IX. Wanting to know the breadth of a river in whose mouth are several islands; from a cliff on the eastern side, I sailed NNW. 90 miles; and then running SSW. 35 miles, I reached the western shore: Required the breadth between those places?

Here, $AB = 50$ m. $BC = 35$ m. and $\angle B = 4$ points, Required AC?

ANSWER. The breadth between A and C is 35,35 mil.s.

QUESTION X. *From two ports lying under the same meridian, and distant 236 miles, two ships sail towards the east, and meet when each has run 372 miles: Required the course of each?*

In Fig. X. A and B are the ports sailed from, and c the place where the ships met. Here are given the three sides; but two of them being equal, the solution is easily found by drawing the perpendicular CD.

ANSWER. A's course is S. $71^{\circ} 30'$ E. and B's course N. $71^{\circ} 30'$ E. (III. 49)

QUESTION XI. *Two ships meeting at sea, find they had sailed from two ports in the same latitude, and distant 576 miles: The ship from the western port had run between the N. and E. 328 miles, and the other had sailed 416 miles: Required the course steered by each ship?*

A and B are the ports sailed from, and c the place where the ships met. From the three given sides, the angles may be found by B. III. 49 or 50.

ANSWER. The course from A is N. $44^{\circ} 49'$ E. and from B is N. $55^{\circ} 59'$ W.

QUESTION XII. *A sloop in three days coasted round a triangular-shaped island: the first day she ranged along one side, and her courses reduced to one was SW. distance 64 miles; the second day's distance was 106 miles between the S. and E.; and that of the third day was 83 miles: Required the direct courses of the second and third days?*

In Fig. XII. the first day's distance is AB, the second BC, the third AC. And the figure is thus constructed.

Having described the horizon, and drawn the meridian and parallel (22), their intersection A is taken for the place the ship sailed from; draw the SW. rhumb AB = 64 miles; from A and B describe arcs with the radii 106 and 83; to their intersection C, draw the lines BC, AC.

Draw the line Ar parallel to BC, and continue CA to o.

Then the arc sr measures or gives the second day's course.

And the arc No the course of the third day.

Now the angles being found by either method, as in Book III. 62; the courses will be thus obtained.

From $\angle BAC$ take 4 points = $\angle SAB$, leaves $\angle SAC$.

Then arc sr, or $\angle sar = \angle SAC + \angle C (= \angle Car$ by II. 95) for the second course.

And arc No, or $\angle Nao = \angle SAC$ (II. 93) is the third day's course.

ANSWER. The second day's course is S. $83^{\circ} 29'$ E. and the third day's N. $46^{\circ} 21'$ W.

The directions given for this 12th question may be applied to the two preceding ones.

QUEST.

QUESTION XIII. *Sailing into a harbour I observed two landmarks, the one bore E. the other W.; and after running S. 3,3 miles, the eastern one bore N. 58° 30' E. and the western one N. 47° 20' W.: Required my distance from those landmarks at each time of setting them?*

The ship is supposed first at A, and sees the landmarks c and D in the lines AC, AD; then sailing to B, she sees the same marks in the lines BC, BD, drawn parallel to the given rhumbs An, Ao.

Here are two right angled triangles BAC, BAD, to work in; there being known in each, the side AB, and all the angles; whence the solution will be as in Book III. 45.

ANSWER. $AD=3,58$ m. $AC=5,385$ m. $BD=4,87$ m. $BC=6,316$ m.

QUESTION XIV. *Coasting along shore I observed two islands, the one bore E. and the other W.; then sailing SSE. $\frac{1}{2}$ E. 16 $\frac{1}{2}$ miles, the western island bore NW. b. W. $\frac{1}{2}$ W. and the eastern bore NE. $\frac{1}{2}$ E.: Required their distance at each time of setting them?*

Here are two oblique angled triangles to work in; and the solution will be as in Book III. 54; there being one side AB and all the angles known.

ANSWER. $AC=22,99$ m. $AD=23,85$ m. $BC=34,03$ m. $BD=21,67$ m.

QUESTION XV. *Three ships, B, C, D, sail from the same road A, and bound to different ports lying in the same parallel of latitude; B's course is NW. b. N. distance 12 miles; C's course NE. b. E.; and D's course is ENE. $\frac{1}{2}$ E.: Required the distance of c and D from their ports, and also the distance of those ports from one another?*

Having described the horizon, and drawn the meridian and parallel, let their intersection c represent C's port; draw the given rhumbs Co, cN, cR; make Co=12 miles; produce nc towards A, and parallel to nA draw oB, meeting the west line in B; draw BA parallel to oC, and draw AD parallel to cR, meeting the east line in D: Then A is the port sailed from, and B, C, D, are the ports bound to.

The solutions of these triangles are as in Book III. 54; there being given one side and all the angles.

Thus $\angle BAC=8$ points, $\angle B=5$ points, $\angle BCA=3$ points.

The $\angle BAD=9\frac{1}{2}$ points, $\angle D=1\frac{1}{2}$ point.

ANSWER. $AC=17,96$ m. $AD=34,36$ m. $BC=21,6$ m. $BD=39,56$ m.

QUESTION XVI. *Two ships, A and B, sail from the port C; A sails SSE. 50 miles, and B SW. b. S. 40 miles: Required their bearing and distance; and how far A is to the southward and eastward of B?*

Describe the horizon, draw the meridian and parallel, and their intersection c represents the port sailed from.

Draw the given rhumbs, in which take $CA=50$, $CB=40$: then A and B represent the places the ships are come to.

Draw AB; through B draw the meridian BD, meeting the parallel AD drawn through A: Then BD is the difference of latitude, DA the departure, and BA shews the bearing and distance sought.

In the triangle CBA, find the unknown angles, and side BA by art. 60. Book III.

Draw CN parallel to BA; then $\angle sca + \angle cas (= \angle acn$ (II. 95) gives $\angle scn$ the bearing from B to A: And $\angle scn$ taken from 90° leaves $\angle BAD$: Hence BD, DA, are found by art. 52. Book III.

ANSWER. $BA=43,33$ miles, $BD=12,93$ miles, $DA=41,36$ miles.

And the bearing from B to A is S. $72^\circ 38'$ E.

QUESTION XVII. *At sun-rise, being four o'clock, a ship saw a sloop to the WNW. and a brig to the E. b. N.; they all came to anchor along-side one another at nine o'clock, each having run since sun-rise five knots an hour, and the ship's course was due north: Required the courses sailed by the sloop and brig, and their distance from the ship in the morning when she set them?*

Draw the WNW. and E. b. N. rhumbs ao , an ; assume any point D in the meridian line for their anchoring place, and in the same line take $DA=25$ miles—the distance each sailed; through A draw AC, AB, parallel to ao , an , and on D with the radius DA describe a circle cutting AC, AB, in C, B: Then A is the ship's place, C the sloop's, and B the brig's, at four in the morning.

Now $\angle c = \angle dac = \angle dao = 6$ points; therefore $\angle adc = 4$ points.

And $\angle b = \angle dab = \angle dan = 7$ points; therefore $\angle adn = 2$ points.

The sides AC, AB, will be found by art. 56. Book III.

Answer $\begin{cases} CA=19,13 \text{ miles, and c's course is NE.} \\ BA=9,755 \text{ miles, and b's course is NNW.} \end{cases}$

QUESTION XVIII. *Having dropped anchor, in very deep water in a river, with a buoy and rope of 50 fathoms fixed to it, I fell down the stream till I had veered out 140 fathoms of cable, and then the buoy was 120 fathoms a-head of me: Required the depth of water?*

Construct the triangle ABC with the three given sides, letting CA=120 be horizontal to represent the surface of the water.

Draw

Draw BD perpendicular to CA continued; the B represents the anchor, A the buoy, C the ship, and BD the depth of the water.

In the triangle ABC find (III. 50) the angle CAB ; then its supplement DAB will also be known; and in the right angled triangle ADB , the side AB and all the angles being known, the side BD may be found, as in art. 52. Book III.

ANSWER. The depth of water DB is 48,71 fathoms.

QUESTION XIX. *Coming within sight of two headlands bearing North and South of each other, the southern one bore from the ship due East, and the other NE. b. E.; and after sailing due East 5 miles, the northern headland bore NE. b. N. $\frac{1}{2}$ E.: Required the distance of those headlands from one another, and their distance from the ship at each time of setting them?*

In the east and west line take $BA = 5$ miles; draw the NE. b. E. and NE. b. N. $\frac{1}{2}$ E. rhumbs BN , BD ; through A draw AD parallel to BN , meeting BD in D ; draw DC perpendicular to AB continued.

Then D and C are the headlands; A , B , are the ship's places at the times of setting them, and AB the distance sailed.

Now $\angle A = (\angle CBN \text{ by II. 95}) = 3$ points; the $\angle DBC = 4\frac{1}{2}$ points, Then $\angle ABD = 11\frac{1}{2}$ pts. (II. 91); $\angle ADB = 1\frac{1}{2}$ pt. and $\angle BDC = 3\frac{1}{2}$ pts.

In the triangle ABD having the side AB , and all the angles given, find the sides AD , BD , by art. 52. Book III.

And in the right angled triangle BCD , knowing the side BD , and all the angles, find the sides BC , CD .

ANSWER. $AD = 13,31$ m. $BD = 9,565$ m. $BC = 6,07$ m. $CD = 7,395$ m.

QUESTION XX. *Having cast anchor, I observed the bearings of a tower, a mill, and a lighthouse, known to lie due east and west of one another; the tower bore WSW. the mill SW. b. S. $\frac{1}{2}$ W. and the lighthouse S. b. W. $\frac{1}{2}$ W.; the distance of the tower from the mill is $5\frac{1}{2}$ miles: How far are each of those objects distant from the place where I anchored?*

Draw the given rhumbs BT , Bo , BN ; make $BA = 5\frac{1}{2}$ miles; continue BT till it meets AD , drawn parallel to BT , in D ; draw DC parallel to BN : Then D represents the ship's place, A the tower, B the mill, and C the lighthouse.

The angles are made out, and the triangles are solved, as in the last question.

ANSWER. $DA = 9,021$ m. $DB = 4,456$ m. $DC = 3,608$ miles.

SECTION VII.

Of Sailing to Windward.

42. When the wind is directly, or partly, against a ship's direct course to the place she is bound to, she reaches her port by a kind of zigzag, or z like course; which is made by sailing with the wind first on one side the ship, and then on the other side.

In a ship, and looking towards the *stem* or *head*,

STARBOARD signifies the right hand side.

LARBOARD the left hand side.

FORWARDS or AFORE, is towards the head or stem of the ship.

AFT or ABAFT, is towards the hinder part or *stern*.

THE BEAM signifies *abwart* or across the middle of the ship.

43. When a ship fails the same way the wind blows, she is said to fail or run before the wind; and the wind is said to be *right aft*, or *right astern*; and her course is then 16 points from the wind.

When a ship fails with the wind blowing directly across her, she is said to have the *wind on the beam*; and her course is eight points from the wind.

When the wind blows obliquely across the ship, the wind is said to be *aboft the beam*, or *afore the beam*, according as her course is more or less than eight points from the wind.

When a ship endeavours to fail towards that point of the compass from whence the wind blows, she is said to *fail on a wind*, or to *ply to windward*.

A vessel failing as near as she can to the point from whence the wind blows, is said to be close hauled: The generality of ships will lie within about six points of the wind; but sloops, and some other vessels, will lie much nearer.

44. The *windward*, or *weather side*, is that side of the ship on which the wind blows; and the other side is called the *leeward*, or *lee side*.

Tacks and *sheets* are large ropes made fast to the lower corners of the fore and main sails, by which either of these corners are hauled fore or aft.

When a ship fails on a wind, the windward tacks are always hauled forwards, and the leeward sheets aft.

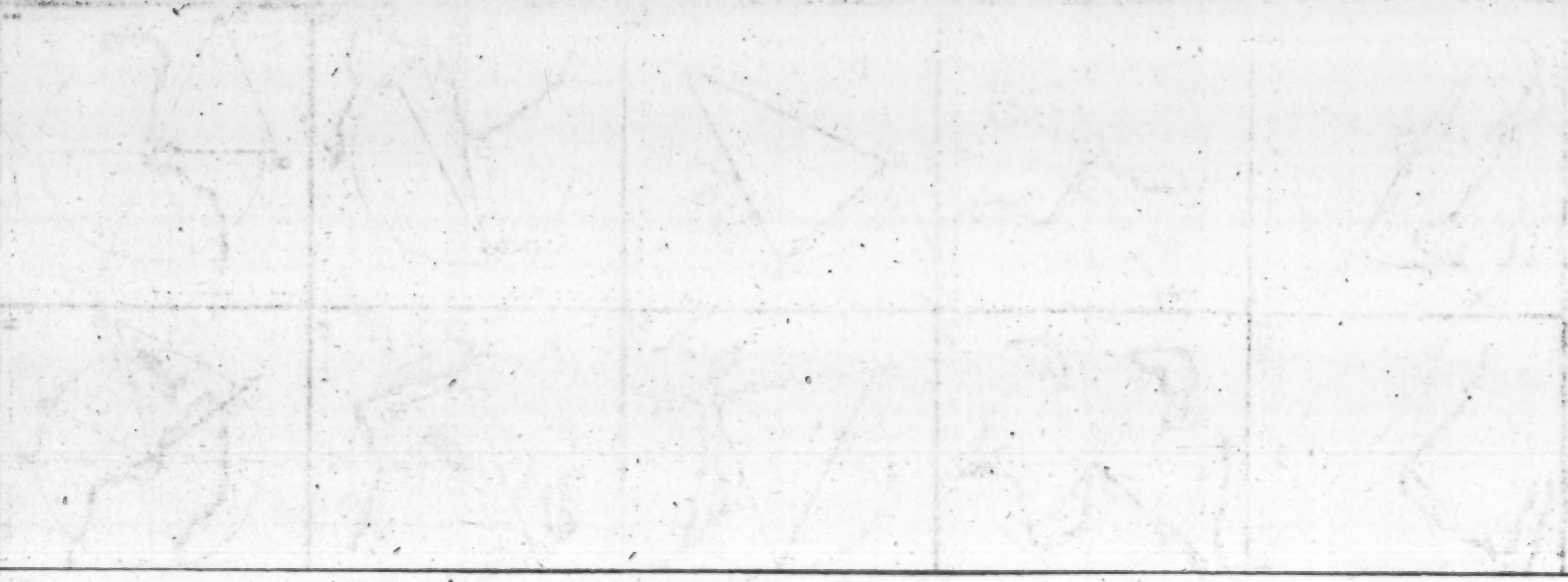
The *starboard tacks* are *aboard* when the starboard side is to windward, and the larboard to leeward.

And the *larboard tacks* are *aboard* when the larboard side is to windward, and the starboard to leeward.

45. To know how near the wind a ship will lie; observe the course she goes on each tack, when she is close hauled; then half the number of

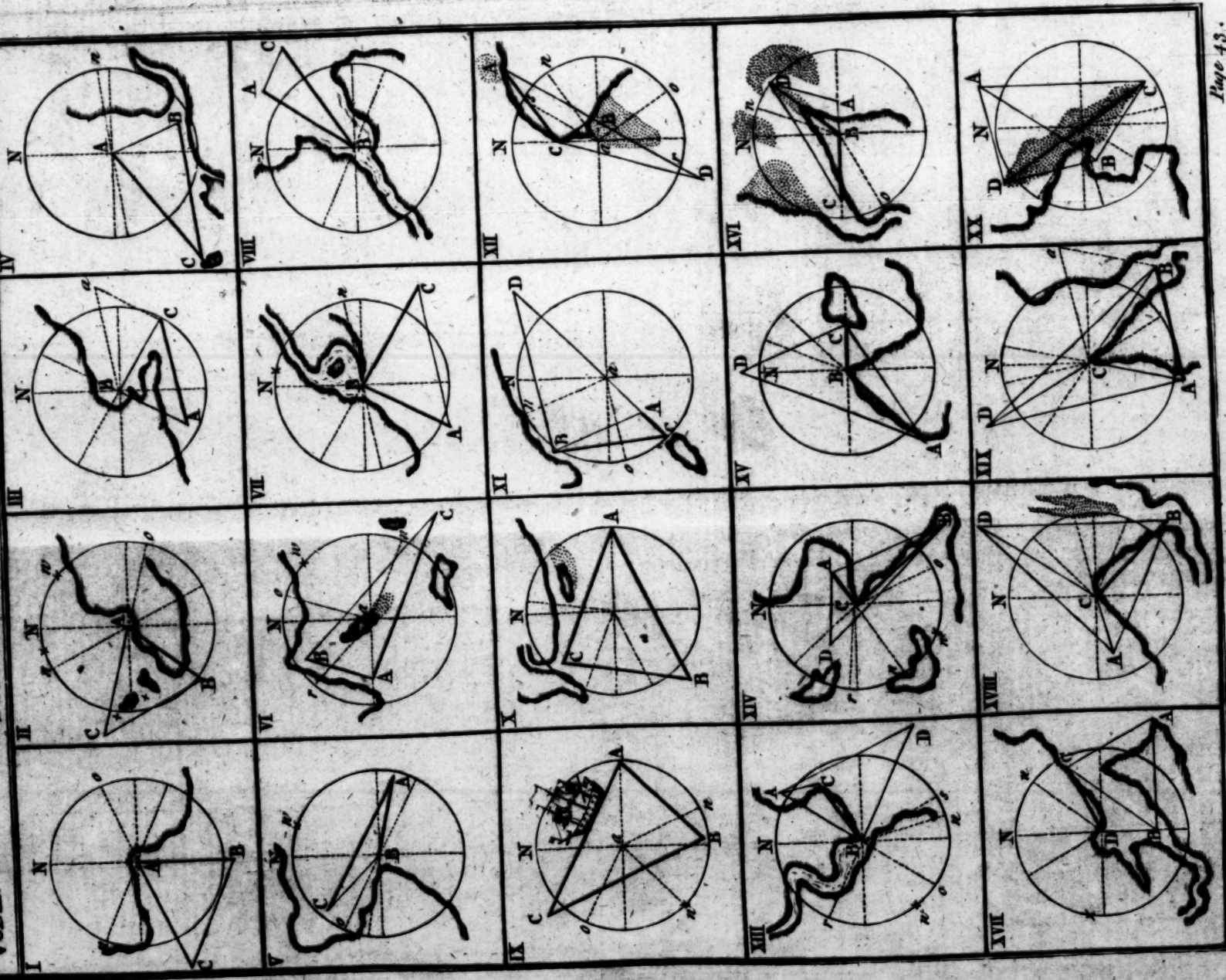
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of points between the two courses, will shew how near the wind that ship will lie.

46. The most common cases in turning to windward may be constructed by the following precepts.

Having drawn the meridian and parallel of latitude (or east and west line) in a circle representing the horizon of the place, mark in the circumference the place of the wind; draw the rhumb passing through the place bound to, and lay thereon the distance of that place from the center.

On each side of the wind, lay off in the circumference, the points or degrees shewing how near the wind the ship can lie; and draw these rhumbs.

Now the first course will be on one of these rhumbs, according to the tack the ship leads with: Draw a line through the place bound to, parallel to the other rhumb and meeting the first, and this will shew the course and distance on the other tack.

The figures referred to in this section are in Plate IX. at the end; wherein they are marked by the number of the question.

QUESTION I. The wind at north, and I am bound to a port 25 miles directly to windward, being on my starboard tacks, and can lie within 6 points of the wind: What must be my course and distance on each of two tacks to reach my port?

The construction being done as above directed, N is the place of the wind, A the port bound to, and B the place where the ship is supposed to be, when the port is 25 miles to windward of her.

Now as the ship leads with her starboard tacks, that is, has the wind blowing on her starboard side, she must first sail along the line BC drawn parallel to the rhumb AN , which is 6 points from the wind; and where the other rhumb AQ , also 6 points from the wind, being produced from A towards C, meets with BC in the point C, is the place where she must tack about to reach her port within 6 points of the wind on her larboard tacks.

Here BC being parallel to AN , the $\angle B = (\angle NAn, \text{ by II. } 95. =) 6 \text{ points.}$

And the $\angle BAC = (\angle NAO, \text{ by II. } 93. =) 6 \text{ points; hence } \angle C = 4 \text{ pts.}$

Therefore, as $\angle B = \angle BAC$; the side $BC = \text{side } CA$, (II. 104)

Now as one side AB, and all the angles are given, the solution falls under Article 45. Book III.

Therefore, As $\sin. \angle C : AB :: \sin. \angle BAC : BC$.

Or, As $\sin. 45^\circ \text{ co}' : 25 \text{ m.} :: \sin. 67^\circ 30' : 32.66$.

ANSWER, the ship sails { on her starboard tacks *WNW*. 32,66 miles.
on her larboard tacks *ENE*. 32,66 miles.

QUESTION II. *A ship that is bound to a port 80 miles directly to windward, then at NE. b. N. $\frac{1}{2}$ E. proposes to reach her port at two boards each within 6 points of the wind: Required her course and distance on each tack, so as to double a rocky shelf lying in the NW. quarter?*

As the wind is in the NE. quarter, and the ship is to double a shelf in the NW. quarter, therefore the must lead with her starboard tacks; and running from B, her present place, to c in a parallel to the rhumb Aⁿ, 6 points from w the place of the wind, she then tacks for her port A, in the direction of the rhumb Aa, 6 points from w.

The manner of solution, is the same as in the last question.

ANSWER, she runs on her { starboard tacks NNW. $\frac{1}{2}$ W. 104,5 miles.
 { larboard tacks ESE. $\frac{1}{2}$ E. 104,5 miles.

QUESTION III. *A ship bound to a port bearing NNE. $\frac{1}{2}$ E. must double a cape to the eastward: Now she having run on her larboard tack 15 leagues within 6 points of the wind then at N. b. E. the port bore NW. b. W. $\frac{1}{4}$ W.: Required how near the wind she must lie on her starboard tack, and the distance of the port at each time of setting it?*

Here A is the place of the ship at first, B the port bound to; AC is the run on the larboard tack, and CB that on the starboard tack.

ANSWER { The distance AB from the port is 10,08 leagues.

{ The run CB is 11,61 leagues, and $6\frac{1}{2}$ points from the wind.

QUESTION IV. *A ship being bound to a port bearing SSE. and directly to windward, having run on her larboard tacks $83\frac{1}{4}$ miles within $6\frac{1}{2}$ points of the wind; on tacking about falls in with a rock at the same distance from her port: What course must she run on her starboard tacks, how near the wind, and what was the port's distance at first?*

Here the side AC=CB; therefore $\angle B = \angle BAC = 6\frac{1}{2}$ points.

Hence $\angle C = 2\frac{1}{2}$ pts. Also $\angle B = \angle BAN = 6\frac{1}{2}$ pts. Therefore $\angle NAB = 7\frac{1}{2}$ points.

ANSWER { Starboard course E. b. N. $\frac{1}{2}$ E. $6\frac{1}{2}$ points from the wind.
 { The port's distance at first was 40,47 miles.

QUESTION V. *A privateer lying by with the wind at NNE. sees a sloop on her starboard tacks that had just doubled a point 18 miles to the W. b. N.; the sloop run for her port within 6 points of the wind: The privateer gave chase at the rate of eight knots an hour, and in four hours came up with her: Required the sloop's rate of sailing, and also the courses of both vessels?*

Draw BA to the E. b. S. and equal to 18 miles; on A with 32m. = 8×4 , cu; the rhumb BC drawn six points from the wind, at w: Then the

the privateer is first at A, sees the sloop at B, gives chase along the line AC, and takes the sloop at C; and BC drawn parallel to AC shews the privateer's course from the north.

Here are known two sides, AB, AC, and the $\angle ABC = 13$ points.

ANSWER. $\left\{ \begin{array}{l} \text{The sloop sails NW. at the rate of 3,86 m. per hour} = \frac{BC}{4} \\ \text{The privateer's course N. } 63^{\circ} 12' \text{ W.} \end{array} \right.$

QUESTION VI. *Being within sight of my port bearing N. b. E. $\frac{1}{2}$ E. distant 18 miles, a fresh gale sprung up at NE.; then running 48 miles on my larboard tacks within six points of the wind, I tacked about: Required the course and distance to my port?*

From any point A draw AB, AC, Parallel to ao , an , the N. b. E. $\frac{1}{2}$ E. and ESE. rhumbs, the latter being six points from the wind at NE.

Make $AB = 18$, $AC = 48$, and join BC. Then the ship at A first sees her port at B, and sailing from A to c on the larboard, must go from c to B on the starboard tacks.

Here are are known two sides, and the included angle $A = 8\frac{1}{2}$ points. Therefore the solution will be as in Article 48. Book III.

ANSWER. The ship has to sail 52,89 miles on a course N. $47^{\circ} 42'$ W.

QUESTION VII. *A ship bound to a port met a sloop that had sailed from thence SSW. $\frac{1}{2}$ W. 325 miles, the wind being at N. b. E.; afterwards, the ship having plied on her larboard tack for three days, she met a vessel who had sailed on a direct course SE. b. E. $\frac{1}{2}$; E. 412 miles since she departed from that port: Required the ship's course and distance between her meeting those vessels, and how near the wind she lay?*

Here B is the port bound to, A and c are the points where the ship met the vessels, and AC the distance run; parallel to which, a line B n being drawn, will shew the course and distance from the wind.

ANSWER. The distance $AC = 524,8$ m. The course is N. $79^{\circ} 51'$ E. $6\frac{1}{4}$ points from the wind.

QUESTION VIII. *Being on my starboard tacks, I met a snow that had sailed NE. b. N. 284 miles from the port I am bound to, and next day I met a brig which had sailed from thence 326 miles, with the wind, then at S. b. E., two points abt the beam: Required my course and distance between the meeting those vessels, and how near the wind?*

Here B is the port, A and c the places of meeting.
There are known two sides and the contained angle.

ANSWER. $\left\{ \begin{array}{l} \text{The ship had sailed S. } 64^{\circ} 6' \text{ E. } 4\frac{1}{4} \text{ pts. from the wind.} \\ \text{The distance run was 125,9 miles.} \end{array} \right.$

QUESTION

QUESTION IX. *The wind at SW. and a ship plying to windward, after running 450 miles on her larboard tacks, and 450 on her starboard tacks, finds she has got 300 miles directly to windward: What were her course, and how near the wind?*

From any point A draw $AB = 300$ miles, and parallel to the wind rhumb aw ; on A, B, with 450 miles, describe arcs cutting in C; then the lines AC, CB being drawn, will shew the distances run on each tack.

The point C is taken towards the *NW.* quarter, as the ship leads with her larboard tacks; but had she led with her starboard, the point C must have been taken towards the *SE.* quarter.

The rhumbs ao, an , being drawn parallel to AC, CB, will shew the courses on each tack.

Here are the three sides given to find the angles: But two of the sides being equal, the solution is readily performed, as in Art. 64. Book III.

ANSWER { Course on larb. is N. $64^{\circ} 28' W.$ } $6\frac{1}{4}$ pts. from the wind.
 { Course on starb. is S. $25^{\circ} 32' E.$ }

QUESTION X. *A pirate being in chase of a sloop 15 miles ahead to the WSW. both on the starboard tacks, the wind at NW.; the sloop finding she lost way, tacked about for an island in the NW. quarter, 11 miles distant from her, and 13 from the pirate: Required how near the wind must the sloop run, and whether the pirate can continue the chase?*

ANSWER { The sloop's course is N. $10^{\circ} 05' E.$ 5 pts. from the wind.
 { The pirate's course is N. $66^{\circ} 54' W.$ 2 pts. from the wind.
 Therefore he cannot continue the chase.

QUESTION XI. *Being on a NE. course with my larboard tacks, an island bore SW. distant 12 miles, and my port bore NNW.; the distance of the island and port is 40 miles: Now the wind being at NNW. $\frac{1}{2}$ W. and I can lie no nearer than $6\frac{1}{2}$ points: Required how far I must run on my present course before I tack about, and also the course and distance I have then to steer?*

In a *SW.* rhumb, from any point A take $AC = 12$ miles; and on C with a distance of 40 miles cut AB drawn parallel to the *NNW.* rhumb an ; now the rhumb ao being taken $6\frac{1}{2}$ points from the wind, through B draw BD parallel to ao , meeting CA continued in D.

Then A is the ship's place, C the island, B the port bound to; AD is the distance to run on her larboard tacks, and BD on the starboard.

In the triangle CBA there are known CA, CB, and $\angle CAB = \angle Can = 10$ pts.; then finding AB (III. 57) in the triangle ABD, one side and all the angles being known, the sides AD, DB are found, as in Art. 54. B. III.

The $\angle D = Cao = 3$ pts. $\angle BAD = \angle Dan = 6$ pts. $\angle ABD = 7$ pts.

ANSWER. { The distance run on the NE. course is 59.75 miles.
 { On the starboard she must run W. b. S. dist. 56.28 miles.

QUESTION

QUESTION XII. *A ship sailing close by a point, sees two lighthouses, one 18 miles to the NE. b. E. the other eight miles to the SE. b. S.; the ship is on her larboard tacks, within $6\frac{1}{2}$ points of the wind, then at SE. b. E.: Required how far she must run to bring the lighthouse to bear in one line; and what will be the bearing and distances of them from the ship at that time?*

From any point *c* draw *CA* parallel to *an* the NE. b. E. rhumb, and equal to 18 miles; also draw *CB* parallel to *ao* the SE. b. S. rhumb, and equal to eight miles; join *AB*, which produce till it meets in *D* the line *CD* drawn parallel to *ar*, a rhumb $6\frac{1}{2}$ points from the wind at SE.

Then *c* is the ship's place at first; *A*, *B*, are the lighthouses; *CD* the distance run to see them in one right line *DBA*; then a rhumb *as*, drawn parallel to *DA*, shews their bearing at that time.

In the triangle *ACB*, knowing two sides and the included $\angle c = \angle na o = 8$ points, the other angles and side *AB* is found, as in Art. 58. B. III.

In the triangle *ACD*, knowing *CA*, the $\angle ACD = \angle nar = 13\frac{1}{4}$ points; also the $\angle A$ found before; the distance *CD*, *AD* are found, as in Art. 55. Book III.

The ship runs on her larboard S. b. W. $\frac{1}{2}$ W. dist. 23,38 m.
 Then the lighthouses bear N. 32° $17'$ E.
 The distance of *A* being 38,66 miles, and of *B*, 18,96 miles.

QUESTION XIII. *A ship wants to reach on two tacks a port bearing SW. b. S. but must double a point bearing S. b. E. the point being 25 miles to the NE. b. E. of the port; the ship can lie no nearer the wind, now at SW. than $6\frac{1}{2}$ points: Required her course and distance on each tack?*

Take *c* the point, 25 miles to the NE. b. E. of *B* the port; through *c* draw *CA* parallel to *Bn* the S. b. E. rhumb, and its meeting with *o B* the SW. b. S. rhumb produced northward, gives *A* the ship's place; then *AD* and *DB* being drawn in the direction of the rhumbs *B s*, *B r*, $6\frac{1}{2}$ points from the wind at *w*, shew the runs on both tacks.

In the triangle *ABC*, there is known *BC*, $\angle BAC = \angle o B n = 4$ points, $\angle ABC = 2$ points, $\angle C = 10$ points; then *AB* is found, as in Art. 55. B. III.

In the triangle *ABD*, *AB* is now known, $\angle BAD = \angle o B s = 5\frac{1}{2}$ points, $\angle ABD = \angle o B r = 7\frac{1}{2}$ points, $\angle D = 3$ points; hence *AD*, *DB* are found, as in Article 55. Book III.

ANSWER. { Course on starboard is SSE. $\frac{1}{2}$ E. distance 58,51 miles.
 { Course on starboard is NW. $\frac{1}{2}$ W. dist. 51,85 miles.

QUESTION XIV. *A privateer lying behind a low point 6 miles to the SW. b. W. sees a ship to the SSE. coming out of a port 18 miles distant from the point; the ship steered NW. with a brisk gale at SSW.; the privateer slip his cable and gave chase within $6\frac{1}{2}$ points of the wind, and takes her: How far had each run when they met?*

Take *A*, 6 miles to the NE. b. E. of *c* the point; from *c* with 18 miles cut *AB* drawn parallel to *co*, the SSE. rhumb, and *B* is the port the ship

ship failed from; then AD drawn $6\frac{1}{2}$ points from the wind meeting BD drawn NW. the intersection D is the place where the privateer meets the ship.

In the triangle ACB, is given AC, CB, and $\angle CAB = \angle NEC = 7$ points; find AB.

Then in $\triangle AED$, where $\angle DAB = \angle rco = 10\frac{1}{2}$ points, $\angle DBA = 2$ points, $\angle ADB = 3\frac{1}{2}$ points; find the sides AD, BD, as in Article 55. Book III.

ANSWER. { The ship had failed NW: distance 25, 27 miles.

{ The privateer failed W. b. N. $\frac{1}{2}$ W. distance 10, 96 miles.

QUESTION XV. *A ship that can lie within six points of the wind, sees a headland 21 miles to the NE. which she is to weather in order to reach an island 9 miles to windward thereof, the wind being at East: What course and how far must she go on each tack, she leading with the starboard?*

Draw BA = 21 m. to the SW. BC = 9 m. to the East, join AC; draw AD, CD each 6 points from the wind; then A is the ship's place, B the headland, C the island, AD the run on the starboard tacks, and DC that on the larboard.

In the $\triangle ABC$, is given BA, BC, $\angle ABC = 12$ points; find the other $\angle s$, and AC.

Then in the $\triangle ACD$, is known AC, and all the angles, to find AD, DC. Thus $\angle D = 4$ pts.; $\angle DCA = \angle DCB + \angle BCA$; $\angle CAD = \angle BAD + \angle BAC$.

ANSWER. { On the starboard she runs 30, 19 miles on the NNE. rhumb.
{ On the larboard she runs 23, 12 miles on the SSE. rhumb.

QUESTION XVI. *There is a certain port, which to sail into you must bring the port and a mill 4 miles to the east thereof, to bear due west; at the same time a beacon 5 miles to the NE. b. N. $\frac{1}{2}$ E. of the mill, bears N. b. E. $\frac{1}{4}$ E.; and then steer directly for the beacon, and thence to the port: Required the best wind in the NW. quarter for a ship that goes equally well on both tacks, together with the distance she must run on each tack from the said situation to the port?*

In the W. rhumb take BC = 4 miles, and in the NE. b. N. $\frac{1}{2}$ E. rhumb take BD = 5 miles, join DC, and parallel to the N. b. E. $\frac{1}{4}$ E. rhumb BN, draw DA meeting CB continued in A.

Then C is the port, B the mill, D the beacon, and A the ship's place. In the $\triangle BAD$, is given BD and all the angles, to find AD.

In the $\triangle CBD$, is given BC, BD, and $\angle CBD$, to find the other angles and DC.

Draw the rhumb Bo parallel to DC; then the $\angle nBo$ being bisected, will give the place of the wind as required.

ANSWER. { She will run on her larb. N. b. E. $\frac{1}{4}$ E. distance 4, 105 miles.
{ And on her starboard S. 61° 40' W. distance 8, 145 miles.
{ The best wind is that blowing from the NW. $\frac{1}{2}$ W. nearly.

QUESTION

QUESTION XVII. *A ship in doubling a point directly to the east of her port, sees a lighthouse bearing NNW. $\frac{1}{2}$ W. which she knows lies NE. b. N. $\frac{1}{2}$ E. from the port, and is 10 miles to the NE. b. E. $\frac{3}{4}$ E. of a beacon bearing due north of her port: The wind is at W. b. N. $\frac{1}{2}$ W. and she can lie within five points: Required her distance on each tack, she leading with the larboard, and keeping close to the lighthouse when she tacks?*

Suppose D the beacon, make DC=10 miles in the NE. b. E. $\frac{1}{2}$ E. rhumb, and C is the lighthouse; draw CB parallel to the NE. b. N. $\frac{1}{2}$ E. rhumb D n, and its intersection B with the S. rhumb DB is the port; through B and C, lines being drawn parallel to the E. and NNW. $\frac{1}{2}$ W. rhumbs, their intersection A is the place of the ship.

In the $\triangle$ CBD is given DC, and all the angles; hence BC is found. (III. 55.)

In the $\triangle$ CBA there is known CB, and all the angles; hence AC is found, as in Article 55, Book III.

Here $\angle$ DBC=3 $\frac{1}{4}$ points, $\angle$ BDC=10 $\frac{1}{4}$ points, $\angle$ DCB=2 $\frac{1}{4}$ points.

And $\angle$ ABC=4 $\frac{1}{4}$ points, $\angle$ BAC=5 $\frac{1}{4}$ points, $\angle$ ACB=6 points.

ANSWER. { The distance on the larboard tacks is 12,49 miles.
The distance on the starboard tacks is 14,25 miles.

QUESTION XVIII. *A ship that can lie on either tack within six points of the wind then at ESE., having her starboard tacks aboard, sees her port directly to windward, and a headland that lies 6 miles to the NW. of her port, bore E. b. N.: Now she wanting to reach her port at two boards: Required her course and distance on each, and also the bearing and distance of the headland when she tacks?*

Let C be the headland, and B, six miles to the SE. be the port; through C and B draw lines parallel to the E. b. N. and ESE. rhumbs, their intersection A is the ship's place; lines drawn through A and B parallel to the rhumbs, six points from the wind, will meet in C, the place where the ship must tack; draw DC, and this shews the position and distance of the headland from the ship when she tacks.

In the $\triangle$ ACB is given BC and all the angles; find AB=8,979.

In the $\triangle$ ABD is known AB and all the angles; find AD, BD.

In the $\triangle$ BCD is known, BC, BD, and $\angle$ CBD; find the other angles and CD.

Here $\angle$ ACB=11 points, $\angle$ CAB=3 points, $\angle$ ABC=2 points.

The $\angle$ BAD=6 points, $\angle$ ABD=6 points, $\angle$ ADB=4 points, and $\angle$ CBD=4 points.

ANSWER. { Ship runs on starboard 11,73 miles NE.

on larboard 11,73 miles S.

Headland bears when she tacks S. 29° 32' W. dist. 8,607 m.

QUESTION XIX. *A ship on her starboard tacks passing by a point, saw her port eight miles to the E. b. N. and at the same time a cliff, which lay to the NW. b. W. of the port, bore N. b. E.; the ship on her starboard could lie within $6\frac{1}{2}$ points of the wind, then at NE. b. E. $\frac{1}{2}$ E., and within seven points on the larboard; she proposes to reach the port on two boards: Required her course and distance on each, together with the bearing and distance of the cliff when she tacks about?*

Here the point c is taken for the cliff, in order to bring the figure within the limits; but if A, representing the ship's place, was taken at the center, the construction would be more simple.

In the E. b. N. rhumb, take $c a = 8$ miles; draw $a b$ parallel to the S. b. W. meeting $c b$ drawn SE. b. E. in B, which is the port bound to; draw BA parallel to $a c$, meeting the S. b. W. rhumb in A, which is the ship's place; through A draw AD parallel to a rhumb $6\frac{1}{2}$ points from the wind, and through B draw BD parallel to a rhumb seven points from the wind, their meeting D is the place where the ship is to tack; then DC being drawn, shews the bearing and distance of the cliff from the ship when she tacks.

In the $\triangle CAB$, is given $BC = AB$ and all the angles; find $CA = 6, 123$.

In the $\triangle ABD$, is given AB and all the angles; find AD, BD.

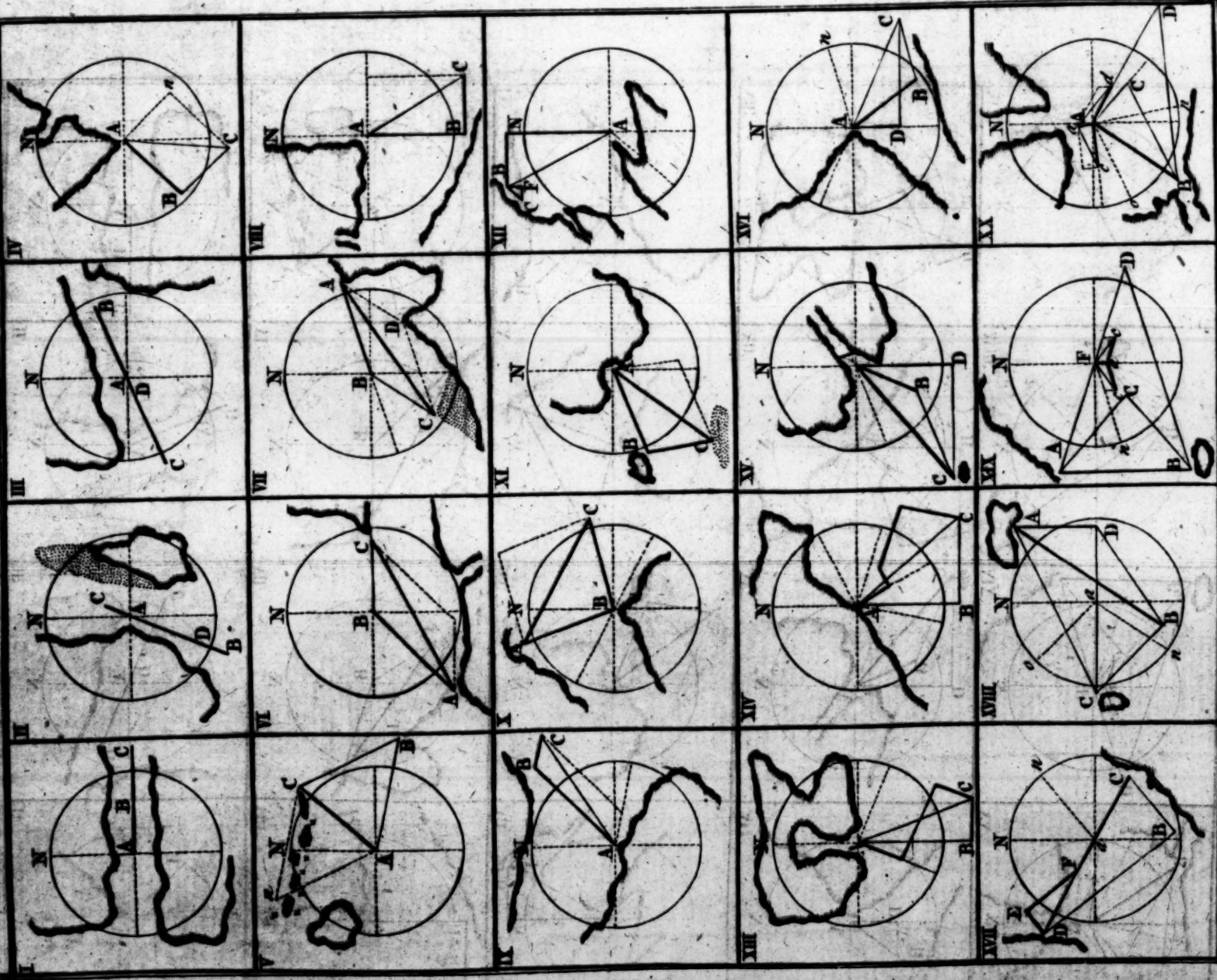
In the $\triangle ACD$, is known AC, AD, and $\angle CAD$; find the other angles and CD.

ANSWER. { Course on starboard is N. b. W. distance 14,96 miles.
 { Course on larboard is SE. b. S. $\frac{1}{2}$ E. distance 16,97 miles.
 { The cliff on tacking bore S. $25^{\circ} 22'$ E. dist. 9,596 miles.

QUESTION XX. *A sloop at sea discovers two beacons at the extremities of a shoal, the one bearing W. b. S. and the other which bore S. was nine miles to the SE. b. S. of the northern beacon; the port bound to lay NW. b. W. of one beacon, and S. b. E. of the other: Now the vessel can lie within $5\frac{1}{2}$ points of the wind then in the SW. quarter, and it being high water, she can tack close to the southern beacon: Required the place of the wind, with the course and distance on each of two boards to gain her port; and also her bearing and distances from the port when she first saw the beacons?*

The construction and solution of this question being not much unlike the two last, they are here omitted to exercise the learner's skill.

ANSWER. { The Wind is SW. b. W. $\frac{1}{2}$ W.
 { Course on starboard is S. distance 8,478 miles.
 { Course on larboard is NW. b. W. distance 4,87 miles.
 { Port bore S. $35^{\circ} 3'$ W. distance 7,05 miles.



SECTION VIII. Of Sailing in Currents.

47. A CURRENT or TIDE is a progressive motion of the water, causing all floating bodies to move that way towards which the stream is directed.

The *setting of a tide*, or *current*, is that point of the compass towards which the waters run ; and the *drift of a current* is the rate it runs per hour.

The setting and drift of the most remarkable tides and currents are pretty well known : But in unknown currents, the usual way to find the setting and drift, is thus :

Let three or four men take a boat a little way from the ship ; and by a rope, fastened to the boat's stem, let down a heavy iron pot, or loaded kettle, into the sea to the depth of 80 or 100 fathoms, when it can be ; whereby the boat will ride almost as steady as at anchor : Then heave the log, and the number of knots run out in half a minute will give the miles which the current runs per hour ; and the bearing of the log shews the setting of the current.

48. A body moving in a current may be considered in three cases : Namely, Moving with, or the same way the current sets.

Moving against, or the contrary way the current sets.

Moving obliquely to the current's motion.

When a ship sails with the current, its velocity will be equal to the sum of its proper motion, and the current's drift.

When a ship sails against a current, its velocity will be equal to the difference of the current's drift and its own motion.

So that if the current drives stronger than the wind, the ship will drive astern, or lose way.

When the course set is oblique to the drift of the current, the ship's real course, or that made good, will be somewhere between that in which the ship endeavours to go, and the track in which the current tries to drive it : That is, it will always be along the diagonal of a parallelogram, wherein one side represents the ship's course set, and the other adjoining side is the current's drift.

For suppose $ABDC$ be a parallelogram, whose diagonal is AD .

Now if the wind alone would drive the ship from A to B , in the same time as the current alone would drive it from A to C .

Then as the wind neither helps nor hinders the ship from coming towards the line CD , the current will bring it there in the same time as if the wind did not act.

And as the current neither helps nor hinders the ship from coming towards the line BD , the wind will bring it there in the same time as if the current did not act.

E 2

Therefore



Therefore the ship must, at the end of that time, be found in both those lines ; that is, in their meeting D.

Consequently the ship must have passed from A to D, in the diagonal AD.

49. When the ship's course and distance by account, and the setting and drift of the current are given ; the true course and distance of the ship may be found, either by working a traverse, in which the setting and drift of the current are used as a course and distance ; or by a trigonometrical solution of the triangles forming the figure : Which is thus constructed.

From the center of a circle draw the ship's proper course and distance, and the current's setting and drift ; with these two lines form a parallelogram ; and from the center draw a diagonal, which will shew the ship's real course and distance.

50. It may frequently happen, that a ship is plying to windward a-crofs a tide's way. In such cases,

First construct the figure for the plying to windward.

Then with the course on each tack, and the setting of the current, form parallelograms, one of whose sides in each must be the ship's rate, and the other the current's drift.

And lines drawn to meet one another, from the center and port bound to, parallel to the diagonals of those parallelograms, will shew the courses and distances the ship must run on each tack.

The figures referred to in this section are at the end, in Plate X, wherein they are marked by the number of the question.

QUESTION I. *A ship sails E. five miles an hour, in a tide setting the same way four miles an hour : Required the ship's course, and the distance made good ?*

The ship's motion is 5 m. E.

The current's motion is 4 m. E.

The ship's true run is 9 m. E.

In this case, as the ship sails from A, the same way the tide sets ; therefore her motion will be quickened by as much as is the tide's motion.

QUESTION II. *A ship with a brisk gale sails SSW. at the rate of nine miles an hour, in a current setting NNE. two miles an hour : Required the ship's course, and the distance made good ?*

The ship's motion is SSW. 9 m.

The current's mot. is NNE. 2 m.

The ship's true run is SSW. 7 m.

Here as the ship sails in a contrary direction to the tide, its motion will be slower by as much as the tide acts against the motion given by the wind.

QUESTION III. *At ten o'clock in the forenoon, a ship that was running four miles an hour, comes in sight of her port bearing ENE.; but finding, at three o'clock in the afternoon, that she had made no way, suspects a current, which on trial was found to set WSW. five miles an hour: What has been the course and real run of the ship during this time?*

In this case, had there been no current, the ship in five hours would have got a-head 20 miles from A to B: And had she been becalmed, she would have drove with the current 25 miles from A to c: Consequently the difference AD of these motions, is the way the ship has run; that is, she has drove aftern five miles.

QUESTION IV. *A ship in doubling a cape meets a strong tide setting SE.; and after running SW. 18 miles by the log, the cape bore N. $\frac{3}{4}$ E.: Required the cape's distance, and the drift of the current?*

From A the cape, and center of the horizon, draw AB in the SW. rhumb, and equal to 18 miles; through B draw BC parallel to the setting of the tide in the SE. rhumb A n; then AC drawn S. $\frac{3}{4}$ W., its intersection c with BC shews the ship's place; and that she has run the line AC while the tide runs the length of BC, or A n.

In the triangle ABC is given AB and all the angles, to find AC, BC.

The $\angle B = 8$ points, $\angle BAC = 3\frac{1}{4}$ points, $\angle ACB = 4\frac{1}{4}$ points.

ANSWER. { The ship has run 22.41 miles S. $\frac{3}{4}$ W.
The tide has run 13.35 miles SE.

QUESTION V. *A ship falling in with a current setting NNW. $\frac{1}{2}$ W. discovers a riff of rocks on the larboard, the eastern point whereof is nine miles to the NE. b. N. $\frac{1}{2}$ E.; which can be weathered by steering E. b. S.: Required the distance the ship must run by the log, and also the drift of the current?*

Here A is the ship's place, c the eastern point of the rocks to be weathered, AB the course she must shape in order to run along the line AC, which is the diagonal of the parallelogram formed by the supposed way AB of the ship, and A n the current's drift.

Answer { The apparent run by the log is 10.76 miles.
And the drift of the current is 10.27 miles.

QUESTION VI. *A ship in crossing the mouth of a river, into which the tide set due east, sails from a buoy on the south side, NE. 10 miles, and then falls in with another buoy on the north side, distant from the first 15 miles: Required the ship's course, with the drift of the current?*

L 3

In

In the *SW.* rhumb take $BA = 10$ miles; on *A* with 15 miles cut the *E.* rhumb *BC* in *c*: Then *A* represents the southern buoy, *c* the northern one; *AB* is the ship's supposed run, *AC* the real run, and *BC* the drift of the tide.

Here is given $AB = 10$, $AC = 15$, the $\angle B = 12$ points.

ANSWER. { The ship's course is $N. 61^{\circ} 52' E.$
The tide's drift is 6,161 miles.

QUESTION VII. At 11 o'clock in the evening *beve* weathered a cape on my starboard, a lighthouse bore *SW. b. S.* and a strong tide was setting *WSW. $\frac{1}{2} W.$* at the rate of $3\frac{1}{2}$ miles an hour; at four in the morning we passed over a sandy point $28\frac{1}{2}$ miles from the cape: Required the distance between the lighthouse and cape, also the course the ship has run?

In the *ENE.* $\frac{1}{2} E.$ rhumb take $BA = 17\frac{1}{2}$ miles ($= 3\frac{1}{2} \times 5$) for the tide's drift. On *A* with $28\frac{1}{2}$ miles cut the *SW. b. S.* rhumb in *c* the sandy point; through *A* and *c*, lines drawn parallel to *BC*, *AB*, their intersection *D* is the lighthouse.

ANSWER. { The distance *AD* or *BC* is 12,72 miles.
The ship's course along *AC* is $S. 56^{\circ} 41' W.$

QUESTION VIII. A ship running south at the rate of five miles an hour, in ten hours crosses a current which all that time was setting east at the rate of three miles an hour: Required the ship's true course and distance sailed?

Here the ship is first supposed to be at *A*, her imaginary course is along the line *AB*; but her real course is along the line *AC*, the diagonal of the parallelogram, whose known sides are $AB = 50$ miles ($= 10 \times 5$), and $BC = 30$ ($= 3 \times 10$).

ANSWER. The course is $S. 30^{\circ} 58' E.$ distance 58,31 miles.

QUESTION IX. Suppose a ship in 24 hours sails *NE.* 100 miles by the log, in a current setting *E. b. S. $1\frac{1}{4}$* miles an hour: What is the course and distance made good?

Draw *AB NE.* and equal to 100 miles, through *B* draw *BC* parallel to the *E. b. S.* rhumb, and make it 42 miles ($= 1\frac{1}{4} \times 24$); then draw *AC*, which shews the true run of the ship from *A*.

Here are given two sides *AB*, *BC*, and $\angle B = 11$ points.

ANSWER. The course is $N. 60^{\circ} 49' E.$ distance 128,2 miles.

QUESTION

QUESTION X. *A ship departs from her port at 8 o'clock in the evening, and sails before the wind then at NNW. at the rate of $3\frac{1}{2}$ knots an hour, in a tide running at the same rate towards the eastward; and next day at noon she found herself 84 miles distant from her port: Required the ship's true course and the setting of the tide?*

In the NNW. rhumb take $BA = 56$ miles ($= 3\frac{1}{2} \times 16$ h.), on A, with 84 miles, cut an arc described from B with 56 miles, their intersection c is the place the ship is come to, A being the port failed from, AB the imaginary course before the wind, BC the run of the current, and AC the real run of the ship.

Here $BA = BC$, and the solution is as in Article 63. Book III.

ANSWER. { The ship's true course is S. $63^{\circ} 55'$ E.
The setting of the current is N. $74^{\circ} 40'$ E.

QUESTION XI. *A ship in the evening departs for an island 46 miles to the WSW. but by a strong tide running to the southward, she found herself next morning on a shoal 96 miles from her departed port, and 34 miles from the island: Required the bearing of her port, and the setting of the tide?*

Here A is the port failed from, B the island bound to, c the shoal come to; the construction as in the last, and the solution as in Art. 62. III.

ANSWER. { The port bears N. $37^{\circ} 37'$ E.
The current set S. $4^{\circ} 46'$ E.

QUESTION XII. *A ship in latitude $41^{\circ} 18'$ is bound to a port in latitude $46^{\circ} 30'$ N. lying 258 miles to the west; where being arrived, finds by her dead reckoning that she has made 384 miles of northing, and 206 miles of westing: Required the setting and drift of the current that occasioned this difference?*

Here by the dead reckoning the ship should have gone from A to B; but she being arrived at the port c, the setting and drift of the current will be represented by the line BC; which is the hypothenuse of the triangle BFC, whose legs are the differences between the real and imaginary differences of latitude and departures.

For the current setting southward will cause the ship to be longer in getting to the norward, and thereby increase her account of northing; and by its setting westward, quickens the ship's motion that way more than her account shews.

ANSWER. The current's course is S. $35^{\circ} 50'$ W. and drift 88.82 m.

QUESTION XIII. *A ship from a place in latitude $37^{\circ} 40' N$. sails SSW. distance 48 miles in 10 hours; then ESE. 72 miles in 14 hours; and has all the time been in a current setting SSW. $1\frac{1}{2}$ mile an hour: Required the ship's present latitude and departure, together with the direct course and distance she has sailed?*

As the current has run $1\frac{1}{2}$ mile an hour for 24 hours, its whole drift is 36 miles.

The best way of solving this question is to consider the current as another course and distance, and then with the three courses and distances form a traverse table, as shewn in Article 36.

TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
SSW.	48		44,3		18,4
ESE,	72		27,6	66,5	
SSW.	36		33,3		13,8
			105,2	66,5	32,2
				32,2	
					34,3

The fouthing and easting the ship has made, being thus known; the direct course and distance will be found, as in Article 33.

Here the ship departs from A, and is come to C; AB being the diff. lat., and BC the departure.

ANSWER. { The present latitude is $35^{\circ} 55' N$. departure 34,3 miles.
Direct course is S. $18^{\circ} 3' E$. distance 110,7 miles.

QUESTION XIV. *Yesterday noon we were in latitude $28^{\circ} 14' N$. and have sailed till this day noon these courses, viz. SSE. $\frac{1}{2} E$. 24 miles; NE. $b. E. \frac{1}{2} E$. 15 miles; ESE. $\frac{1}{4} E$. 45 miles; and all this time we were in a current setting S. b. W. $1\frac{1}{2}$ miles an hour: Required the ship's present latitude, departure, direct course, and distance?*

TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
SSE. $\frac{1}{2} E$.	24		21,2	11,3	
NE. $b. E. \frac{1}{2} E$.	15	7,1		13,2	
ESE. $\frac{1}{4} E$.	45		15,2	42,4	
S. b. W.	42		41,2		8,2
		7,1	77,6	66,9	8,2
			7,1	8,2	
		D.lat.	70,5	58,7	Dep.

Here the current being considered as a course, the traverse table gives the diff. lat. = 70,5 miles.

And the departure = 58,7 miles.

And hence the direct course is S. $39^{\circ} 47' E$. and the distance is 91,73 miles.

In any question, where there are given one or more courses and distances sailed, and also the setting and drift of the current, the work may be performed as above.

The reason of this is pretty evident; for as each course and distance is affected by the current, therefore instead of correcting them one by one, as shewn in most of the preceding questions, they may be all allowed for together, by making a course and distance of the setting and drift of the current.

QUESTION

QUESTION XV. *At eight o'clock in the evening, a ship sailed from her port on a SSW. course, four miles an hour; a strong tide was then setting between the S. and W.; and at six o'clock in the morning she passed by a rock 70 miles to the westward, and 60 miles to the southward of her port: Required the setting and drift per hour of the tide?*

With 60 miles as a diff. lat. and 70 miles as a departure, find the place of the rock at c; A being the place sailed from; in the SSW. rhumb take $AB = 40$ ($= 4 \times 10$); draw BC, AC: Then AB is the supposed course, AC the real course, and BC the setting and drift of the tide.

In the $\triangle ADC$, is given AD, DC, $\angle D$; to find the $\angle s$ and $AC = 92, 2$.

In the $\triangle ABC$, is known AC, AB, $\angle CAB$; to find $\angle s$ and $BC = 59, 37$.

ANSWER. The tide sets S. $67, 09$ W. and runs 5.937 miles an hour.

QUESTION XVI. *At six o'clock in the forenoon a ship weathered a cape in latitude $46^{\circ} 32' N.$; and having run SE. b. S. 24 miles by the log in a current, found herself at noon in latitude $46^{\circ} 17' N.$ and the cape bore WNW.: Required the setting and drift of the current?*

Make $AD =$ the diff. lat.; draw the departure DC, meeting the ESE. rhumb in c; draw AB, SE. b. S. and equal to 24 miles, and join BC.

Then A is the cape, c the place come to, AB the supposed run, BC the drift of the current, and AC the real run.

In the $\triangle ADC$, is given AD and the angles; to find $AC = 39, 2$.

In the $\triangle ABC$, is known AB, AC, $\angle BAC$; to find the angles and $BC = 23, 41$ m.

Then drawing the rhumb A n parallel to BC, it shews the setting of the current.

ANSWER. The current sets N. $77^{\circ} 47'$ E. and runs 3.9 miles an hour.

QUESTION XVII. *A ship bound from Dover to Calais, lying 21 miles to the SE. b. S. $\frac{1}{2} E.$; and the tide of flood setting NE. $\frac{1}{2} E.$ 2 $\frac{1}{2}$ miles an hour: Required the course she must steer, and the distance run by the log at the rate of six miles an hour, to reach her port?*

In the position of the SE. b. E. $\frac{1}{2} E.$ rhumb draw $DC = 21$ miles; and parallel to the NE. $\frac{1}{2} E.$ rhumb draw $DE = 2 \frac{1}{2}$ miles; on E with six miles cut DC in F; draw DB parallel to EF, meeting CB drawn parallel to DE.

Then D is Dover, c Calais; and the ship by trying to sail along the line DB, will be driven between it and the current in the line DC.

ANSWER. The ship must sail S. $39, 14$ E. and will run by the log 19.4 m.

QUESTION

QUESTION XVIII. Yesterday noon we were in latitude $42^{\circ} 20' N.$ and this day at noon find ourselves in latitude $41^{\circ} 10' N.$; by our account we have sailed *SW.* $b. S. 189$ miles, and all the time have been in a current setting *NW.*: Required the ship's direct course and distance, with the current's drift per hour?

Take $AD=70$ miles, the diff. lat.; draw AB parallel to an , the *SW.* $b. S.$ rhumb, and equal to 189 miles; through B draw BC parallel to the *NW.* rhumb ao , meeting the departure DC , and join AC .

Then the ship from A supposes herself sailing along AB ; but by the action of the current is driven along AC .

In the $\triangle ADB$, is given AD , AB , $\angle BAD$; required the angles and $DB=136,4$.

In the $\triangle DBC$, is known DB and the angles; to find $DC=192$ and $BC=123,1$.

In the $\triangle ADC$, is known AD , DC , $\angle ADC$; to find the angles and $AC=204,4$.

ANSWER. { The course is $S. 69^{\circ} 58' W.$ distance 204,4 miles.
The current's drift per hour is 5,13 miles.

QUESTION XIX. There are two ports which lie 270 miles to the north and south of one another: a ship sails from the northern port with her starboard tacks aboard within 72 degrees of the wind then at $S.$ at the rate of four miles an hour, in a current setting south two miles an hour: Required her course and distance on each tack to reach the southern port?

From a point A in a line AD drawn $S. 72^{\circ} E.$ draw $AB=270$ miles to the south; through B draw BD parallel to Fn , a rhumb 72° from the wind; then was there no current, the ship must sail from A to D on the starboard, and from D to B on the larboard tacks.

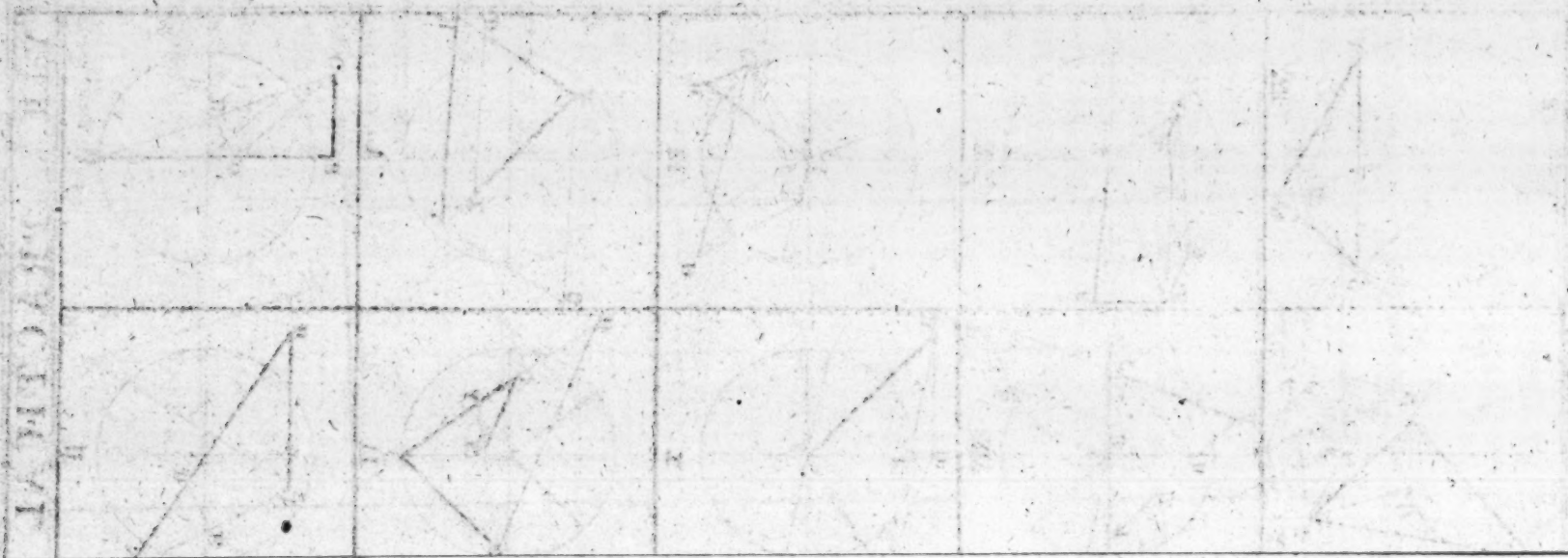
Take $Fa=2$ miles, draw ac, ac , parallel to FD, Fn , and make ac, ac , each of four miles; draw the diagonals Fc, Fc , of those parallelograms; then through A and B , lines drawn parallel to Fc, Fc , meeting in C , will shew the run of the ship on each tack.

Here the triangles Fac, Fac , being congruous by construction, therefore the $\angle CAB (= \angle aFc)$ is equal to $\angle CBA (= \angle aFc)$.

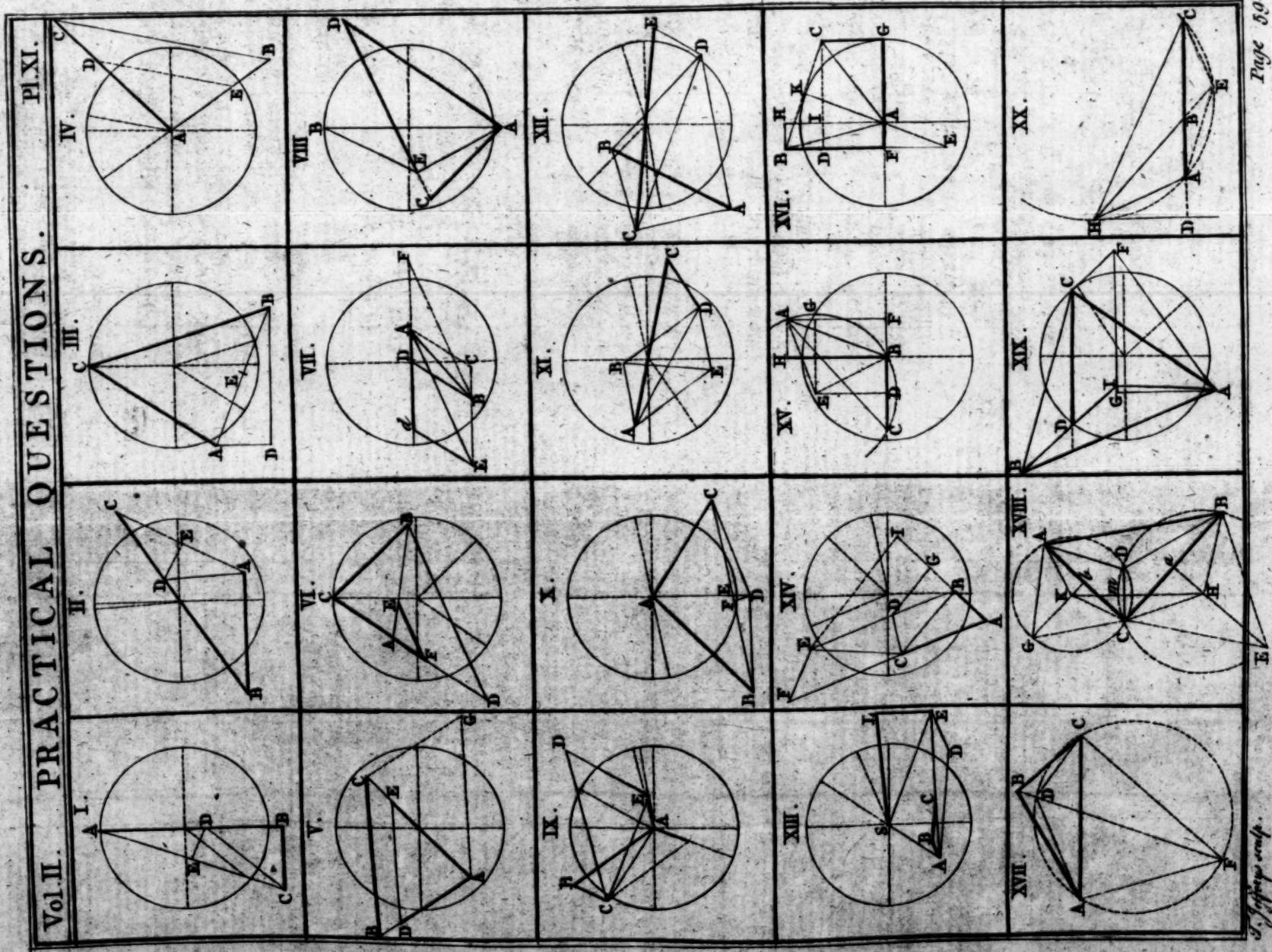
Now in the triangle aFc , is given aF, ac , $\angle Fac=108^{\circ}$; from whence the other angles are known; viz. $\angle aFc=49^{\circ} 37'$.

Then in the triangle ABC , is known the side AB , and all the angles, whence the sides AC and CB are easily found.

ANSWER. { The course on starb. is $S. 49^{\circ} 37' E.$ distance 208,4 miles.
The course on larb. is $S. 49^{\circ} 37' W.$ distance 208,4 miles.



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QUESTION XX. *A ship that makes her way good within $6\frac{1}{2}$ points of the wind, is bound to a port bearing SW. b. S. $\frac{1}{2}$ W. distant 476 miles: Now the wind bearing S. b. W. a current setting S. b. E. $2\frac{1}{2}$ miles an hour, and the ship sailing $7\frac{1}{2}$ miles an hour: Required the course and distance she must sail on her starboard and larboard tacks to gain the port?*

In the SW. b. S. $\frac{1}{2}$ W. rhumb, take $AB=476$ miles; and through A and B, draw the lines AD, BD, in the direction of rhumbs $6\frac{1}{2}$ points from the wind at S. b. W.; then, was there no current, the ship would sail on the lines AD, DB.

With the run of the current $aA=2\frac{1}{2}$, and the run of the ship $7\frac{1}{2}$ miles an hour, taken in rhumbs AC, Ad, $6\frac{1}{2}$ points from the wind, complete the parallelograms, whose diagonals ac, ad, shew the courses the ship will move in, with the compound motions of the current's and ship's runs.

Then AC, BC, drawn parallel to ad, ac, will shew the run of the ship on the starboard and larboard tacks.

In the $\triangle Aad$ are given Aa , Ad, $\angle Aad$; to find the $\angle daA=37^{\circ} 41'$.

In the $\triangle Aac$ are given Aa , Ac, $\angle aAc$; to find the $\angle caA=74^{\circ} 54'$.

Then $\angle can=\angle daA$; $\angle oan=\angle caA$; and $\angle bac=\angle ban+\angle can=85^{\circ} 30'$.

Also $\angle nao-\angle nab=BAO=\angle ABC=27^{\circ} 05'$. (II. 93)

Then in the triangle ACB, there is known one side AC, and all the angles; by which the sides AC, CB, may be found.

ANSWER. { The course on the starb. is S. $48^{\circ} 56'$ E. dist. 234,7 miles.
 { The course on the larb. is S. $63^{\circ} 39'$ W. dist. 514 miles.

Most of the Questions in the three preceding sections, although not of such common use at sea, as those in sections the IVth and Vth, yet have they their uses on various occasions, as will evidently appear from many of the questions themselves: But there is another excellent use to be derived from their solution, that does not appear at first sight so evident to learners, which is, the furnishing them with an agreeable opportunity of exercising their talents in the application of Geometry and Plain Trigonometry: For it should be well observed, that as all the operations relating to sailing depend chiefly on the principles of Trigonometry; so the persons, to whom these principles are most familiar, will with the greatest readiness not only apply them to the cases of sailing, but also see more clearly the reason of every step they shall find themselves necessitated to take: And as a farther exercise for the understandings of those who are desirous of extending their notions this way, it has been thought proper to annex the Questions in the following section with their solutions.

SECTION IX.

51. *A Collection of Practical Questions.*

The figures belonging to the questions in this section are in Pl. XI. and numbered the same as the questions.

QUESTION I. *A ship in latitude $47^{\circ} 30'$ N. is bound to a port in latitude $42^{\circ} 00'$ N.; and 120 miles to the west, proposes to reach her port by running equal distances on each tack, the first to be on the meridian: What must the course be on the second tack, and how far on each?*

SOLUTION. With the given difference of latitude $= 330$, and the departure $= 120$, construct the triangle ABC; from E, the middle of AC, draw ED at right angles to AC: Then drawing DC, the lines AD, DC, will be the distances the ship must run on each tack.

For ADC is by construction an isosceles triangle. (II. 93)

In the triangle ABC, find the angles (III. 46), $\angle A = 19^{\circ} 59' = \angle ACD$; the $\angle ACB - \angle ACD = DCB = 50^{\circ} 02'$; and $90^{\circ} - \angle DCB = \angle BDC = 39^{\circ} 58'$.

In the triangle CBD, find (III. 45) $DC = 186.8$.

ANSWER. The first course is S., the 2d S. $39^{\circ} 58'$ W.; and the dist. on each tack 186.8 miles.

QUESTION II. *A ship having sailed NE. b. E. from a port in latitude $42^{\circ} 18'$ N., met a sloop who had sailed from a port in the same latitude, lying 92 miles to the west of the ship's port: The sum of their distances made 159 miles: Required their respective courses and distances?*

SOLUTION. In and east and west line take $BA = 92$ m., from B draw on the NE. b. E. rhumb $BC = 150$ m. join CA; then ED, drawn at right angles to CA from E the middle, gives D, so that $DA = DC$. (II. 93)

In the triangle ABC, find (III. 48) the $\angle BAC = 114^{\circ} 27'$, and $\angle ACD = \angle CAD = 31^{\circ} 47'$.

Then $\angle BAC - \angle CAD = \angle BAD = 82^{\circ} 40'$; hence $\angle BDA = 63^{\circ} 25'$.

In the triangle BAD, find (III. 45) $BD = 101.9$ m., and $AD = 57.7$ m.

ANSWER. The ship sailed NE. b. N. 101.9 miles, and the sloop sailed N. $7^{\circ} 20'$ W. 57.7 miles.

QUESTION III. *Two ships, A, B, sail from one road, their courses making an angle of $50^{\circ} 00'$; A sails between the S. and W. 40 leagues, B sailing between the S. and E. 50 leagues, has got 15 leagues to the southward of A: Required the course of each ship, also the bearing, distance, and departure from A to B?*

SOLUTION. With the given sides and included $\angle A$, construct the triangle ACB: On A, with 15 leagues, describe an arc at D, and from E, the middle of AB, with the radius EA, cut the former arc in D (II. 131); draw the given diff. lat. AD, and the required departure DB: Then C is the port sailed from, A one ship's place, B the other's.

In the triangle ACB, find (III. 48) the $\angle CAB = 78^{\circ} 24'$, and $\angle CBA = 51^{\circ} 56'$; the side AB will be found 39.1 miles. In the triangle ADB, where AB, AD, are known, find $\angle DBA = 22^{\circ} 34'$, $\angle DAB = 67^{\circ} 26'$, and $BD = 36.1$.

ANSWER. A's course is S $34^{\circ} 10'$ W. B's is S. $15^{\circ} 10'$ E.; B bears from A S. $67^{\circ} 26'$ E. departure BD is 36.1 m. the dist. AB is 39.1 miles.

QUESTION

QUESTION IV. *From two ports bearing NW. b. N. and SE. b. S. two ships sail, the one NE. the other N. b. E.; they meet, and find that the sum of their distances added to the distance of the ports made 148 miles: Required the distance of the ports, and the run of each ship?*

SOLUTION. Make a triangle ADE equiangular to the triangle required, wherein one side, as AD, shall be of any assigned length, as 30; and by Art. 45. Book III. find $DE=41,61$; $AE=23,57$; and the sum of the three sides of the triangle ADE will be 95,18: Then say,

$$\text{As } 95,18 : 148 :: AD=30 : AC=46,65.$$

$$\text{And as } 95,18 : 148 :: DE=41,61 : CB=64,70.$$

Then the other side AB will be 36,25 miles.

ANSWER. { The distance of the ports is 36,25 miles.

{ One ship's run is 46,65 miles, the other's 64,7 miles.

QUESTION V. *Two ships meet at sea bound to two ports distant from one another 12 miles: Now if one sails NW. b. N. six miles an hour; and the other NE. eight miles an hour, they will arrive at their respective ports at the same time: Required the distance each ship must run, and the bearing of those ports from one another?*

SOLUTION. With the courses and rates of sailing make the triangle ADE, A representing the place of meeting; parallel to DE draw $AG=12$ miles; through G draw GC parallel to AD, meeting AE produced in C; through C draw CB parallel to AG, meeting AD produced in B; then B and C are the ports. For $AB:AC::6:8$; and $BC=12=AG$, BG being a parallelogram.

In the triangle ADE, knowing AD, AE, and $\angle A$, find $\angle D=60^\circ 30'$, $\angle E=40^\circ 45'$, and $DE=9,015$ miles; then $DE:BC::AE:AC=10,65::AD:AB=7,988$.

ANSWER { One runs NE. 10,65 miles; the other NW. b. N. 7,988 m.
{ C bears from B, N. $85^\circ 45'$ E.

QUESTION VI. *A fleet of ships steering SW. b. S. four miles an hour, on seeing a sail, detached a cruiser, who gave chase SE. five hours at the rate of seven miles an hour, comes up with and takes the chase (being a privateer); and after an hour's time spent in adjusting matters on board the prize, she steers for the fleet, who still kept on the same course and rate: Required the course the cruiser must shape, and the distance she must run at seven miles an hour, to join them?*

SOLUTION. Let c be the first place of the fleet, draw CD, SW. b. S. and CB, SE.; make $CB=35$ ($=7 \times 5$), $CA=24$ ($=4 \times 6$), and draw AB; then B is the place where the cruiser took the chase, and A the place where the fleet was when she left the chase: Make $AF=4$, $FE=7$, draw BD parallel to FE, and D is the place where the cruiser joined the fleet.

In the triangle ACB, find $\angle CAB=63^\circ 25'$, and $\angle CBA=37^\circ 49'$, and $AB=38,39$. Then (II. 91) $\angle FAE=116^\circ 35'$. In the triangle FAE, find $\angle E=30^\circ 44'$ and $\angle F=32^\circ 41'$.

In the triangle ADB, similar to AFE, find $BD=63,58$ miles.

ANSWER. The cruiser must steer S. $66^\circ 26'$ W. and run 63,58 miles.

QUESTION

QUESTION VII. *A ship on a WSW. course can sail from the port A to the port B in six days : But sailing at the same rate by the log on a NE. b. N. course, she goes from B to A in three days, a current all the way between the ports setting east, half a mile an hour : Required her dead reckoning outwards and homewards, also the bearing and distance of A from B ?*

SOLUTION. In the nine days outward and homeward bound, the current runs at $\frac{1}{2}$ mile an hour, 108 miles east, that is 72 m. for outwards, and 36 homewards. In an east and west line take $AD=36$, $Dd=72$; through A draw a WSW. rhumb, and through D draw a NE. b. N. and SW. b. S. rhumb, continued to cut the former in E : Through D, E draw parallels to dE , dA , cutting in B; join AB, and draw BF parallel to EA, meeting dA produced in F; then A and B represent the two ports, AE the outward dead reckoning, BD the homeward, and AB the true distance.

For if $AF=BE=Dd=72$ is the drift of the current in the outward account, and the course shaped is along AE, the ship must run between AF and AE : Also if $BC=AD=36$ is the current's drift homeward, and the course shaped is along BB, the ship must run between BC and BD : But AB is the only common diagonal to the parallelograms FE and DC; consequently A and B are the ports.

In the triangle A d E, find (III. 45) $AE=161.6$; and $Ed=BD=74.39$ m.

In the triangle ADB, find (III. 48) $\angle DAB=39^\circ 39'$, and $\angle DBA=17^\circ 36'$. Also (III. 45) find $AB=99.04$.

ANSWER. { The dead reckoning outw. = 161.6 m. homew. = 74.39 m.
The true course is S. $21^\circ 21'$ W. distance 99.04 miles.

QUESTION VIII. *A ship, after doubling a cape and sailing NE. b. N. 45 miles, receives in the night considerable damage from a storm; she then bore directly towards a lighthouse lying 24 miles to the NW. of the cape, and having run 40 miles, and the day breaking, she discovers a port 42 miles to the north of the cape : What was her course and distance to that port ?*

SOLUTION. Draw AD, NE. b. N. 45 m.; AB, N 42 m.; AC, NW. 24 miles; join DC, make $DE=40$ m. and draw AE, EB; then A is the cape, C the lighthouse, and B the port.

In the triangle CAD, with the given sides and included angle A, find the $\angle C=70^\circ 58\frac{1}{2}'$, $\angle D=30^\circ 16\frac{1}{4}'$, and the side $CD=46.69$ miles.

In the triangle ADE, with the given sides and included angle D, find the $\angle DEA=87^\circ 08'$, $\angle DAE=62^\circ 36'$, and $AE=22.71$ miles.

In the triangle BAE, with the known sides AB, AE, and included $\angle EAB$, find $\angle BEA=124^\circ 47'$, $\angle B=26^\circ 22'$, and $EB=24.67$ miles.

ANSWER. { The course from E to B is N. $26^\circ 22'$ E.
And the distance EB is 24.67 miles.

QUESTION IX. *A ship running at the rate of $4\frac{1}{2}$ miles an hour, with the wind, then at E. b. N. blowing two points abaft the beam; at five in the evening, being two leagues to the W. b. S. of a headland, she met the tide of flood setting SSW. at the rate of $1\frac{1}{2}$ miles an hour, and at nine that evening, a lighthouse, which lay to the NNE. $\frac{1}{2}$ E. of the headland, bore ENE. $\frac{1}{4}$ E.: Required the ship's course and her distance from the lighthouse?*

SOLUTION. From five to nine is four hours, in which time the ship runs 18 miles, and the tide seven miles: Also two points abaft the beam, or 10 points from the wind, is NW. b. N.

Suppose the ship at A; then in the E. b. N. rhumb take AE=6, and E is the headland; in the NW. b. N. take AB=18; through B draw BC parallel to the SSW. rhumb, and equal to 7, join CA, CE; through E draw a NNE. $\frac{1}{2}$ E. rhumb, meeting in D a ENE. $\frac{1}{4}$ E. rhumb drawn from C: Then C is the ship's place at last, AC her real run, and D the place of the lighthouse.

In the triangle ABC, where two sides, AB, BC, and the included $\angle B$, are known; find $\angle BCA=101^{\circ} 19'$, $\angle BAC=22^{\circ} 25'$, and the side AC=15.26 miles.

In the triangle CAE, where are known AE=6, AC=15.26, $\angle CAE=134^{\circ} 55'$; find $\angle AEC=32^{\circ} 47'$, $\angle ACE=12^{\circ} 18'$, and the side CE=19.94 miles.

In the triangle CED, where are known CE=19.94 m. $\angle D=47^{\circ} 49'$, $\angle DCE=35^{\circ} 36'$, and $\angle DEC=96^{\circ} 35'$; the side ED will be found 15.67 miles, and the side CD=26.74 miles.

ANSWER. The ship's true course is N. $56^{\circ} 10'$ W.; dist. from the lighthouse 26.74 miles.

QUESTION X. *Two ships departed from a port at the same time, after one had sailed SE. b. E. 40 miles, and the other SW. 50 miles, they were equally distant from a rock lying directly south of their port: Required their bearing and distance from the rock?*

SOLUTION. From A, the port, draw AB SW. 50 miles, AC SE. b. E. 40 miles, join BC, and from F, the middle thereof, draw at right angles to BC the line FD, meeting the meridian in D, the place of the rock, and join DB, DC, which are equal by Article 103, Book II.

In the triangle BAC, where are known AB=50, AC=40, $\angle BAC=101^{\circ} 15'$; find $\angle ACB=44^{\circ} 35'$, $\angle ABC=34^{\circ} 10'$, and BC=69.86 miles.

In the triangle AEC, where are known AC=40, $\angle EAC=56^{\circ} 15'$, $\angle ACE=44^{\circ} 35'$, $\angle AEC=79^{\circ} 10'$; find EC=33.86 miles.

Then $CF=(\frac{1}{2} BC)-EC=FE=1.07$.

In the right angled triangle FED, where are known FE=1.07, $\angle FED=79^{\circ} 10'$; find the side DF=5.591.

In the right angled triangle DFC, where DF=5.591, EC=34.93; find the $\angle FDC=80^{\circ} 54'$, and DC=35.38 miles.

ANSWER. { The ship at B is 35.38 miles to the S. $88^{\circ} 16'$ W. the rock.
The ship at C is 35.38 miles to the N. $70^{\circ} 04'$ E. the rock.

QUESTION

QUESTION XI. Sailing *E. b. S.* with a brisk gale $7\frac{1}{2}$ knots an hour, we saw two islands, one bearing *SE. b. S.* the other *E. b. N.*; the tide was then running *SW. b. W.* $2\frac{1}{2}$ miles an hour: Two hours after, the first island bore *W. b. S.* and the other *NW. b. N.*: Required the ship's course and distance, together with the bearing and distance of those islands from one another?

SOLUTION. In an *E. b. S.* rhumb, take $AC = 15$ miles, ($= 7\frac{1}{2} \text{ m.} \times 2$); through A draw AB, AE , parallel to the *E. b. N.* and *SE. b. S.* rhumbs; draw CD *SW. b. W.* and equal to five miles, which the tide runs in two hours; through D lines being drawn parallel to the *NW. b. N.* and *W. b. S.* rhumbs, their intersections with AB, AE , give E, the place of one island, and B, the place of the other; join AD, BE.

In the triangle ADC, where $AC = 15$, $DC = 5$, and $\angle ACD = 45^\circ 00'$; find the $\angle CDA = 117^\circ 52'$, $\angle CAD = 17^\circ 08'$, and $AD = 12$ miles.

Now as ED is parallel to AB, and BD parallel to AE, therefore ABDE is a parallelogram; and so $AB = ED$, $AE = BD$.

In the triangle AED, where $AD = 12$ miles, $\angle AED = 112^\circ 30'$, $\angle DAE = 27^\circ 52'$, and $\angle ADE = 39^\circ 38'$; find $AE = 8,286$ miles, and $ED = 6,072$ miles.

In the triangle ABE, where $AE = 8,286$ miles, $AB = 6,072$ miles, and $\angle EAB = 67^\circ 30'$; find $\angle ABE = 69^\circ 15'$, the $\angle AEB = 43^\circ 15'$, and the side $BE = 8,186$ miles.

ANSWER. { The ship's course was *S. $61^\circ 37'$ E.* distance 12 miles.

{ The *N.* island bore from the *S.* island, *N. $9^\circ 30'$ E.* dist 8,186 m.

QUESTION XII. Coming out of port, into which the tide of flood was then setting, I saw two headlands, the northern one, which bore *ENE. $\frac{1}{2}$ E.* was known to be 15 miles to the *NNE. $\frac{1}{2}$ E.* of the other, which then bore *S. b. E.*: After running *E. b. S. $\frac{1}{2}$ E.* 24 miles by the log, the northern headland bore *NW.* and the other *W. b. S.*: Required the setting and drift of the tide?

SOLUTION. In a *NNE. $\frac{1}{2}$ E.* rhumb take $AB = 15$ miles; through A draw AC *N. b. W.* and AD *E. b. N.*; also through B draw BC *WSW. $\frac{1}{2}$ W.* and BD *SE.*; then C, the intersection of AC, BC, is the ship's place at first, and D, the meeting of BD and AD, is the ship's place at last: From C draw an *E. b. S. $\frac{1}{2}$ E.* rhumb $CE = 24$ miles, and draw AD, ED.

In the triangle ABC, where $AB = 15$ miles, $\angle ACB = 95^\circ 38'$, and the $\angle CBA = 45^\circ 00'$; find $AC = 10,66$ miles.

In the triangle ABD, where $AB = 15$ miles, $\angle ADB = 56^\circ 15'$, and the $\angle ABD = 73^\circ 08'$; find $AD = 17,26$ miles.

In the right angled triangle CAD, where $AC = 10,66$, and $AD = 17,26$; find $\angle ACD = 58^\circ 19'$, $\angle ADC = 31^\circ 41'$, and $CD = 20,29$ m.

In the triangle CDE, where $CE = 24$, $CD = 20,29$, and $\angle DCE = 14^\circ 49'$; find $\angle CDE = 112,29$, $\angle CED = 52^\circ 42'$, and $DE = 6,522$ m.

ANSWER. The tide set *S. $42^\circ 55'$ W.* and its drift is 6,522 miles.

QUESTION

QUESTION XIII. *A ship running five knots an hour, discovers Scilly lights bearing NE. b. N. distant six leagues, the flood tide being just made, and setting ENE. two miles an hour: Required what course she must steer, and how far by the log, to bring herself five leagues to the south of the Lizard, which lies 11 leagues to the E. b. N. $\frac{1}{2}$ E. of Scilly?*

SOLUTION. Draw SL E. b. N. $\frac{1}{2}$ E. 33 miles, LE S. 15 miles, and SA SW. b. W. 18 miles; join SE, AE: Draw AB ENE. two miles, and on B with BC = 5 miles cut AE in C, join BC; draw AD parallel to BC; and ED parallel to AB; then A represents the ship's place at first, S, Scilly, L the Lizard, E the ship's place at last, and AD the rhumb and distance she must sail.

In the triangle SLE, where SL = 33 miles, LE = 15, and the $\angle SLE = 84^{\circ} 22'$; find $\angle LES = 70^{\circ} 18'$, $\angle LSE = 25^{\circ} 20'$, and SE = 34.89 m.

In the triangle ASE, where AS = 18, SE = 34.89, and the $\angle ASE = 104^{\circ} 02'$; find $\angle SAE = 51^{\circ} 59'$, $\angle SEA = 23^{\circ} 59'$, and AE = 42.96 m.

In the triangle ABC, where AB = 2, BC = 5, and $\angle BAC = 18^{\circ} 14'$; find $\angle C = 7^{\circ} 11'$.

Now in the triangle ADE, similar to the triangle ABC, are known AE = 42.96 miles, $\angle DAE = 7^{\circ} 11'$, $\angle AED = 18^{\circ} 14'$, and the $\angle D = 154^{\circ} 35'$; hence the side AD will be found = 31.32 miles.

ANSWER. The ship must sail S. $87^{\circ} 05'$ E. distance 31.32 miles.

QUESTION XIV. *At six o'clock in the evening a privateer saw a ship directly to windward bearing NE. about three leagues distance, they were both close hauled on their starboard tacks; the privateer having sailed 18 miles, at nine o'clock the ship bore E. b. N. $\frac{1}{2}$ E.: Now the privateer finding he got a-head, would speak with the ship at two o'clock in the morning: Required at what rate the privateer must sail, and at what time tack about, keeping six points from the wind, supposing the other ship does not alter her course and rate of sailing?*

SOLUTION. Six points from the wind at NE. on starboard tacks is NNW; then in a line AF parallel to a NNW. rhumb take AC = 18 m. draw AI NE. in which take AB = 9; also BG = 3, GI = 5, representing the times between six and nine, and nine and two o'clock; draw BE parallel to AF, and through C draw an E. b. N. $\frac{1}{2}$ E. rhumb CB; join DG, draw IE parallel to DG; through E draw EF six points from the wind, and join EC, CB: Then the privateer tacking at F, will meet the ship at E.

In the triangle ABC, find $\angle ABC = 82^{\circ} 46'$, $\angle ACB = 29^{\circ} 44'$, and CE = 16.76.

In the triangle CBD, find BD = 12.04, and DC = 8.689.

Then BG : GI :: BD : DF = 20.07 miles, the ship sails from nine to two.

In the triangle CDE, find $\angle C = 81^{\circ} 31'$, $\angle E = 25^{\circ} 21'$, and EC = 19.42 miles.

In the triangle CEF, find FE = 11.76, and FC = 25.87 miles.

F

Now

Now $FE + FC = 37,63$ m. which run in five hours, is $7,12$ m. an hour. And FC at $7,12$ m. per hour gives $3h. 37 m. 48 f.$ after nine o'clock.

ANSWER. The privateer must tack at $12 h. 37 m. 48 f.$ and sail $7,12$ miles an hour.

QUESTION XV. *Two ships steering between the S. and W. parted company in latitude $20^{\circ} 15' N.$ one going $2\frac{1}{2}$ points more westerly than the other; next day at noon they arrived at two ports in latitude $19^{\circ} 16' N.$ and distant 45 miles: Required the course and distance which each ship sailed after they parted?*

SOLUTION. In an east and west line take $CB = 45$ miles; on CB describe (II. 89) the segment CBA of a circle that shall contain an angle equal to $30^{\circ} 56' = 2\frac{1}{2}$ points; in the meridian take $BH = 59$ miles, the diff. lat.; through H draw HA parallel to CB , meeting the circle CBA in A ; draw AF parallel to HB , meeting CB produced in F , join AB , AC , and from E , the center of the circle CBA , draw EA , EB ; EG parallel to CB , and ED parallel to BH .

In the right angled triangle EBD , where $\angle DEB = (\angle CAB =) 30^{\circ} 56'$, and $DB = 22,5$ miles; find $ED = FG = 37,54$, and $EB = EA = 43,77$ m.

In the right angled triangle AEG ; find $\angle AEG = 29^{\circ} 22'$, and $EG = 38,15$.

In the right angled triangle AFB , where $AF = 59$ miles, $BF = DF (= EG) = DB = 15,65$; find $\angle BAF = 14^{\circ} 51'$, and $AB = 61,06$ miles.

In the right angled triangle AFC , where $A = 59$, $CF = CB + BF = 60,65$ miles, and $\angle CAF = 45^{\circ} 47'$; find $AC = 84,62$ miles.

ANSWER. { The course of one ship is $S. 14^{\circ} 51' W.$ dist. $61,06$ miles.
The course of the other is $S. 45^{\circ} 47' W.$ dist. $84,62$ miles.

QUESTION XVI. *Two ships A and B, both going at the rate of six knots an hour, parted company in the latitude of $45^{\circ} N.$ A steering in the NE. and B in the NW. quarter; next day at noon they both arrived at their ports, A in lat. $45^{\circ} 58' N.$ B in lat. $45^{\circ} 36' N.$ and the sum of their departures was 64 miles: At what hour did they part, and what was the course and distance each ship had sailed?*

SOLUTION. Make the departures $FG = 64$ miles, the diff. lat. $FB = 48$, and $GC = 36$ miles; draw BC , and from K , the middle of BC , draw at right angles to BC , KE , meeting FG in A , and BF produced in E ; join AC , AB ; then draw AH parallel to FB ; and CD , BH , parallel to FG , meeting AH in I and H .

In the right angled triangle BDC , where $DC = 64$, $DB = 22$; find $\angle DBC = 71^{\circ} 02'$, $\angle DCB = 18^{\circ} 58'$, and $BC = 67,69$ miles: Hence $BK = KC = 33,84$.

In the right angled triangle BKE , where $\angle E = 18^{\circ} 58'$, $KBE = 71^{\circ} 02'$, and $KB = 33,84$ miles; find $BE = 104,1$: Hence $BE - BF = FE = 46,1$ miles.

In

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In the right angled triangle EFA , find $FA=15,84$ miles: Hence $AG=48,16$.

In the right angled triangle AIC , where $AI=(GC=) 36$, $IC=48,16$; find $\angle IAC=53^{\circ} 13'$, and $AC=60,13$ miles.

In the right angled triangle AHB , find $\angle BAH=15^{\circ} 17'$.

Now AC or $AB=60,13$ divided by 6 gives $10,02$ hours $= 10$ h. 2 m. 12 f.

ANSWER. $\left\{ \begin{array}{l} \text{The ships parted at 1 h. 58 m. 48 f. in the morning.} \\ \text{A's course is N. } 15^{\circ} 17' \text{ W.; B's is N. } 53^{\circ} 13' \text{ E.; dist. of} \\ \text{each is 60,13 miles.} \end{array} \right.$

QUESTION XVII. *Passing by a rock lying off the mouth of a harbour whose breadth is 12 miles, the western point thereof bearing N.N.W. is nine miles distant from the town; and the eastern point of the mouth bearing N.E. b. N. is six miles from the town: Required the distance of the rock from the western and eastern points, and also from the town which bore N. b. E.?*

SOLUTION. With the three given sides, 12, 9, 6, having constructed a triangle where A and C are the western and eastern points of the harbour's mouth, and B the town; make the $\angle ACD=3$ points, $=\angle$ between the bearings of A and B ; also make the $\angle CAD=2$ points $=\angle$ between the bearings of C and B ; through ADC describe a circle; then the right line BDF being drawn will give in the circumference the point F , the place of the rock.

For the $\angle AFD=\angle ACD=3$ pts.; and $\angle CFD=\angle CAD=2$ pts. (II. 129)

In the triangle ABC , find $\angle BAC=28^{\circ} 57'$, the $\angle BCA=46^{\circ} 33'$ and the $\angle ABC=104^{\circ} 30'$.

In the triangle ACD , where $\angle ADC=123^{\circ} 45'$, $\angle DCA=3$ points, $\angle DAC=2$ points; find $DA=8,018$ miles.

In the triangle ADB , where $AB=9$, $AD=8,018$, and $\angle BAD=6^{\circ} 27'$; find the $\angle ABD=41^{\circ} 05\frac{1}{2}'$: Hence $\angle CBD=63^{\circ} 24'$.

In the triangle ABF , find $AF=10,65$ miles, and $FB=15,64$ miles.

In the triangle FBC , find $FC=14,02$ miles.

ANSWER. $FA=10,65$, $FB=15,64$, $FC=14,02$ miles.

QUESTION XVIII. *At night casting anchor among a cluster of islands, I observed three lighthouses, A, B, C; A bore NNE. B bore SSE. $\frac{1}{2}$ E. and C bore West; the distance from A to B is 94 miles, from B to C 78 miles, and from A to C is 59 miles: Required my distance from each lighthouse?*

SOLUTION. Let ABC be the triangle formed by the distances of the three lighthouses; on CB , CA , describe (II. 89) segments of circles that shall contain the angles included between the bearings of C and B , and of C and A ; and the intersection D of these circles, is the point where the ship cast anchor: Through the centers H , K , draw aE , bG , at right angles to CB , CA ; and draw EB , BC ; HB , HC ; GC , GA ; KC , KA ; and join KH cutting CD in m .

F 2

Now

Now because cd is a chord common to both circles, therefore HK bisects cd in m : Also, because $\angle CDB$ and $\angle CEB$ together are measured by half the circle (II. 128); therefore $\angle CEB = 180^\circ - \angle CDB$; and for the same reason, the $\angle CGA = 180^\circ - \angle CDA$; therefore the $\angle s$. CEB , CGA , are known; and (II. 127) so are their equals $\angle CH a = 61^\circ 52\frac{1}{2}'$, $CK b = 67^\circ 30'$.

In the triangle CAB , from the given sides find $\angle CAB = 55^\circ 48'$, $\angle CBA = 38^\circ 44'$, and the $\angle ACB = 85^\circ 28'$.

In the right angled triangle caH , where $ca = 39$ m. $\angle CH a = 61^\circ 52\frac{1}{2}'$; find $CH = 44,22$ miles.

In the right angled triangle cbK , where $cb = 29\frac{1}{2}$ m. $\angle CK b = 67^\circ 30'$; find $CK = 31,93$ miles.

In the triangle KCH , where $KC = 31,93$, $CH = 44,22$, the $\angle KCH = 136^\circ 06'$ ($= \angle KCA + \angle ACB + \angle BCH$); find $\angle CKH = 25^\circ 40'$, and $\angle CHK = 18^\circ 14'$.

In the right angled triangle $CH m$, where $\angle CH m = 18^\circ 14'$, and $CH = 44,22$ m.; find $cm = 13,84$ miles, which doubled gives $CD = 27,68$ miles.

In the triangle ACD , where $CD = 27,98$ m. $\angle CAD = 25^\circ 40\frac{1}{2}'$, and $\angle ACD = 41^\circ 49\frac{1}{2}'$; find $AD = 42,6$ miles.

In the triangle ADB , where $AB = 94$ miles, $\angle BDA = 129^\circ 22\frac{1}{2}'$, $\angle DAB = 30^\circ 07\frac{1}{2}'$; find $DB = 61,03$ miles.

ANSWER. The distance from D to $\begin{cases} A \text{ is } 42,6 \text{ miles.} \\ B \text{ is } 51,03 \text{ miles.} \\ C \text{ is } 27,68 \text{ miles.} \end{cases}$

QUESTION XIX. In latitude $48^\circ 50'$ N . we were attacked by a privateer, who upon receiving our broadside lay to, and we stood away NNW .; some time after the privateer bore away NE . b . N . and two hours after, he bearing by our estimation run ten miles, was seven miles directly east of us: Now by our account we had sailed four leagues since the action, and the tide all the time had run two miles an hour: Required the setting of the tide, the time the privateer lay to, our rate of sailing, and each ship's direct course and distance?

SOLUTION. Suppose the action at A , then parallel to the NNW . and NE . b . N . rhumbs, draw $AB = 12$ m. $AC = 10$ m. and join BC ; through c draw $CD = 7$ miles due west; join BD , and in this line produced take $DI = 4$ miles, the tide's drift in two hours; draw CF parallel and equal to DI ; through F draw a west line FG , meeting BI in I , and a meridian through A , in G ; and draw AF , AI , which represent the runs of the two ships, F being the place of the privateer when she was seven miles east of the other ship then at I .

In the triangle ABC , where $AB = 12$, $AC = 10$, and $\angle BAC = 56^\circ 15'$; find $\angle ACB = 71^\circ 31\frac{1}{2}'$, $\angle ABC = 52^\circ 13\frac{1}{2}'$, and $BC = 10,52$ miles.

In the triangle BCD , where $BC = 10,52$ m. $CD = 7$ m. $\angle BCD = 15^\circ 16\frac{1}{2}'$ ($71^\circ 31\frac{1}{2}' - 56^\circ 15'$); find $\angle BDC = 138^\circ 40'$, $\angle DBC = 26^\circ 04'$, and $BD = 4,197$ miles, which is the tide's run while the privateer lay by.

In

In the triangle ACF , where $CA=10$ m. $CF=4$ m. and $\angle ACF=82^\circ 25' (= \angle DCF - \angle DCA)$; find $\angle AFC=74^\circ 52\frac{1}{2}'$, $\angle CAF=22^\circ 42\frac{1}{2}'$ and $AF=10.27$ miles, the distance sailed by the privateer.

In the right angled triangle AGF , where $AF=10.27$, the $\angle FAG= (\angle GAC + \angle CAF =) 56^\circ 27\frac{1}{2}'$; find $AG=5.675$, and $GF=8.561$ miles.

In the right angled triangle AGI , where $AG=5.675$, $GI=(GF - IF=) 1.561$; find $\angle GAI=15^\circ 23'$, and $AI=51.886$ miles.

Now 4.197 m. divided by 2 m. per hour, gives 2.098 h. $= 2$ h. 5 m. 52 f.

The privateer lay by 2 h. 5 m. 52 f. nearly.

The tide sets $S. 48^\circ 40'$ E.

ANSWER. { The privateer sailed $N. 56^\circ 27'$ E. distance 10.27 miles.

{ The other ship sailed $N. 15^\circ 23'$ E. distance 5.886 miles.

QUESTION XX. *From the ports A, B, C, lying under the same parallel of latitude, B being 60 miles, and C 160 miles to the east of A, three ships sail towards the NW. quarter and meet together, B having sailed two points, and C $3\frac{1}{2}$ points farther from the meridian than A had: Required the course and distance each ship has run?*

SOLUTION. In an east and west line CD, take $CA=160$ m. $AB=60$ m. make the angle $ACE=2$ points $=22^\circ 30'$, the angle $CAE=1\frac{1}{2}$ points $=16^\circ 52'$; and join EB.

Through the points AEC describe a circle, cutting the line EB produced in H, and join HA, HC.

Now $\angle AHE=\angle ACE$; and $\angle BHC=\angle BAE$. (II. 129)

Therefore ships sailing from A, B, C, and meeting in H, their course will differ by the quantities proposed.

For drawing a meridian HD, the angles DHA, DHB, DHC, will respectively represent the angles which the ships courses make with the meridian.

In the triangle ACE, where $AC=160$ miles, $\angle EAC=1\frac{1}{2}$ points, $\angle ECA=2$ points, and $\angle AEC=12\frac{1}{2}$ points; find the side $EC=73.21$ miles.

In the triangle BCE, where $BC=100$ m. $CE=73.21$, and the included $\angle BCE=22^\circ 30'$; find the angle $CEB=116^\circ 37'$, and $\angle CBE=40^\circ 53'$.

Hence $\angle ABH=40^\circ 53'$ (II. 93); the $\angle HAB=116^\circ 37'$.

In the triangle AHB, where $AB=60$, and all the angles are known, find $HA=102.6$ miles, and $AB=140.2$ miles.

In the triangle AHC where $AC=160$, and all the angles are known, find $HC=225.5$ miles.

ANSWER. { A's course is $N. 26^\circ 37'$ W. distance 102.6 miles.

{ B's course is $N. 49^\circ 07'$ W. distance 140.2 miles.

{ C's course is $N. 69^\circ 59'$ W. distance 225.5 miles.

SECTION X.

Of the Surveying of Coasts and Harbours.

From what has been already said in the preceding sections, an intelligent reader would see how the business of taking the bearings of part of a coast, and of plotting or delineating it, might be done: But as there are some particulars, which can be gained only by experience in the art of surveying, it will not be improper to apprise the learner concerning them, and thereby qualify him to go more readily to work.

52. *To take the draught of part of a coast in sailing along it.*

1st. Having brought the ship to a convenient place, from whence the principal points of the coast, or bay, may be seen, either cast anchor if it is convenient, or *lie to* as steady as possible; or if the coast is too shoal, let the observations and measures be done in a boat: Then while the vessel is in a stationary situation, take, with the azimuth compass, the bearings in degrees of such points of the coast, as form the most material projections, or hollows; write down these bearings, and make a rough sketch of the appearance of the coast, observing carefully to mark the points, whose bearings were taken, with letters, for the sake of reference.

2d. Then let the ship or boat run in a direct line, which must be very carefully measured by the log, or otherwise, one, two, or three miles, more or less, until she comes into a situation from whence the same points before observed can be seen again; there let the vessel lie steady as at the foregoing station, and again observe the respective bearings in degrees of the same noted points, which are also to be wrote down, and a rough sketch of the coast should also be taken from this station: But while the vessel is running the *base line* from station to station, a more accurate drawing of the appearance of the coast should be made: Then let the several bearings be corrected by the variation, to reduce them to their true positions.

3d, *To map these observations.* In some convenient part of a sheet of paper describe a circle (the larger and better), on which lay the several corrected bearings taken from the first station, and let them be numbered 1, 2, 3, &c. on the outside of the circle: Also lay down the several corrected bearings taken from the second station, let these be numbered 1, 2, 3, &c. on the inside of the circle, observing that the bearings of the same points are numbered with the same figures.

4th. Draw a line to express the ship's run both in length and course; and from that end of the line expressing the first station, draw lines parallel to the respective bearings taken from that end, and marked on the outside of the circle: Also from the other end, draw lines parallel to the bearings taken at that end, and noted on the inside of the circle; mark the intersection of each pair of lines, directed to the same point, with the number annexed to their bearing; and through the intersections so marked, draw by hand a curved line, observing to wave the line in and out, as near as can be like the trending of the coast itself.

5th. Against each part, draw the appearance of the elevated, or low ground as marked in the sketches, distinguishing rocks, cliffs, high-land, low-land, sand hills, &c. If there are any currents, or eddies, express them in their proper places by darts, or arrows, the points being turned that way the current sets: Put in the several soundings at low water in small figures, distinguishing whether fathoms, or feet; shew the time of high-water on the full and change days by Roman figures, and tell the rise in feet: Put in a compass, with a scale of miles or leagues, such as the vessel's run was laid down by; add the name of the place, the coast, and the latitude and longitude, as true as can be obtained.

6th. If there is a shoal, or sand, on the coast, let it be taken by a boat sailing round it, and keeping an account of the courses, distances and soundings: But to put it in the draught, the boat must from some part of the sand, or shoal, take the bearings of two points on the coast, whose bearings have been taken from the ship: Or, the bearing of the boat at some part of the shoal, or of some beacon in that place, must be taken by the ship at the stations where she takes the bearings of the shore; for by either of these means, one point of the sand being obtained, the rest of it can be laid down from the boat's account.

7th. If the coast to be drawn is a bay or harbour, winding in such a manner that all its principal points cannot be seen at two stations, let as many bases, or lines, be run and exactly measured, as may be found necessary; observing that the several distances run should join to one another in the nature of a traverse, that each new set of objects, or points, observed, should be taken from two stations at the end of a known distance, and that the objects, whose bearings are taken, do not extend so much beyond the limits of the base, as to make angles with it of less than about $\frac{1}{2}$, or $\frac{3}{4}$, of a point; but rather reserve such objects for the next measured base; for when lines lie very oblique to one another, their intersection is not easily ascertained.

8th. If any particular parts of the harbour cannot be conveniently seen from either of the stations, take the boat into those places, and having well examined them, make sketches thereof, estimating the lengths and breadths of the several inlets, either by the rowing, or sailing of the boat, taking as many bearings, soundings, and other notes, as may be thought necessary: then annex these particular views in their proper places in the general draught.

9th. If there are any dangerous sands, or rocks, besides inserting them in their proper places, there should be a double line drawn through that point and one or more objects ashore; and for this purpose, chuse a church, mill, house, noted tree, a cliff, or any other remarkable thing that can be distinctly seen at sea, and which can be brought to bear in the same right line with the point to be avoided: But if that point is under water, there must be two land-marks brought to bear with the danger, either in the same right line when it can be, or in two lines; and that those land-marks may be put down in their proper places, their bearings must also be taken from two of the ship's stations.

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8th. If any particular parts of the harbour cannot be conveniently seen from either of the stations, take the boat into those places, and having well examined them, make sketches thereof, estimating the lengths and breadths of the several inlets, either by the rowing, or sailing of the boat, taking as many bearings, soundings, and other notes, as may be thought necessary: then annex these particular views in their proper places in the general draught.

9th. If there are any dangerous sands, or rocks, besides inserting them in their proper places, there should be a double line drawn through that point and one or more objects ashore; and for this purpose, chuse a church, mill, house, noted tree, a cliff, or any other remarkable thing that can be distinctly seen at sea, and which can be brought to bear in the same right line with the point to be avoided: But if that point is under water, there must be two land-marks brought to bear with the danger, either in the same right line when it can be, or in two lines; and that those land-marks may be put down in their proper places, their bearings must also be taken from two of the ship's stations.

10th. It should be remarked in the draught what places, if any, are unfit for anchorage, and what are fit, by writing *rocky ground*, *soul anchorage*, *good anchorage*; and in the latter to draw the figure of an anchor: Also, if there is any particular channel more convenient to sail through than another, it is to be pointed out by lines drawn to its entrance from two or more noted marks ashore.

The foregoing method of surveying a coast, supposes in general that it is taken by a ship in her passage along, not having an opportunity of going ashore: But when the circumstances will permit the measures and observations to be made on land, the survey can be taken more accurately than on the water.

53. *To survey a Harbour by Observations on Shore.*

1st. Make an eye-draught of the place to be surveyed; and in going round its coasts fix, in the most remarkable points and bends of the shore, station staves, or straight poles, tall enough to be seen at a considerable distance; but if at any of those places there is a noted tree, house, or any other remarkable thing, that object may serve instead of a station staff; and it will be convenient to black the staves, and tie a piece of white bunting to the top of each: Then in the eye-draught put letters at the noted points, or marks, for distinction sake.

2d. Choose the most level spot of ground, wherein a base line may be measured of one or more half miles in length; or of a length not less than a tenth part of the distance of the two extreme objects marked for observing; and let the direction of the measured base line be so laid out, that from both ends of it, as many as possible of the station staves before planted, or the objects before remarked, may be seen: The bearing, or position, of this base must be well determined in degrees and minutes; and also its length must be accurately measured to feet and parts, either by a measuring chain, or by a piece of log-line of 100 feet long, properly marked at the end of every 10 feet, and each end-length marked at every foot.

3d. From one end of the base, observe (with any instrument proper to take bearings) the positions or bearings, in degrees and minutes, of all the staves or objects within view; and write them down orderly; do the same from the other end of the base; and let all the bearings be corrected by the variation of the compass. See Book IX.

4th. Then these measures and corrected bearings being plotted, or laid down, will give the most conspicuous points of the shore; the intermediate spaces are to be filled up from the sketches of them made on the spot, as in 8th of Article 52.

5th. But if any of these objects should spread on either hand so far beyond the limits of the base, that at either end thereof, the other end and those objects or staves should appear nearly in the same direction, or to make angles not exceeding about 10 degrees: Or, if some of the remarked objects can be seen from only one end of the base; then let the bearings of such objects be taken from a place whose position has been

determined

determined from both ends of the measured base: Or, if there are several remarked objects which cannot be seen from neither end of the base line; let the bearings of such objects be taken from each of two points whose positions have been taken from both ends of the base: Or, it may on some occasions be proper to chuse another place, on which another base of a convenient length may be measured, and from the extremities of which, the ends of the first base may be seen; and also, as many as can be of the remaining objects which lay too oblique for the first base, or which could not be seen from it: In such manner proceed until the bearings are taken of all the points judged necessary for completing the survey of the limits of the harbour.

If a base of a sufficient length cannot be measured in one right line, it may be taken in two adjoining lines, as the two sides of a triangle; the included angle being accurately taken, and the bearing of either line.

The positions of objects taken by a magnetic compass are liable to great uncertainties, as is well known to those who have had any experience, especially at sea; and therefore the mariner should be informed, that the angular positions of objects are best taken on shipboard, or even on land, by the Hadley's quadrant, the description and use of which will be given in the IXth Book.

54. When the outlines or limits of an harbour, bay, road, &c. are delineated by the preceding precepts; the appearances of the shores with the adjacent remarkable objects correctly drawn; a compass with the variation and scale properly fitted to the plan; the isles, rocks, sands, &c. marked in their proper places; the setting and drift of the currents and tides: the times of high-water on the days of full and new moon, with the rise of water at those times, and whether whole tide, or tide and part; the best anchoring places, with their soundings at low-water, and the winds open to them; the best track, with the soundings all the way to those anchoring places; the proper sailing marks to avoid dangers; the winds, if any troublesome ones, which prevail, and at what seasons; the places where fresh water can be got, and the time in which a boat's crew can fill a number of hogheads at those places; the name of the place, the country in, on what sea, the latitude and longitude; a sketch of the appearance the place makes at sea upon a known rhumb, and at an estimated distance; and whatever else a judicious seaman shall think proper to insert; then is the plan fit for all nautical purposes: But as it is usual to embellish such drawings with proper colours, the following observations may be useful to beginners.

55. A draught's-man should be furnished with some, if not all of these particulars, viz. 1st. Black-lead pencils: 2d. One pair of hair pencils, with the quill about the size of a common pen, and the hairs not shorter than $\frac{1}{2}$ an inch, nor longer than $\frac{2}{3}$ inch; another pair of a size or two smaller; they should all draw to a fine point when wetted; and that wet point be drawn through the margin of the flame of a candle to take off the ends of straggling hairs, if any: 3d. A marble of five or six inches square, and about $\frac{1}{4}$ inch thick, with four or six or more little round cavities, or cells of about $1\frac{1}{4}$ inch diameter, and $\frac{1}{4}$ inch deep, to
put

put liquid colours in when using ; or, for want of a marble, a few horse muscles, or such like, shells : 4th. A cake of good Indian ink : 5th. A little paper of carmine, which is a fine red colour : 6th. A piece of prepared indigo, which is a blue : 7th. A piece of prepared yellow : 8th. A piece of a sap-green : 9th. A piece of Spanish liquorice, or some liquid brown made by a strong solution of tobacco in boiling water : 10th. A small vial of distilled verdigrise, which is a fine sea-green : 11th. Some gum Arabic dissolved in water : These particulars are to be had at colour-shops ; where the colours in lumps, or liquids, are generally prepared with a sufficient quantity of gum in them ; but those in powders must have a pencil full or two of gum-water put to them when used.

The colours in lumps are best used by gently rubbing one end of the lump in a cell in the marble (or on a tile,) till as much is wore off as is judged necessary for the present use ; and it is best to rub off fresh colour as it is wanted, and not have too much at a time : If the colour first rubbed up is too strong, as it generally is (unless for finishing touches,) put one or two pencils full of that colour into another cell, and lighten it with one or more pencils full of water ; remove some of this second colour into a third cell, and lighten this with water ; and so to a fourth cell ; whereby four different tints of that colour will be had, which will be found quite sufficient in the most finished piece ; in most cases two tints will do.

In the using of liquid colours, the pencil should be well wetted with the colour, and then gently drawn across the edge of the cell, to bring the pencil to a point, and free it of some of the colour ; for the pencil should not be quite full, unless it be to cover a large spot of the drawing.

56. When a person, unaccustomed to the use of colours, begins to work with them, he may acquire by degrees a facility in laying them well on paper : Thus, with black lead pencil, draw two parallel lines at about $\frac{1}{4}$ inch apart, and with the lightest of the aforesaid tints (suppose of Indian ink,) let him try to colour the space between these lines ; and repeat the trial with other lines, until he finds he can cover the space with an uniform tint, and keep exactly between the lines : Let him try in like manner with all the tints gradually to the deepest (which is about as black as good writing ink,) and the most difficult to use ; and when he finds he can succeed in all the tints, and his hand accustomed to keep the colour exactly to a line, which he may acquire in a few days, he may proceed thus : With a pair of pencils fitted on a stick, one for colour, and the other for fair water, let him lay a tint close to a single line, and soften off the other margin with the water-pencil ; let him try to do thus in several examples, some on one side, and some on both sides, of the colour, taken of different tints, until he finds that he can succeed so, that the colour shall from the deepest gradually become fainter and fainter, until it loses itself in the whiteness of the paper : Observing, 1st. not to lay on a tint longer than $1\frac{1}{2}$ or 2 inches, before the water-pencil is applied to soften its margin ; lest the colour should dry too fast. 2d. Not to have the water-pencil too wet ; lest it run into the tint too much : 3d. At each length the water-pencil is used to, let its point be stroked on the hand, or on a waste-paper, to take off the colour it had imbibed in washing that length ; otherwise the next length might be of a different tint.

The

The foregoing observations being well attended to, will enable a beginner to work properly with colours, in plans: But it must be confessed, that half an hour employed in seeing these operations performed by an artist, will furnish more knowledge in these matters than can be drawn from a multitude of verbal instructions only.

57. The young artist knowing how to use his colours, and graduate their tints, he may proceed to colour the plans of harbours, and their coasts, thus:

The shores on the land-side are to be shaded with Indian ink; the breadth of the tint being between the limits of about $\frac{1}{4}$ and $\frac{1}{2}$ of an inch, and laid on at three times; viz. 1st, a light tint laid close to the shore, with the margin on the land-side softened off: 2d. A deeper middle tint, laid also close to the shore, but of a less breadth than the former, and the inside margin also softened off, within the first washing: 3d. A fine line traced on the said shore, with a hair pencil charged with the deepest tint, and not softened on either side.

The shores on the water-side are to be shaded with distilled verdigrise; the breadth of the tint about $\frac{1}{4}$ inch, and laid on at twice: For the first tint, about half water and half colour, laid close to the shore, and the water-side margin washed; the second tint of verdigrise only, narrower than the former, laid close to the shore, and not washed.

Sands, dry at low water, to be covered pretty thick with fine black points (see figures 12, 16, 18, 20, in Plate IX. the sand coloured with brown, and the borders, on the water-side, shaded with verdigrise washed, or softened.

The plans of buildings on the land to be coloured with a light tint of carmine, and the limit, on the inside, touched round with a deeper tint.

Fresh waters to be expressed by a colour of half verdigrise and half fair water, shaded with a light tint of indigo: Or, with a tint of indigo alone of a light sky-colour, interspersed with fine streaks of white.

Green colours of various tints, may be made with sapgreen, yellow and blue rubbed together, with a pencil full of verdigrise among them; and with such colours, woods and grass lands may be expressed, when properly broke in with trees, bushes, and little irregular dashes with Indian ink.

Arable or ploughed land is expressed by covering the spot in the drawing with a kind of broken parallel lines, done by a hair pencil dipt in a brown, or clay colour.

Muddy shores at low-water are shewn by laying on a middling tint of Indian ink, and laying over that a tint of light verdigrise.

The drawing and colouring of rocky shores, steep banks, hills, marshy ground, and the manner of breaking the ground in plans, &c. are best gained from examining a coloured plan done by a proper proficient; but where that is wanting, a pretty good taste may be gained by consulting printed plans and views, such as are in Lord Anson's voyage round the world.

Sea-

Sea-draughts taken according to the preceding precepts, and neatly drawn and coloured, besides the real use they may be of, cannot fail to recommend the young mariner, who surveys and constructs them, to the notice of his superiors.

SECTION XI.

To estimate Distances.

Besides the method of finding the distances of remote objects by trigonometrical computations, there are two other means which may be usefully applied when the former cannot; and these are the *motion of sound*, and the *curvature of the earth*.

58.

Of the Motion of Sound.

It is a well known observation, that the noise or sound arising from any blow, or shock, made at a distance, is always heard some time after the stroke is seen, or known to be given; and hence it is generally concluded, that the seeing of any act done within view is instantaneous, but that sound travels at a perceptible rate.

Some of the most eminent philosophers, judging that the knowledge of the flight of sound might be of use on various occasions, have been at extraordinary pains and expence to measure the rate at which it moved, and the result of their experiments, particularly of those which were best conducted, are as follow.

1st. The velocity of sound is the same, whether by sea or by land, in dry or in rainy weather, by day or by night, in winter or summer.

2^d. That sound, whether more or less strong, flies with the same swiftness: For by experiments, a cannon fired with a half-pound charge of powder was heard at about the distance of $17\frac{1}{2}$ miles in the same time after the flash was seen, as when fired with a charge of 6 lb.

3^d. That the times in which sound is heard is proportional to the distance; that is, at a double distance it is heard in twice the time, at a triple distance in thrice the time, &c.

4th. That sound flies quicker or slower by just as much as is the velocity of the wind, according as it is with or against the motion of the sound.

5th. That sound travels at the rate of about 1142 feet, or 380 yards in one second of time; so that

Sound moves in 1 second, 1142 feet, or 380 yards.

1 minute

22840 yards, or about 13 miles.

Which is about a mile in 4,6 seconds, or about a league in 14 seconds.

But sea miles are to land miles, nearly as 7 to 6.

Therefore sound runs a sea league in about 12 seconds of time.

It is a common observation, that persons in good health have about 75 pulsations, or beats, of the artery at the wrist in a minute.

59. There-

59. Therefore in 75 pulsations, sound flies about 13 land miles, and about $11\frac{1}{2}$ sea miles; which is about one land mile in six pulses, and about one sea mile in near seven pulses, or a league in 20 pulses.

And hence the distance of objects may be found, by knowing the time which sound takes to move from those objects to an observer.

EXAMPLE. Upon seeing the flash of a gun at sea, I counted 56 beats of the pulse at the wrist before I heard the report: How far was that gun from me?

Now 56 divided by 20 gives 2,8 leagues, or about eight miles.

60. Of the Course steered by a Ship seen at a distance.

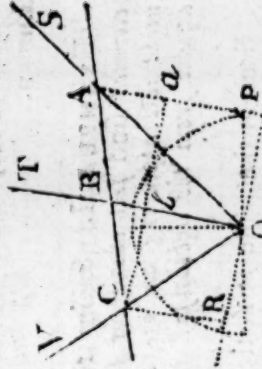
A ship lying to (at O .) may discover the course (AC .) which a vessel seen at a distance is going on; by taking, at two known intervals of time, three bearings (OS , OR , OV) of that vessel; thus.

In a line POR , drawn at right angles to the middle bearing OR , take OP , OR , as the given intervals of time, in minutes, taken from any scale of equal parts; and through P , R , draw lines parallel to OR , cutting OS , OV , in A , C .

Then a right line drawn through A and C , cutting OR in B , will shew the position, or the course, that vessel was sailing.

For drawing parallel to RP , the line ca , cutting PA , OB , in a , b ; then ab , bc , equal to PO , OR , are as the times; And so are the lines AB , BC , by similar triangles.

Although by this method the apparent course may be determined, and also the relative distances at any proposed times; and among others, the nearest or least distance: Yet the absolute distances are not hence to be found; and if the observing ship should in the mean time change its place, the position AC will become uncertain.



61. Of the Curvature of the Earth.

Most persons know, that if they are raised above the surface of the adjacent land or water, they can not only see distant objects that lie on that surface better, but also see those more and more remote, as they advance higher. The irregularity of the surface of the land will not be subjected to any one rule that will give the distance to which objects may be seen at different elevations; but at sea, where there is generally an uniform curvature of the surface of the water, upon the supposition of the spherical figure of the Earth, those distances may be easily computed.

62. Let (see fig. 12. Pl. XIV.) the eye at E be raised the height Es above the surface s of the water; then will it see an object at f on the surface, in the tangent Ef , which is the distance of that object from the eye.

Now

Now in the triangle Efc , right angled at f , there are known, the side cf = semidiameter of the Earth, and the side ce equal to the semidiameter cs increased by se the height of the eye: To find the tangent ef .

$$\begin{aligned}\text{Then, by II. 113. } ef &= \sqrt{ce^2 - cf^2} = \sqrt{cs^2 + se^2 - cf^2} \\ &= \sqrt{2cs + se \times se}.\end{aligned}$$

That is, To the Earth's diameter add the height of the eye, multiply the sum by that height; then the square root of the product is the distance at which an object on the surface of the water can be seen by an eye so elevated.

And by this Rule was the table at Article 65 computed, the diameter of the Earth being taken at 41798117 feet, according to Sir Isaac Newton's numbers.

This table may be usefully applied to estimate the distance of an object at sea, the elevation of that object above its horizon being known.

EXAMPLE I. *Sailing towards a headland on which is a lighthouse elevated 600 feet above the surface of the water, we saw the lights at night just appearing in the horizon: How far were we at that time distant from the lighthouse?*

Seek in the table at Article 65 for 600 feet in the column signed height in feet, and right against it in the column signed distance in miles, stands 29,994; so that the distance may be reckoned about 30 miles.

EXAMPLE II. *Being in company with some merchants walking on a sandy shore on the look-out for a vessel which was then expected, whose top-gallant mast was 140 feet above the surface, allowance being made for her immersion in the water; we observed through a telescope a ship's vane just appearing in the horizon: How far off is that ship, supposing it the vessel expected?*

ANSWER. Against 140 feet, the height, stands 14,448; that is, her distance is about 14½ miles.

63. Here is no allowance made for the height of the eye above the horizon; but it is obvious, that the higher the eye, the farther it can see: Now as objects are seen in a straight line, and that line is a tangent to the earth's surface; therefore it follows, that to find the distance of two elevated objects when the right line joining them touches the surface of the Earth between those objects; seek the distance answering to each height, and the sum is the distance sought.

Thus in Example II. suppose the eye is raised six feet above the water's edge, it can see an object on the surface 2,999, or three miles off; this distance added to the 14½ miles, makes the distance of the ship to be 17½ miles.

EXAM.

EXAMPLE III. *A man being on the main-top gallant mast of a man of war, 200 feet above the water, sees a 100 gun ship she had engaged the day before, hull to: How far were those ships distant?*

A ship of 100 guns, or a first rate man of war, is about 60 feet from the keel to the rails; from which deduct about 20 feet for the draught of water, leaves 40 feet for the height of her quarter above water: Now a ship is seen *hull to*, when her upper works just disappear.

Then 200 feet high, gives
And against 40 feet stands

17,316 miles.
7,744 miles.

Their sum, which is 25,060 miles, is their distance.

64. It might perhaps hereafter be found useful to mariners, could a list of the most considerable lighthouses, and other elevated objects in different parts of the world, be obtained; together with their heights above the water: For as some of these objects are so high as to be seen at the distance of 30 or 40 miles, and even more; and as their distances upon being first seen could be near enough known by the table at art. 65. were their heights known; therefore such a list might, on several occasions, not only give much satisfaction to a seaman, but even become of great use to know his distance from those objects, as he could thereby know how to increase or slacken sail in order to save his tide, or other emergency; also he would know better how to lay down the distances of rocks or shoals that lie off a coast, &c. But as such a list is not easy to be come at, unless skilful mariners would be at the trouble of procuring such as occurred to them in their voyages, and on their return communicate them to some public body; therefore the curious in those matters must be contented at present with what is here already said.



restrial Objects at Sea.

66. A

66. A TRAVERSE TABLE

To every POINT and QUARTER POINT of the

COMPASS or HORIZON;

IN FOUR PAGES.

AND ALSO

67. A TRAVERSE TABLE

To every DEGREE and QUARTER DEGREE of the

COMPASS or HORIZON;

IN FORTY-FIVE PAGES.

TRAVERSE TABLE. 66.

Distance	0 Points			1 Point			2 Points									
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.								
1	1,00	0,05	0,99	0,10	0,99	0,15	0,98	0,20	0,97	0,24	0,96	0,29	0,94	0,34	0,92	0,38
2	1,00	0,10	0,99	0,20	0,98	0,29	0,96	0,39	0,94	0,49	0,91	0,58	0,88	0,67	0,85	0,77
3	1,00	0,15	0,99	0,29	0,97	0,44	0,94	0,59	0,91	0,73	0,85	0,87	0,82	0,81	0,77	0,71
4	1,00	0,20	0,98	0,39	0,96	0,59	0,92	0,78	0,88	0,97	0,85	1,16	0,77	1,35	0,76	1,53
5	1,00	0,25	0,98	0,49	0,95	0,73	0,90	0,98	0,85	1,17	1,45	1,45	0,74	1,68	0,72	1,91
6	1,00	0,30	0,97	0,59	0,93	0,85	0,98	1,17	1,02	1,46	1,74	1,74	0,65	2,02	0,54	2,10
7	1,00	0,35	0,97	0,69	0,92	1,03	0,98	1,27	1,17	1,70	2,02	2,02	0,59	2,36	0,47	2,68
8	1,00	0,40	0,96	0,78	0,91	1,17	0,98	1,36	1,27	1,94	2,24	2,24	0,69	2,69	0,39	3,06
9	1,00	0,45	0,96	0,88	0,90	1,32	0,98	1,46	1,36	2,24	2,54	2,54	0,79	3,03	0,32	3,44
10	1,00	0,50	0,95	0,98	0,89	1,47	0,98	1,55	1,47	2,54	2,84	2,84	0,89	3,37	0,24	3,83
11	1,00	0,55	0,95	1,08	0,88	1,61	0,98	1,64	1,57	2,84	3,14	3,14	0,99	3,71	0,16	4,21
12	1,00	0,60	0,94	1,18	0,87	1,76	0,98	1,73	1,66	2,94	3,24	3,24	1,09	4,04	0,09	4,59
13	1,00	0,65	0,94	1,28	0,86	1,91	0,98	1,82	1,79	3,14	3,44	3,44	1,20	4,38	0,01	4,97
14	1,00	0,70	0,93	1,37	0,85	2,05	0,98	1,91	1,88	3,34	3,64	3,64	1,31	4,72	0,00	5,36
15	1,00	0,75	0,93	1,47	0,84	2,20	0,98	2,01	1,95	3,54	3,84	3,84	1,42	5,05	0,00	5,74
16	1,00	0,80	0,92	1,57	0,83	2,34	0,98	2,11	2,04	3,74	4,04	4,04	1,53	5,39	0,00	6,12
17	1,00	0,85	0,92	1,67	0,82	2,49	0,98	2,22	2,15	3,94	4,24	4,24	1,64	5,73	0,00	6,51
18	1,00	0,90	0,91	1,76	0,81	2,64	0,98	2,33	2,26	4,14	4,44	4,44	1,75	6,06	0,00	6,89
19	1,00	0,95	0,91	1,86	0,80	2,79	0,98	2,44	2,37	4,34	4,64	4,64	1,86	6,40	0,00	7,27
20	1,00	1,00	0,90	1,96	0,78	2,93	0,98	2,55	2,48	4,54	4,84	4,84	1,97	6,74	0,00	7,65
21	1,00	1,05	0,90	2,06	0,77	3,08	0,98	2,66	2,59	4,74	5,04	5,04	2,08	7,07	0,00	8,04
22	1,00	1,10	0,89	2,16	0,76	3,23	0,98	2,77	2,72	4,94	5,24	5,24	2,19	7,41	0,00	8,42
23	1,00	1,15	0,88	2,25	0,75	3,37	0,98	2,88	2,83	5,14	5,44	5,44	2,30	7,75	0,00	8,80
24	1,00	1,20	0,88	2,35	0,74	3,52	0,98	2,99	2,94	5,34	5,64	5,64	2,41	8,08	0,00	9,18
25	1,00	1,25	0,87	2,44	0,73	3,67	0,98	3,10	3,05	5,54	5,84	5,84	2,52	8,42	0,00	9,57
26	1,00	1,30	0,87	2,54	0,72	3,81	0,98	3,21	3,16	5,74	6,04	6,04	2,63	8,76	0,00	9,95
27	1,00	1,35	0,86	2,64	0,71	3,96	0,98	3,32	3,27	5,94	6,24	6,24	2,74	9,10	0,00	10,33
28	1,00	1,40	0,86	2,74	0,70	4,11	0,98	3,43	3,38	6,14	6,44	6,44	2,85	9,44	0,00	10,72
29	1,00	1,45	0,85	2,84	0,69	4,25	0,98	3,54	3,49	6,34	6,64	6,64	2,96	9,78	0,00	11,10
30	1,00	1,50	0,85	2,94	0,68	4,40	0,98	3,65	3,60	6,54	6,84	6,84	3,07	10,11	0,00	11,48
31	1,00	1,55	0,84	3,04	0,66	4,55	0,98	3,76	3,71	6,74	7,04	7,04	3,18	10,45	0,00	11,86
32	1,00	1,60	0,84	3,14	0,65	4,70	0,98	3,87	3,82	6,94	7,24	7,24	3,29	10,79	0,00	12,25
33	1,00	1,65	0,83	3,24	0,64	4,84	0,98	3,98	3,93	7,14	7,44	7,44	3,40	11,12	0,00	12,63
34	1,00	1,70	0,83	3,34	0,63	4,99	0,98	4,09	4,04	7,34	7,64	7,64	3,51	11,45	0,00	13,01
35	1,00	1,75	0,82	3,44	0,62	5,14	0,98	4,20	4,15	7,54	7,84	7,84	3,62	11,78	0,00	13,39
36	1,00	1,80	0,82	3,54	0,61	5,28	0,98	4,31	4,26	7,74	8,04	8,04	3,73	12,11	0,00	13,77
37	1,00	1,85	0,81	3,64	0,60	5,43	0,98	4,42	4,37	7,94	8,24	8,24	3,84	12,44	0,00	14,15
38	1,00	1,90	0,81	3,74	0,59	5,58	0,98	4,53	4,48	8,14	8,44	8,44	3,95	12,77	0,00	14,53
39	1,00	1,95	0,80	3,84	0,58	5,72	0,98	4,64	4,59	8,34	8,64	8,64	4,06	13,10	0,00	14,91
40	1,00	2,00	0,80	3,94	0,57	5,87	0,98	4,75	4,69	8,54	8,84	8,84	4,17	13,43	0,00	15,29
41	1,00	2,05	0,79	4,04	0,56	6,02	0,98	4,86	4,79	8,74	9,04	9,04	4,28	13,76	0,00	15,67
42	1,00	2,10	0,79	4,14	0,55	6,16	0,98	4,97	4,89	8,94	9,24	9,24	4,39	14,09	0,00	16,05
43	1,00	2,15	0,78	4,24	0,54	6,31	0,98	5,08	5,03	9,14	9,44	9,44	4,50	14,42	0,00	16,43
44	1,00	2,20	0,78	4,34	0,53	6,46	0,98	5,19	5,14	9,34	9,64	9,64	4,61	14,75	0,00	16,81
45	1,00	2,25	0,77	4,44	0,52	6,60	0,98	5,30	5,25	9,54	9,84	9,84	4,72	15,08	0,00	17,19
46	1,00	2,30	0,77	4,54	0,51	6,75	0,98	5,41	5,36	9,74	10,04	10,04	4,83	15,41	0,00	17,57
47	1,00	2,35	0,76	4,64	0,50	6,90	0,98	5,52	5,47	9,94	10,24	10,24	4,94	15,74	0,00	17,95
48	1,00	2,40	0,76	4,74	0,49	7,04	0,98	5,63	5,58	10,14	10,44	10,44	5,05	16,07	0,00	18,33
49	1,00	2,45	0,75	4,84	0,48	7,19	0,98	5,74	5,69	10,34	10,64	10,64	5,16	16,40	0,00	18,71
50	1,00	2,50	0,75	4,94	0,47	7,34	0,98	5,85	5,80	10,54	10,84	10,84	5,27	16,73	0,00	19,09
51	1,00	2,55	0,74	5,04	0,46	7,48	0,98	5,96	5,91	10,74	11,04	11,04	5,38	17,06	0,00	19,47
52	1,00	2,60	0,74	5,14	0,45	7,63	0,98	6,07	6,02	10,94	11,24	11,24	5,49	17,39	0,00	19,85
53	1,00	2,65	0,73	5,24	0,44	7,78	0,98	6,18	6,13	11,14	11,44	11,44	5,60	17,72	0,00	20,23
54	1,00	2,70	0,73	5,34	0,43	7,92	0,98	6,29	6,24	11,34	11,64	11,64	5,71	18,05	0,00	20,61
55	1,00	2,75	0,72	5,44	0,42	8,07	0,98	6,40	6,35	11,54	11,84	11,84	5,82	18,38	0,00	20,99
56	1,00	2,80	0,72	5,54	0,41	8,22	0,98	6,51	6,46	11,74	12,04	12,04	5,93	18,71	0,00	21,37
57	1,00	2,85	0,71	5,64	0,40	8,36	0,98	6,62	6,57	11,94	12,24	12,24	6,04	19,04	0,00	21,75
58	1,00	2,90	0,71	5,74	0,39	8,51	0,98	6,73	6,68	12,14	12,44	12,44	6,15	19,37	0,00	22,13
59	1,00	2,95	0,70	5,84	0,38	8,66	0,98	6,84	6,79	12,34	12,64	12,64	6,26	19,70	0,00	22,51
60	1,00	3,00	0,70	5,94	0,37	8,80	0,98	6,95	6,90	12,54	12,84	12,84	6,37	20,03	0,00	22,89

TRAVERSE TABLE. 66.

83

Distance	0 Points			1 Point			2 Points			Distance							
	$\frac{1}{4}$			$\frac{1}{2}$			$\frac{3}{4}$										
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.									
61	60.93	2.99	60.71	5.98	60.34	8.95	59.83	11.90	59.17	14.82	58.37	17.71	57.43	20.55	56.36	23.34	61
62	61.93	3.04	61.70	6.08	61.33	9.10	60.81	12.09	60.14	15.06	59.33	18.00	58.38	20.89	57.28	23.73	62
63	62.92	3.09	62.70	6.17	62.32	9.24	61.79	12.29	61.13	15.31	60.29	18.29	59.33	21.22	58.21	24.11	63
64	63.92	3.14	63.69	6.27	63.31	9.39	62.77	12.48	62.08	15.55	61.24	18.58	60.26	21.56	59.13	24.49	64
65	64.92	3.19	64.69	6.37	64.30	9.54	63.75	12.68	63.05	15.79	62.20	18.87	61.10	21.90	60.05	24.87	65
66	65.92	3.24	65.68	6.47	65.28	9.68	64.73	12.88	64.02	16.04	63.16	19.16	62.14	22.23	60.98	25.26	66
67	66.92	3.29	66.68	6.57	66.27	9.83	65.71	13.07	64.99	16.28	64.11	19.45	63.08	22.57	61.90	25.64	67
68	67.92	3.34	67.67	6.66	67.26	9.98	66.69	13.27	65.96	16.52	65.07	19.74	64.02	22.91	62.82	26.02	68
69	68.92	3.39	68.67	6.76	68.25	10.12	67.67	13.46	66.93	16.77	66.03	20.03	64.97	23.24	63.75	26.40	69
70	69.92	3.44	69.66	6.86	69.24	10.27	68.66	13.66	67.90	17.01	66.99	20.32	65.91	23.58	64.67	26.79	70
71	70.92	3.48	70.66	6.96	70.23	10.42	69.64	13.85	68.87	17.25	67.94	20.61	66.85	23.92	65.60	27.17	71
72	71.91	3.53	71.65	7.06	71.22	10.56	70.62	14.05	69.84	17.49	68.90	20.90	67.79	24.26	66.52	27.55	72
73	72.91	3.58	72.65	7.15	72.21	10.71	71.60	14.24	70.81	17.74	69.86	21.19	68.73	24.59	67.44	27.94	73
74	73.91	3.63	73.64	7.25	73.20	10.86	72.58	14.44	71.78	17.98	70.81	21.48	69.67	24.93	68.37	28.32	74
75	74.91	3.68	74.64	7.35	74.19	11.00	73.56	14.63	72.75	18.22	71.77	21.77	70.62	25.27	69.29	28.70	75
76	75.91	3.73	75.63	7.45	75.18	11.15	74.54	14.83	73.72	18.47	72.73	22.06	71.56	25.60	70.21	29.08	76
77	76.91	3.78	76.63	7.55	76.17	11.30	75.52	15.02	74.69	18.71	73.68	22.35	72.50	25.94	71.14	29.47	77
78	77.91	3.83	77.62	7.64	77.15	11.45	76.50	15.22	75.66	18.95	74.64	22.64	73.44	26.28	72.06	29.85	78
79	78.91	3.88	78.62	7.74	78.14	11.59	77.48	15.41	76.63	19.20	75.60	22.93	74.38	26.61	72.99	30.23	79
80	79.90	3.93	79.61	7.84	79.13	11.74	78.46	15.61	77.60	19.44	76.55	23.22	75.32	26.95	73.91	30.61	80
81	80.90	3.98	80.61	7.94	80.12	11.88	79.44	15.80	78.57	19.68	77.51	23.51	76.27	27.29	74.83	31.00	81
82	81.90	4.02	81.60	8.04	81.11	12.03	80.43	16.00	79.54	19.92	78.47	23.80	77.21	27.62	75.76	31.38	82
83	82.90	4.07	82.60	8.14	82.10	12.18	81.41	16.19	80.51	20.17	79.43	24.09	78.15	27.96	76.68	31.76	83
84	83.90	4.12	83.59	8.23	83.09	12.32	82.39	16.39	81.48	20.41	80.38	24.38	79.09	28.30	77.61	32.14	84
85	84.90	4.17	84.59	8.33	84.08	12.47	83.37	16.58	82.45	20.65	81.34	24.67	80.03	28.63	78.53	32.53	85
86	85.90	4.22	85.58	8.43	85.07	12.62	84.35	16.78	83.42	20.90	82.30	24.96	80.97	28.97	79.45	32.91	86
87	86.90	4.27	86.58	8.53	86.06	12.76	85.33	16.97	84.39	21.14	83.25	25.25	81.91	29.31	80.38	33.29	87
88	87.89	4.32	87.58	8.63	87.05	12.91	86.31	17.17	85.36	21.38	84.21	25.54	82.86	29.65	81.30	33.67	88
89	88.89	4.37	88.57	8.72	88.04	13.06	87.29	17.36	86.33	21.62	85.17	25.83	83.80	29.98	82.23	34.06	89
90	89.89	4.42	89.57	8.82	89.02	13.21	88.27	17.56	87.30	21.87	86.12	26.12	84.74	30.32	83.15	34.44	90
91	90.89	4.47	90.56	8.92	90.01	13.35	89.25	17.75	88.27	22.11	87.08	26.41	85.68	30.66	84.07	34.82	91
92	91.89	4.52	91.56	9.02	91.00	13.50	90.23	17.95	89.24	22.35	88.04	26.70	86.62	30.99	85.00	35.21	92
93	92.89	4.56	92.55	9.12	91.99	13.65	91.21	18.14	90.21	22.60	89.00	27.00	87.56	31.35	85.92	35.59	93
94	93.89	4.61	93.55	9.21	92.98	13.79	92.19	18.34	91.18	22.84	89.95	27.29	88.51	31.67	86.85	35.97	94
95	94.89	4.66	94.54	9.31	93.97	13.94	93.18	18.54	92.15	23.08	90.91	27.58	89.45	32.00	87.77	36.35	95
96	95.88	4.71	95.54	9.41	94.96	14.09	94.16	18.73	93.12	23.33	91.87	27.87	90.39	32.34	88.69	36.74	96
97	96.88	4.76	96.53	9.51	95.95	14.23	95.14	18.93	94.09	23.57	92.82	28.16	91.33	32.68	89.62	37.12	97
98	97.88	4.81	97.53	9.61	96.94	14.38	96.13	19.12	95.06	23.81	93.78	28.45	92.27	33.01	90.54	37.50	98
99	98.88	4.86	98.52	9.70	97.93	14.55	97.10	19.31	96.03	24.05	94.74	28.74	93.21	33.35	91.46	37.88	99
100	99.88	4.91	99.52	9.80	98.92	14.67	98.08	19.51	97.00	24.30	95.69	29.03	94.15	33.69	92.39	38.27	100
101	100.9	4.96	100.5	9.90	99.91	14.82	99.06	19.70	97.97	24.54	96.65	29.32	95.10	34.03	93.31	38.65	101
102	101.9	5.01	101.5	10.00	100.9	14.97	100.0	19.90	98.94	24.78	97.61	29.61	96.04	34.36	94.24	39.03	102
103	102.9	5.05	102.5	10.10	101.9	15.11	101.0	20.09	99.91	25.03	98.56	29.90	96.98	34.70	95.16	39.42	103
104	103.9	5.10	103.5	10.19	102.9	15.26	102.0	20.28	100.9	25.27	99.52	30.19	97.92	35.04	96.08	39.80	104
105	104.9	5.15	104.5	10.29	103.9	15.41	103.0	20.48	101.9	25.51	100.5	30.48	98.86	35.37	97.01	40.18	105
106	105.9	5.20	105.5	10.39	104.9	15.55	104.0	20.68	102.8	25.76	101.4	30.77	99.80	35.71	97.93	40.56	106
107	106.9	5.25	106.5	10.49	105.8	15.70	104.9	20.88	103.8	26.00	102.4	31.06	100.7	36.05	98.86	40.95	107
108	107.9	5.30	107.5	10.59	106.8	15.89	105.9	21.07	104.8	26.24	103.4	31.35	101.7	36.38	99.78	41.33	108
109	108.9	5.35	108.5	10.68	107.8	15.99	106.9	21.27	105.7	26.48	104.3	31.64	102.6	36.72	100.7	41.71	109
110	109.9	5.40	109.5	10.78	108.8	16.14	107.9	21.46	106.7	26.73	105.3	31.93	103.6	37.06	101.6	42.09	110
111	110.9	5.45	110.5	10.88	109.8	16.29	108.9	21.66	107.7	26.97	106.2	32.22	104.5	37.39	102.6	42.48	111
112	111.9	5.50	111.5	10.98	110.8	16.43	109.8	21.85	108.6	27.21	107.2	32.51	105.5	37.73	103.5	42.86	112
113	112.9	5.55	112.5	11.08	111.8	16.58	110.8	22.05	109.6	27.46	108.1	32.80	106.4	38.07	104.4	43.24	113
114	113.9	5.59	113.5	11.17	112.8	16.73	111.8	22.24	110.6	27.70	109.1	33.09	107.3	38.44	105.3	43.63	114
115	114.9	5.64	114.4	11.27	113.8	16.87	112.8	22.44	111.6	27.94	110.0	33.38	108.3	38.74	106.2	44.01	115
116	115.9	5.69	115.4	11.37	114.7	17.02	113.8	22.63	112.5	28.19	111.0	33.67	109.2	39.08	107.2	44.39	116
117	116.9	5.74	116.4	11.47	115.7	17.17	114.8	22.83	113.5	28.43	112.0	33.96	110.2	39.42	108.1	44.77	117
118	117.9	5.79	117.4	11.57	116.7	17.31	115.7	23.02	114.5	28.67	112.9	34.25	111.1	39.75	109.0	45.16	118
119	118.9	5.84	118.4	11.66	117.7	17.46	116.7	23.22	115.4	28.91	113.9	34.54	112.0	40.09	109.9	45.54	119
120	119.9	5.89	119.4	11.76	118.7	17.61	117.7	23.41	116.4	29.16	114.8	34.83	113.0	40.43	110.9	45.92	120
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance
	$\frac{3}{4}$		$\frac{1}{2}$		$\frac{1}{4}$		0		$\frac{1}{4}$		$\frac{1}{2}$		$\frac{3}{4}$		0		Distance

G 2

Distance	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	Distance
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TRAVERSE TABLE. 66.

Distance	2 Points						3 Points						4 Points					
	$\frac{1}{2}$			$\frac{1}{4}$			O			$\frac{1}{4}$			$\frac{1}{2}$			$\frac{3}{4}$		
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Distance
1	0.90	0.43	0.88	0.86	0.51	0.83	0.56	0.80	0.60	0.77	0.63	0.74	0.67	0.71	0.71	0.71	0.71	1
2	1.81	0.85	1.76	1.71	1.03	1.66	1.11	1.61	1.19	1.55	1.27	1.48	1.34	1.41	1.41	1.41	1.41	2
3	2.71	1.28	2.65	2.57	1.54	2.49	1.67	2.41	1.79	2.32	1.90	2.22	2.01	2.12	2.12	2.12	2.12	3
4	3.62	1.71	3.53	3.43	2.06	3.33	2.22	3.21	2.38	3.19	2.54	2.96	2.69	2.83	2.83	2.83	2.83	4
5	4.52	2.14	4.41	4.31	2.57	4.16	2.78	4.02	2.98	3.86	3.17	3.70	3.36	3.53	3.53	3.53	3.53	5
6	5.42	2.56	5.29	5.19	3.08	4.99	3.33	4.82	3.57	4.64	3.81	4.45	4.03	4.24	4.24	4.24	4.24	6
7	6.33	2.99	6.17	6.00	3.60	5.82	3.89	5.62	4.17	5.41	4.44	5.19	4.70	4.95	4.95	4.95	4.95	7
8	7.23	3.42	7.05	6.86	4.11	6.65	4.44	6.43	4.77	6.18	5.07	5.93	5.37	5.66	5.66	5.66	5.66	8
9	8.14	3.85	7.94	7.72	4.63	7.48	5.00	7.23	5.36	6.96	5.71	6.67	6.04	6.36	6.36	6.36	6.36	9
10	9.04	4.28	8.82	8.58	5.14	8.32	5.56	8.03	5.96	7.73	6.34	7.41	6.72	7.07	7.07	7.07	7.07	10
11	9.94	4.70	9.70	9.43	5.65	9.15	6.11	8.83	6.55	8.59	6.98	8.15	7.39	7.78	7.78	7.78	7.78	11
12	10.85	5.13	10.58	10.29	6.17	9.98	6.67	9.64	7.15	9.28	7.61	8.89	8.06	8.49	8.49	8.49	8.49	12
13	11.75	5.56	11.46	11.15	6.68	10.81	7.22	10.44	7.74	10.05	8.25	9.61	8.73	9.19	9.19	9.19	9.19	13
14	12.66	5.99	12.35	12.01	7.20	11.64	7.78	11.24	8.33	10.82	8.88	10.37	9.40	9.90	9.90	9.90	9.90	14
15	13.56	6.41	13.23	12.87	7.71	12.47	8.39	12.05	8.93	11.59	9.51	11.11	10.07	10.61	10.61	10.61	10.61	15
16	14.46	6.84	14.11	13.73	8.22	13.30	8.99	12.85	9.52	12.37	10.15	11.85	10.75	11.31	11.31	11.31	11.31	16
17	15.37	7.27	14.99	14.58	8.74	14.13	9.45	13.65	10.12	13.14	10.78	12.60	11.42	12.02	12.02	12.02	12.02	17
18	16.27	7.70	15.87	15.44	9.25	14.96	10.00	14.46	10.71	13.91	11.42	13.34	12.09	12.73	12.73	12.73	12.73	18
19	17.18	8.13	16.76	16.30	9.77	15.80	10.56	15.26	11.31	14.69	12.05	14.08	12.76	13.43	13.43	13.43	13.43	19
20	18.08	8.55	17.64	17.15	10.28	16.63	11.12	16.06	11.91	15.48	12.99	14.82	13.43	14.14	14.14	14.14	14.14	20
21	18.98	8.98	18.52	18.01	10.80	17.46	11.67	16.87	12.51	16.21	13.32	15.56	14.10	14.85	14.85	14.85	14.85	21
22	19.89	9.41	19.40	18.87	11.31	18.29	12.22	17.67	13.11	17.01	13.95	16.30	14.77	15.56	15.56	15.56	15.56	22
23	20.79	9.83	20.28	19.73	11.81	19.12	12.78	18.47	13.70	17.78	14.59	17.04	15.45	16.26	16.26	16.26	16.26	23
24	21.69	10.26	21.17	20.59	12.34	19.95	13.34	19.28	14.30	18.55	15.22	17.78	16.12	16.97	16.97	16.97	16.97	24
25	22.60	10.69	22.05	21.44	12.85	20.78	13.89	20.08	14.89	19.33	15.86	18.52	16.79	17.68	17.68	17.68	17.68	25
26	23.50	11.12	22.93	22.30	13.37	21.62	14.45	20.88	15.49	20.10	16.49	19.26	17.46	18.38	18.38	18.38	18.38	26
27	24.41	11.55	23.81	23.16	13.88	22.45	15.00	21.68	16.09	20.87	17.12	20.01	18.13	19.09	19.09	19.09	19.09	27
28	25.31	11.97	24.69	24.02	14.39	23.28	15.56	22.49	16.68	21.64	17.76	20.75	18.81	19.80	19.80	19.80	19.80	28
29	26.22	12.40	25.58	24.88	14.91	24.11	16.12	23.29	17.28	22.42	18.39	21.49	19.48	20.51	20.51	20.51	20.51	29
30	27.12	12.81	26.46	25.73	15.42	24.94	16.67	24.10	17.87	23.19	19.03	22.23	20.15	21.21	21.21	21.21	21.21	30
31	28.02	13.25	27.34	26.59	15.94	25.78	17.22	24.90	18.46	23.96	19.67	22.97	20.82	21.92	21.92	21.92	21.92	31
32	28.93	13.68	28.23	27.45	16.45	26.61	17.78	25.70	19.06	24.74	20.30	23.71	21.49	22.63	22.63	22.63	22.63	32
33	29.83	14.11	29.11	28.31	16.96	27.44	18.54	26.51	19.66	25.51	20.93	24.45	22.10	23.33	23.33	23.33	23.33	33
34	30.73	14.54	29.99	29.16	17.48	28.27	19.59	27.31	20.25	26.28	21.57	25.19	22.83	24.04	24.04	24.04	24.04	34
35	31.64	14.97	30.87	30.02	17.99	29.10	19.45	28.11	20.85	27.05	22.21	25.93	23.51	24.75	24.75	24.75	24.75	35
36	32.54	15.39	31.76	30.88	18.51	29.93	20.00	28.91	21.45	27.83	22.84	26.67	24.18	25.46	25.46	25.46	25.46	36
37	33.45	15.82	32.64	31.74	19.02	30.76	20.56	29.72	22.04	28.60	23.47	27.42	24.85	26.16	26.16	26.16	26.16	37
38	34.35	16.25	33.52	32.60	19.53	31.59	21.12	30.52	22.64	29.37	24.10	28.16	25.52	26.87	26.87	26.87	26.87	38
39	35.26	16.68	34.40	33.45	20.05	32.42	21.67	31.32	23.23	30.15	24.74	28.90	26.19	27.58	27.58	27.58	27.58	39
40	36.16	17.10	35.28	34.31	20.56	33.26	22.22	32.13	23.83	30.92	25.38	29.64	26.86	28.18	28.18	28.18	28.18	40
41	37.06	17.53	36.16	35.17	21.08	34.09	22.78	32.93	24.42	31.69	26.01	30.38	27.53	28.99	28.99	28.99	28.99	41
42	37.97	17.96	37.04	36.03	21.59	34.92	23.34	33.73	25.02	32.47	26.64	31.12	28.21	29.70	29.70	29.70	29.70	42
43	38.87	18.39	37.92	36.88	22.10	35.75	23.89	34.54	25.62	33.24	27.28	31.86	28.88	30.41	30.41	30.41	30.41	43
44	39.77	18.81	38.80	37.74	22.62	36.58	24.45	35.34	26.21	34.01	27.91	32.60	29.55	31.11	31.11	31.11	31.11	44
45	40.68	19.24	39.69	38.60	23.13	37.41	25.00	36.14	26.82	34.78	28.55	33.34	30.22	31.82	31.82	31.82	31.82	45
46	41.58	19.67	40.57	39.46	23.65	38.24	25.56	36.95	27.41	35.56	29.18	34.08	30.89	32.53	32.53	32.53	32.53	46
47	42.49	20.10	41.45	40.32	24.16	39.08	26.12	37.75	28.01	36.33	29.81	34.82	31.57	33.13	33.13	33.13	33.13	47
48	43.39	20.53	42.33	41.17	24.68	39.91	26.67	38.55	28.61	37.10	30.45	35.57	32.24	33.94	33.94	33.94	33.94	48
49	44.29	20.95	43.21	42.03	25.19	40.74	27.23	39.36	29.20	37.88	31.08	36.31	32.91	34.65	34.65	34.65	34.65	49
50	45.20	21.38	44.10	42.86	25.70	41.57	27.78	40.16	29.79	38.65	31.72	37.05	33.58	35.35	35.35	35.35	35.35	50
51	46.10	21.81	44.98	43.74	26.22	42.40	28.34	40.96	30.38	39.42	32.35	37.79	34.25	36.06	36.06	36.06	36.06	51
52	47.01	22.23	45.86	44.60	26.73	43.23	28.89	41.77	30.98	40.20	32.99	38.53	34.92	36.77	36.77	36.77	36.77	52
53	47.91	22.66	46.74	45.48	27.25	44.07	29.45	42.57	31.57	40.97	33.62	39.27	35.59	37.47	37.47	37.47	37.47	53
54	48.81	23.09	47.62	46.32	27.76	44.90	30.00	43.37	32.17	41.74	34.26	40.01	36.27	38.18	38.18	38.18	38.18	54
55	49.72	23.52	48.51	47.18	28.27	45.73	30.56	44.18	32.76	42.52	34.89	40.75	36.94	38.89	38.89	38.89	38.89	55
56	50.62	23.95	49.39	48.03	28.79	46.56	31.12	44.98	33.36	43.29	35.52	41.49	37.61	39.60	39.60	39.60	39.60	56
57	51.53	24.37	50.27	48.89	29.30	47.39	31.67	45.78	33.96	44.08	36.16	42.23	38.28	40.30	40.30	40.30	40.30	57
58	52.43	24.80	51.15	49.75	29.82	48.22	32.21	46.58	34.55	44.83	36.79	42.98	38.95	41.01	41.01	41.01	41.01	58
59	53.33	25.22	52.03	50.61	30.33	49.05	32.78	47.39	35.15	45.61	37.43	43.72	39.63	41.72	41.72	41.72	41.72	59
60	54.24	25.65	52.92	51.49	30.85	49.89	33.33	48.19	35.74	46.38	38.06	44.46	40.29	42.43	42.43	42.43	42.43	60

TRAVERSE TABLE. 66.

Distance	2 Points				3 Points				4 Points				Distance
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
61	55.14	26.08	53.80	28.75	52.32	31.36	50.72	33.89	49.00	36.34	47.15	38.70	61
62	55.05	26.51	54.68	29.22	53.28	31.87	51.55	34.45	49.80	36.93	47.93	39.13	62
63	54.95	26.94	55.56	29.70	54.04	32.39	52.38	35.00	50.60	37.53	48.70	39.66	63
64	54.85	27.37	56.44	30.17	54.90	32.90	53.21	35.56	51.40	38.13	49.47	40.00	64
65	54.75	27.79	57.33	30.64	55.75	33.42	54.04	36.11	52.21	38.72	50.25	41.24	65
66	54.65	28.22	58.21	31.11	56.61	33.93	54.87	36.67	53.01	39.32	51.02	41.87	66
67	54.55	28.65	59.09	31.58	57.47	34.44	55.71	37.23	53.81	39.91	51.79	42.50	67
68	54.45	29.07	59.97	32.05	58.33	34.96	56.54	37.78	54.62	40.51	52.56	43.13	68
69	54.35	29.50	60.85	32.52	59.19	35.47	57.37	38.34	55.42	41.11	53.34	43.77	69
70	54.25	29.93	61.73	33.00	60.04	35.99	58.20	38.89	56.22	41.70	54.11	44.41	70
71	54.15	30.36	62.62	33.47	60.90	36.50	59.03	39.45	57.03	42.29	54.88	45.04	71
72	54.05	30.78	63.50	33.94	61.76	37.01	59.87	40.00	57.83	42.89	55.66	45.67	72
73	53.95	31.21	64.38	34.41	62.62	37.53	60.70	40.56	58.63	43.49	56.43	46.31	73
74	53.85	31.64	65.26	34.88	63.47	38.04	61.53	41.11	59.44	44.08	57.20	46.94	74
75	53.75	32.07	66.14	35.35	64.33	38.56	62.37	41.67	60.24	44.68	57.98	47.58	75
76	53.65	32.50	67.03	35.82	65.19	39.07	63.21	42.22	61.04	45.27	58.75	48.21	76
77	53.55	32.92	67.91	36.29	66.05	39.58	64.04	42.78	61.85	45.87	59.52	48.84	77
78	53.45	33.35	68.79	36.76	66.91	40.10	64.87	43.34	62.65	46.47	60.29	49.48	78
79	53.35	33.78	69.67	37.24	67.76	40.61	65.70	43.89	63.45	47.06	61.07	50.11	79
80	53.25	34.20	70.55	37.71	68.62	41.13	66.52	44.45	64.26	47.66	61.84	50.75	80
81	53.15	34.63	71.44	38.18	69.48	41.64	67.35	45.00	65.06	48.25	62.61	51.38	81
82	53.05	35.06	72.32	38.65	70.33	42.16	68.18	45.56	65.86	48.85	63.39	52.02	82
83	52.95	35.49	73.20	39.12	71.19	42.67	69.01	46.11	66.67	49.44	64.16	52.65	83
84	52.85	35.92	74.08	39.59	72.05	43.18	69.84	46.67	67.47	50.04	64.93	53.29	84
85	52.75	36.34	74.96	40.07	72.91	43.70	70.67	47.23	68.27	50.64	65.71	53.92	85
86	52.65	36.77	75.85	40.54	73.77	44.21	71.50	47.78	69.08	51.24	66.48	54.55	86
87	52.55	37.20	76.73	41.01	74.62	44.73	72.34	48.34	69.88	51.83	67.25	55.19	87
88	52.45	37.63	77.61	41.48	75.48	45.24	73.17	48.89	70.68	52.42	68.02	55.82	88
89	52.35	38.06	78.49	41.95	76.34	45.75	74.00	49.45	71.48	53.02	68.80	56.46	89
90	52.25	38.49	79.37	42.42	77.20	46.27	74.81	50.00	72.29	53.61	69.57	57.09	90
91	52.15	38.92	80.26	42.90	78.05	46.78	75.66	50.56	73.09	54.21	70.34	57.73	91
92	52.05	39.35	81.14	43.37	78.91	47.30	76.49	51.11	73.90	54.80	71.12	58.36	92
93	51.95	39.78	82.02	43.84	79.77	47.81	77.33	51.67	74.70	55.40	71.89	59.00	93
94	51.85	40.20	82.90	44.31	80.63	48.32	78.16	52.22	75.50	56.00	72.66	59.63	94
95	51.75	40.63	83.78	44.78	81.49	48.84	79.00	52.78	76.30	56.59	73.44	60.26	95
96	51.65	41.06	84.67	45.25	82.34	49.35	79.83	53.34	77.11	57.19	74.21	60.90	96
97	51.55	41.49	85.55	45.72	83.20	49.87	80.66	53.89	77.91	57.78	74.98	61.53	97
98	51.45	41.91	86.43	46.19	84.06	50.38	81.49	54.45	78.71	58.38	75.75	62.17	98
99	51.35	42.34	87.31	46.66	84.92	50.89	82.32	55.00	79.52	58.98	76.53	62.80	99
100	51.25	42.76	88.19	47.14	85.77	51.41	83.15	55.46	80.32	59.57	77.30	63.44	100
101	51.15	43.18	89.07	47.61	86.63	51.92	83.98	56.11	81.12	60.17	78.07	64.07	101
102	51.05	43.61	89.96	48.08	87.49	52.44	84.81	56.67	81.93	60.76	78.85	64.71	102
103	50.95	44.04	90.84	48.54	88.35	52.95	85.64	57.22	82.73	61.36	79.62	65.34	103
104	50.85	44.47	91.72	49.02	89.21	53.47	86.47	57.78	83.53	61.95	80.39	65.97	104
105	50.75	44.90	92.60	49.49	90.06	54.00	87.31	58.34	84.34	62.55	81.17	66.61	105
106	50.65	45.33	93.48	49.96	90.92	54.49	88.14	58.89	85.14	63.15	81.94	67.24	106
107	50.55	45.75	94.37	50.44	91.78	55.01	88.98	59.45	85.94	63.74	82.71	67.88	107
108	50.45	46.18	95.25	50.91	92.64	55.52	89.80	60.00	86.75	64.34	83.48	68.51	108
109	50.35	46.61	96.13	51.38	93.50	56.04	90.63	60.56	87.55	64.93	84.26	69.15	109
110	50.25	47.03	97.01	51.85	94.35	56.55	91.46	61.11	88.35	65.53	85.03	69.78	110
111	50.15	47.46	97.89	52.32	95.21	57.06	92.29	61.67	89.16	66.12	85.80	70.42	111
112	50.05	47.89	98.78	52.79	96.07	57.58	93.12	62.22	89.96	66.72	86.58	71.05	112
113	49.95	48.31	99.66	53.27	96.92	58.09	93.96	62.78	90.76	67.31	87.35	71.68	113
114	49.85	48.74	100.55	53.74	97.78	58.61	94.79	63.34	91.57	67.91	88.12	72.32	114
115	49.75	49.17	101.44	54.21	98.64	59.12	95.62	63.89	92.37	68.51	88.90	72.95	115
116	49.65	49.60	102.33	54.68	99.50	59.63	96.45	64.45	93.17	69.10	89.67	73.58	116
117	49.55	50.03	103.22	55.15	100.40	60.15	97.28	65.00	93.97	69.70	90.44	74.22	117
118	49.45	50.46	104.11	55.62	101.29	60.66	98.11	65.55	94.78	70.29	91.21	74.85	118
119	49.35	50.88	105.00	56.09	102.18	61.18	98.94	66.11	95.58	70.89	91.99	75.49	119
120	49.25	51.31	105.89	56.57	103.07	61.69	99.78	66.67	96.39	71.48	92.76	76.13	120

Distance

Distance

TRAVERSE TABLE. 67.

0 Degrees										1 Degree										88 Degrees									
Distance	15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		Distance	15 Min.		30 Min.		45 Min.		Distance	15 Min.		30 Min.		45 Min.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.		Lat.	Dep.	Lat.	Dep.	Lat.	Dep.		Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	1,00	0,00	1,00	0,01	1,00	0,31	1,00	0,02	1,00	0,02	1,00	0,03	1,00	0,03	1	1,00	0,02	1,00	0,02	1,00	0,02	1	1,00	0,02	1,00	0,03	1,00	0,03	
2	2,00	0,01	2,00	0,02	2,00	0,03	2,00	0,04	2,00	0,04	2,00	0,05	2,00	0,05	2	2,00	0,04	2,00	0,04	2,00	0,04	2	2,00	0,05	2,00	0,06	2,00	0,06	
3	3,00	0,01	3,00	0,03	3,00	0,04	3,00	0,05	3,00	0,05	3,00	0,07	3,00	0,07	3	3,00	0,05	3,00	0,05	3,00	0,07	3	3,00	0,08	3,00	0,09	3,00	0,09	
4	4,00	0,02	4,00	0,03	4,00	0,05	4,00	0,07	4,00	0,07	4,00	0,09	4,00	0,09	4	4,00	0,07	4,00	0,07	4,00	0,09	4	4,00	0,10	4,00	0,12	4,00	0,12	
5	5,00	0,02	5,00	0,04	5,00	0,07	5,00	0,09	5,00	0,09	5,00	0,11	5,00	0,11	5	5,00	0,09	5,00	0,09	5,00	0,11	5	5,00	0,13	5,00	0,15	5,00	0,15	
6	6,00	0,03	6,00	0,05	6,00	0,08	6,00	0,10	6,00	0,10	6,00	0,13	6,00	0,13	6	6,00	0,11	6,00	0,11	6,00	0,13	6	6,00	0,16	6,00	0,18	6,00	0,18	
7	7,00	0,03	7,00	0,06	7,00	0,09	7,00	0,12	7,00	0,12	7,00	0,15	7,00	0,15	7	7,00	0,12	7,00	0,12	7,00	0,15	7	7,00	0,18	7,00	0,21	7,00	0,21	
8	8,00	0,04	8,00	0,07	8,00	0,10	8,00	0,14	8,00	0,14	8,00	0,18	8,00	0,18	8	8,00	0,14	8,00	0,14	8,00	0,18	8	8,00	0,21	8,00	0,25	8,00	0,25	
9	9,00	0,04	9,00	0,08	9,00	0,12	9,00	0,16	9,00	0,16	9,00	0,20	9,00	0,20	9	9,00	0,16	9,00	0,16	9,00	0,20	9	9,00	0,24	9,00	0,28	9,00	0,28	
10	10,00	0,04	10,00	0,09	10,00	0,13	10,00	0,18	10,00	0,18	10,00	0,22	10,00	0,22	10	10,00	0,18	10,00	0,18	10,00	0,22	10	10,00	0,26	10,00	0,30	10,00	0,30	
11	11,00	0,05	11,00	0,10	11,00	0,14	11,00	0,19	11,00	0,19	11,00	0,24	11,00	0,24	11	11,00	0,19	11,00	0,19	11,00	0,24	11	11,00	0,28	11,00	0,32	11,00	0,32	
12	12,00	0,05	12,00	0,11	12,00	0,16	12,00	0,21	12,00	0,21	12,00	0,26	12,00	0,26	12	12,00	0,21	12,00	0,21	12,00	0,26	12	12,00	0,31	12,00	0,35	12,00	0,35	
13	13,00	0,06	13,00	0,12	13,00	0,18	13,00	0,25	13,00	0,25	13,00	0,31	13,00	0,31	13	13,00	0,23	13,00	0,23	13,00	0,28	13	13,00	0,34	13,00	0,38	13,00	0,38	
14	14,00	0,06	14,00	0,12	14,00	0,20	14,00	0,28	14,00	0,28	14,00	0,35	14,00	0,35	14	14,00	0,25	14,00	0,25	14,00	0,31	14	14,00	0,37	14,00	0,41	14,00	0,41	
15	15,00	0,07	15,00	0,13	15,00	0,20	15,00	0,29	15,00	0,29	15,00	0,37	15,00	0,37	15	15,00	0,26	15,00	0,26	15,00	0,33	15	15,00	0,39	15,00	0,43	15,00	0,43	
16	16,00	0,07	16,00	0,14	16,00	0,21	16,00	0,31	16,00	0,31	16,00	0,40	16,00	0,40	16	16,00	0,28	16,00	0,28	16,00	0,35	16	16,00	0,42	16,00	0,46	16,00	0,46	
17	17,00	0,07	17,00	0,15	17,00	0,22	17,00	0,33	17,00	0,33	17,00	0,42	17,00	0,42	17	17,00	0,30	17,00	0,30	17,00	0,37	17	17,00	0,45	17,00	0,49	17,00	0,49	
18	18,00	0,08	18,00	0,16	18,00	0,24	18,00	0,35	18,00	0,35	18,00	0,44	18,00	0,44	18	18,00	0,32	18,00	0,32	18,00	0,39	18	18,00	0,47	18,00	0,51	18,00	0,51	
19	19,00	0,08	19,00	0,17	19,00	0,25	19,00	0,38	19,00	0,38	19,00	0,47	19,00	0,47	19	19,00	0,33	19,00	0,33	19,00	0,40	19	19,00	0,48	19,00	0,52	19,00	0,52	
20	20,00	0,09	20,00	0,17	20,00	0,26	20,00	0,39	20,00	0,39	20,00	0,48	20,00	0,48	20	20,00	0,35	20,00	0,35	20,00	0,42	20	20,00	0,50	20,00	0,54	20,00	0,54	
21	21,00	0,09	21,00	0,18	21,00	0,28	21,00	0,41	21,00	0,41	21,00	0,50	21,00	0,50	21	21,00	0,37	21,00	0,37	21,00	0,44	21	21,00	0,52	21,00	0,56	21,00	0,56	
22	22,00	0,10	22,00	0,19	22,00	0,29	22,00	0,42	22,00	0,42	22,00	0,51	22,00	0,51	22	22,00	0,38	22,00	0,38	22,00	0,45	22	22,00	0,54	22,00	0,58	22,00	0,58	
23	23,00	0,10	23,00	0,20	23,00	0,30	23,00	0,43	23,00	0,43	23,00	0,52	23,00	0,52	23	23,00	0,40	23,00	0,40	23,00	0,47	23	23,00	0,56	23,00	0,60	23,00	0,60	
24	24,00	0,10	24,00	0,21	24,00	0,31	24,00	0,44	24,00	0,44	24,00	0,53	24,00	0,53	24	24,00	0,42	24,00	0,42	24,00	0,49	24	24,00	0,58	24,00	0,62	24,00	0,62	
25	25,00	0,11	25,00	0,22	25,00	0,33	25,00	0,46	25,00	0,46	25,00	0,55	25,00	0,55	25	25,00	0,44	25,00	0,44	25,00	0,51	25	25,00	0,60	25,00	0,64	25,00	0,64	
26	26,00	0,11	26,00	0,23	26,00	0,34	26,00	0,47	26,00	0,47	26,00	0,56	26,00	0,56	26	26,00	0,45	26,00	0,45	26,00	0,52	26	26,00	0,62	26,00	0,66	26,00	0,66	
27	27,00	0,12	27,00	0,24	27,00	0,35	27,00	0,48	27,00	0,48	27,00	0,57	27,00	0,57	27	27,00	0,47	27,00	0,47	27,00	0,54	27	27,00	0,64	27,00	0,68	27,00	0,68	
28	28,00	0,12	28,00	0,24	28,00	0,37	28,00	0,50	28,00	0,50	28,00	0,59	28,00	0,59	28	28,00	0,49	28,00	0,49	28,00	0,56	28	28,00	0,66	28,00	0,70	28,00	0,70	
29	29,00	0,13	29,00	0,25	29,00	0,38	29,00	0,51	29,00	0,51	29,00	0,60	29,00	0,60	29	29,00	0,51	29,00	0,51	29,00	0,58	29	29,00	0,68	29,00	0,72	29,00	0,72	
30	30,00	0,13	30,00	0,26	30,00	0,39	30,00	0,52	30,00	0,52	30,00	0,61	30,00	0,61	30	30,00	0,52	30,00	0,52	30,00	0,59	30	30,00	0,70	30,00	0,74	30,00	0,74	
31	31,00	0,14	31,00	0,27	31,00	0,41	31,00	0,54	31,00	0,54	31,00	0,63	31,00	0,63	31	31,00	0,54	31,00	0,54	31,00	0,61	31	31,00	0,72	31,00	0,76	31,00	0,76	
32	32,00	0,14	32,00	0,28	32,00	0,42	32,00	0,56	32,00	0,56	32,00	0,65	32,00	0,65	32	32,00	0,56	32,00	0,56	32,00	0,63	32	32,00	0,74	32,00	0,78	32,00	0,78	
33	33,00	0,14	33,00	0,29	33,00	0,43	33,00	0,58	33,00	0,58	33,00	0,67	33,00	0,67	33	33,00	0,58	33,00	0,58	33,00	0,65	33	33,00	0,76	33,00	0,80	33,00	0,80	
34	34,00	0,15	34,00	0,30	34,00	0,44	34,00	0,59	34,00	0,59	34,00	0,68	34,00	0,68	34	34,00	0,59	34,00	0,59	34,00	0,66	34	34,00	0,78	34,00	0,82	34,00	0,82	
35	35,00	0,15	35,00	0,31	35,00	0,46	35,00	0,61	35,00	0,61	35,00	0,70	35,00	0,70	35	35,00	0,61	35,00	0,61	35,00	0,68	35	35,00	0,80	35,00	0,84	35,00	0,84	
36	36,00	0,16	36,00	0,32	36,00	0,47	36,00	0,63	36,00	0,63	36,00	0,72	36,00	0,72	36	36,00	0,63	36,00	0,63	36,00	0,70	36	36,00	0,82	36,00	0,86	36,00	0,86	
37	37,00	0,16	37,00	0,33	37,00	0,48	37,00	0,65	37,00	0,65	37,00	0,74	37,00	0,74	37	37,00	0,65	37,00	0,65	37,00	0,72	37	37,00	0,84	37,00	0,88	37,00	0,88	
38	38,00	0,17	38,00	0,33	38,00	0,50	38,00	0,66	38,00	0,66	38,00	0,75	38,00	0,75	38	38,00	0,66	38,00	0,66	38,00	0,73	38	38,00	0,86	38,00	0,90	38,00	0,90	
39	39,00	0,17	39,00	0,34	39,00	0,51	39,00	0,68	39,00	0,68	39,00	0,77	39,00	0,77	39	39,00	0,68	39,00	0,68	39,00	0,75	39	39,00	0,88	39,00	0,92</			

TRAVERSE TABLE. 67.

87

0 Degrees										1 Degree										88 Degrees										89 Degrees																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.	

TRAVERSE TABLE. 67.

2 Degrees										3 Degrees										86 Degrees																			
0 Min.					15 Min.					30 Min.					45 Min.					0 Min.					15 Min.					30 Min.					45 Min.				
Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.	Distance	Lat.	Dep.	Lat.	Dep.					
1	1.00	0.04	1.00	0.04	1	1.00	0.05	1.00	0.05	1	1.00	0.06	1.00	0.06	1	1.00	0.06	1.00	0.06	1	1.00	0.06	1.00	0.06	1	1.00	0.06	1.00	0.06	1	1.00	0.06	1.00	0.06					
2	2.00	0.07	2.00	0.08	2	2.00	0.10	2.00	0.11	2	2.00	0.11	2.00	0.12	2	2.00	0.12	2.00	0.13	2	2.00	0.13	2.00	0.13	2	2.00	0.13	2.00	0.13	2	2.00	0.13	2.00	0.13					
3	3.00	0.11	3.00	0.12	3	3.00	0.14	3.00	0.16	3	3.00	0.16	3.00	0.17	3	3.00	0.17	3.00	0.18	3	3.00	0.18	3.00	0.19	3	3.00	0.19	3.00	0.20	3	3.00	0.20	3.00	0.21					
4	4.00	0.14	4.00	0.16	4	4.00	0.18	4.00	0.21	4	4.00	0.21	4.00	0.23	4	4.00	0.23	4.00	0.25	4	4.00	0.25	4.00	0.27	4	4.00	0.27	4.00	0.29	4	4.00	0.29	4.00	0.31					
5	5.00	0.18	5.00	0.20	5	5.00	0.22	5.00	0.26	5	5.00	0.26	5.00	0.30	5	5.00	0.30	5.00	0.34	5	5.00	0.34	5.00	0.38	5	5.00	0.38	5.00	0.42	5	5.00	0.42	5.00	0.46					
6	6.00	0.21	6.00	0.24	6	6.00	0.26	6.00	0.31	6	6.00	0.31	6.00	0.36	6	6.00	0.36	6.00	0.41	6	6.00	0.41	6.00	0.46	6	6.00	0.46	6.00	0.51	6	6.00	0.51	6.00	0.56					
7	7.00	0.24	7.00	0.28	7	7.00	0.30	7.00	0.35	7	7.00	0.35	7.00	0.40	7	7.00	0.40	7.00	0.45	7	7.00	0.45	7.00	0.50	7	7.00	0.50	7.00	0.55	7	7.00	0.55	7.00	0.60					
8	8.00	0.28	8.00	0.33	8	8.00	0.35	8.00	0.41	8	8.00	0.41	8.00	0.48	8	8.00	0.48	8.00	0.55	8	8.00	0.55	8.00	0.62	8	8.00	0.62	8.00	0.69	8	8.00	0.69	8.00	0.76					
9	9.00	0.31	9.00	0.35	9	9.00	0.39	9.00	0.44	9	9.00	0.44	9.00	0.52	9	9.00	0.52	9.00	0.60	9	9.00	0.60	9.00	0.68	9	9.00	0.68	9.00	0.76	9	9.00	0.76	9.00	0.84					
10	10.00	0.35	10.00	0.39	10	10.00	0.44	10.00	0.48	10	10.00	0.48	10.00	0.58	10	10.00	0.58	10.00	0.68	10	10.00	0.68	10.00	0.79	10	10.00	0.79	10.00	0.90	10	10.00	0.90	10.00	1.01					
11	11.00	0.38	11.00	0.43	11	11.00	0.48	11.00	0.53	11	11.00	0.53	11.00	0.63	11	11.00	0.63	11.00	0.73	11	11.00	0.73	11.00	0.85	11	11.00	0.85	11.00	0.96	11	11.00	0.96	11.00	1.08					
12	12.00	0.42	12.00	0.47	12	12.00	0.52	12.00	0.58	12	12.00	0.58	12.00	0.68	12	12.00	0.68	12.00	0.79	12	12.00	0.79	12.00	0.91	12	12.00	0.91	12.00	1.02	12	12.00	1.02	12.00	1.14					
13	13.00	0.45	13.00	0.51	13	13.00	0.57	13.00	0.62	13	13.00	0.62	13.00	0.72	13	13.00	0.72	13.00	0.83	13	13.00	0.83	13.00	0.95	13	13.00	0.95	13.00	1.06	13	13.00	1.06	13.00	1.18					
14	14.00	0.49	14.00	0.55	14	14.00	0.61	14.00	0.67	14	14.00	0.67	14.00	0.77	14	14.00	0.77	14.00	0.88	14	14.00	0.88	14.00	1.00	14	14.00	1.00	14.00	1.11	14	14.00	1.11	14.00	1.23					
15	15.00	0.52	15.00	0.59	15	15.00	0.65	15.00	0.72	15	15.00	0.72	15.00	0.82	15	15.00	0.82	15.00	0.93	15	15.00	0.93	15.00	1.05	15	15.00	1.05	15.00	1.16	15	15.00	1.16	15.00	1.28					
16	16.00	0.56	16.00	0.63	16	16.00	0.70	16.00	0.77	16	16.00	0.77	16.00	0.87	16	16.00	0.87	16.00	0.98	16	16.00	0.98	16.00	1.10	16	16.00	1.10	16.00	1.21	16	16.00	1.21	16.00	1.33					
17	17.00	0.59	17.00	0.67	17	17.00	0.74	17.00	0.82	17	17.00	0.82	17.00	0.92	17	17.00	0.92	17.00	1.03	17	17.00	1.03	17.00	1.15	17	17.00	1.15	17.00	1.26	17	17.00	1.26	17.00	1.38					
18	18.00	0.63	18.00	0.71	18	18.00	0.79	18.00	0.86	18	18.00	0.86	18.00	0.96	18	18.00	0.96	18.00	1.07	18	18.00	1.07	18.00	1.19	18	18.00	1.19	18.00	1.30	18	18.00	1.30	18.00	1.42					
19	19.00	0.66	19.00	0.75	19	19.00	0.83	19.00	0.91	19	19.00	0.91	19.00	1.01	19	19.00	1.01	19.00	1.12	19	19.00	1.12	19.00	1.24	19	19.00	1.24	19.00	1.35	19	19.00	1.35	19.00	1.47					
20	20.00	0.70	20.00	0.79	20	20.00	0.87	20.00	0.96	20	20.00	0.96	20.00	1.07	20	20.00	1.07	20.00	1.18	20	20.00	1.18	20.00	1.30	20	20.00	1.30	20.00	1.41	20	20.00	1.41	20.00	1.53					
21	21.00	0.73	21.00	0.83	21	21.00	0.92	21.00	1.01	21	21.00	1.01	21.00	1.12	21	21.00	1.12	21.00	1.23	21	21.00	1.23	21.00	1.35	21	21.00	1.35	21.00	1.46	21	21.00	1.46	21.00	1.58					
22	22.00	0.77	22.00	0.86	22	22.00	0.96	22.00	1.06	22	22.00	1.06	22.00	1.17	22	22.00	1.17	22.00	1.28	22	22.00	1.28	22.00	1.40	22	22.00	1.40	22.00	1.51	22	22.00	1.51	22.00	1.63					
23	23.00	0.80	23.00	0.90	23	23.00	1.00	23.00	1.10	23	23.00	1.10	23.00	1.21	23	23.00	1.21	23.00	1.32	23	23.00	1.32	23.00	1.44	23	23.00	1.44	23.00	1.55	23	23.00	1.55	23.00	1.67					
24	24.00	0.84	24.00	0.94	24	24.00	1.05	24.00	1.15	24	24.00	1.15	24.00	1.26	24	24.00	1.26	24.00	1.37	24	24.00	1.37	24.00	1.49	24	24.00	1.49	24.00	1.60	24	24.00	1.60	24.00	1.72					
25	25.00	0.87	25.00	0.98	25	25.00	1.09	25.00	1.19	25	25.00	1.19	25.00	1.30	25	25.00	1.30	25.00	1.41	25	25.00	1.41	25.00	1.53	25	25.00	1.53	25.00	1.64	25	25.00	1.64	25.00	1.76					
26	26.00	0.91	26.00	1.02	26	26.00	1.13	26.00	1.23	26	26.00	1.23	26.00	1.34	26	26.00	1.34	26.00	1.45	26	26.00	1.45	26.00	1.57	26	26.00	1.57	26.00	1.68	26	26.00	1.68	26.00	1.80					
27	27.00	0.94	27.00	1.06	27	27.00	1.18	27.00	1.28	27	27.00	1.28	27.00	1.39	27	27.00	1.39	27.00	1.50	27	27.00	1.50	27.00	1.62	27	27.00	1.62	27.00	1.73	27	27.00	1.73	27.00	1.85					
28	28.00	0.98	28.00	1.10	28	28.00	1.22	28.00	1.32	28	28.00	1.32	28.00	1.43	28	28.00	1.43	28.00	1.54	28	28.00	1.54	28.00	1.66	28	28.00	1.66	28.00	1.77	28	28.00	1.77	28.00	1.89					
29	29.00	1.01	29.00	1.14	29	29.00	1.27	29.00	1.37	29	29.00	1.37	29.00	1.48	29	29.00	1.48	29.00	1.59	29	29.00	1.59	29.00	1.71	29	29.00	1.71	29.00	1.82	29	29.00	1.82	29.00	1.94					
30	30.00	1.05	30.00	1.18	30	30.00	1.31	30.00	1.41	30	30.00	1.41	30.00	1.52	30	30.00	1.52	30.00	1.63	30	30.00	1.63	30.00	1.75	30	30.00	1.75	30.00	1.86	30	30.00	1.86	30.00	1.98					
31	31.00	1.08	31.00	1.22	31	31.00	1.35	31.00	1.45	31	31.00	1.45	31.00	1.56	31	31.00	1.56	31.00	1.67	31	31.00	1.67	31.00	1.79	31	31.00	1.79	31.00	1.90	31	31.00	1.90	31.00	2.02					
32	32.00	1.12	32.00	1.26	32	32.00	1.40	32.00	1.50	32	32.00	1.50	32.00	1.61	32	32.00	1.61	32.00	1.72	32	32.00	1.72	32.00	1.84	32	32.00	1.84	32.00	1.95	32	32.00	1.95	32.00	2.07					
33	33.00	1.15	33.00	1.30	33	33.00	1.44	33.00	1.54	33	33.00	1.54	33.00	1.65	33	33.00	1.65	33.00	1.76	33	33.00	1.76	33.00	1.88	33	33.00	1.88	33.00	1.99	33	33.00	1.99	33.00	2.11					
34	34.00	1.19	34.00	1.34	34	34.00	1.48	34.00	1.58	34	34.00	1.58	34.00	1.69	34																								

TRAVERSE TABLE. 67.

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Distance		2 Degrees						3 Degrees						Distance	
		0 Min.	15 Min.	30 Min.	45 Min.	Lat.	Dep.	0 Min.	15 Min.	30 Min.	45 Min.	Lat.	Dep.		
61	60.96	2.13	60.95	2.40	60.94	2.66	60.93	2.93	3.19	60.90	3.46	60.89	3.72	60.87	3.99
62	61.96	2.16	61.95	2.43	61.94	2.70	61.93	2.98	3.25	61.90	3.51	61.88	3.79	61.87	4.06
63	62.96	2.20	62.95	2.47	62.94	2.75	62.93	3.02	3.30	62.90	3.57	62.88	3.85	62.87	4.12
64	63.96	2.23	63.95	2.51	63.94	2.79	63.93	3.07	3.35	63.90	3.63	63.88	3.91	63.86	4.19
65	64.96	2.27	64.95	2.55	64.94	2.84	64.93	3.12	3.40	64.90	3.69	64.88	3.97	64.86	4.25
66	65.96	2.30	65.95	2.59	65.94	2.88	65.93	3.17	3.45	65.90	3.74	65.88	4.03	65.86	4.32
67	66.96	2.34	66.95	2.63	66.94	2.92	66.93	3.22	3.51	66.90	3.80	66.88	4.09	66.86	4.38
68	67.96	2.37	67.95	2.67	67.94	2.97	67.93	3.26	3.56	67.90	3.86	67.87	4.16	67.85	4.45
69	68.96	2.41	68.95	2.71	68.94	3.01	68.93	3.31	3.61	68.90	3.91	68.87	4.22	68.85	4.51
70	69.96	2.44	69.95	2.75	69.94	3.05	69.93	3.36	3.66	69.90	3.97	69.87	4.28	69.85	4.58
71	70.96	2.48	70.95	2.79	70.94	3.10	70.93	3.41	3.72	70.90	4.03	70.87	4.34	70.85	4.64
72	71.96	2.51	71.95	2.83	71.94	3.14	71.93	3.46	3.77	71.90	4.08	71.87	4.40	71.85	4.71
73	72.96	2.55	72.94	2.87	72.93	3.18	72.92	3.50	3.82	72.90	4.14	72.86	4.46	72.84	4.77
74	73.96	2.58	73.94	2.91	73.93	3.23	73.92	3.55	3.87	73.90	4.20	73.86	4.52	73.84	4.84
75	74.96	2.61	74.94	2.94	74.93	3.27	74.92	3.60	3.93	74.90	4.25	74.86	4.58	74.84	4.91
76	75.96	2.65	75.94	2.98	75.93	3.31	75.92	3.65	3.98	75.90	4.31	75.86	4.64	75.84	4.97
77	76.96	2.69	76.94	3.02	76.93	3.36	76.92	3.70	4.03	76.90	4.37	76.86	4.70	76.84	5.04
78	77.96	2.72	77.94	3.06	77.93	3.40	77.92	3.74	4.08	77.90	4.42	77.85	4.76	77.83	5.10
79	78.96	2.76	78.94	3.10	78.93	3.45	78.92	3.79	4.14	78.90	4.48	78.85	4.82	78.83	5.17
80	79.96	2.79	79.94	3.14	79.93	3.49	79.92	3.84	4.19	79.90	4.54	79.85	4.88	79.83	5.23
81	80.96	2.83	80.94	3.18	80.93	3.53	80.92	3.89	4.24	80.90	4.59	80.85	4.95	80.83	5.30
82	81.96	2.86	81.94	3.22	81.93	3.57	81.92	3.94	4.29	81.90	4.65	81.85	5.01	81.83	5.36
83	82.96	2.90	82.94	3.26	82.93	3.62	82.92	3.99	4.34	82.90	4.71	82.85	5.07	82.83	5.43
84	83.96	2.93	83.94	3.30	83.93	3.66	83.92	4.03	4.40	83.90	4.76	83.84	5.13	83.82	5.49
85	84.96	2.97	84.94	3.34	84.93	3.71	84.92	4.08	4.45	84.90	4.82	84.84	5.19	84.82	5.56
86	85.96	3.00	85.93	3.38	85.92	3.75	85.91	4.13	4.50	85.90	4.88	85.84	5.25	85.82	5.62
87	86.96	3.04	86.93	3.42	86.92	3.79	86.91	4.18	4.55	86.90	4.93	86.84	5.31	86.82	5.69
88	87.96	3.07	87.93	3.46	87.92	3.84	87.91	4.22	4.61	87.90	4.99	87.84	5.37	87.81	5.76
89	88.96	3.11	88.93	3.49	88.92	3.88	88.91	4.27	4.66	88.90	5.05	88.83	5.43	88.81	5.82
90	89.96	3.14	89.91	3.53	89.92	3.93	89.90	4.32	4.71	89.90	5.10	89.83	5.50	89.81	5.89
91	90.96	3.18	90.93	3.57	90.91	3.97	90.90	4.37	4.76	90.85	5.17	90.83	5.56	90.81	5.95
92	91.96	3.21	91.93	3.61	91.91	4.01	91.90	4.41	4.82	91.85	5.22	91.83	5.62	91.80	6.02
93	92.96	3.25	92.93	3.65	92.91	4.05	92.90	4.46	4.87	92.85	5.27	92.83	5.68	92.80	6.09
94	93.96	3.28	93.93	3.69	93.91	4.10	93.89	4.51	4.93	93.85	5.33	93.83	5.74	93.80	6.15
95	94.96	3.32	94.93	3.73	94.91	4.14	94.89	4.56	4.97	94.85	5.39	94.82	5.80	94.80	6.21
96	95.96	3.35	95.93	3.77	95.91	4.19	95.89	4.61	5.03	95.85	5.44	95.82	5.86	95.79	6.28
97	96.96	3.39	96.93	3.81	96.91	4.23	96.89	4.65	5.08	96.84	5.50	96.82	5.92	96.79	6.34
98	97.96	3.42	97.93	3.85	97.91	4.28	97.89	4.70	5.13	97.84	5.55	97.82	5.98	97.79	6.41
99	98.96	3.46	98.93	3.89	98.91	4.32	98.89	4.75	5.18	98.84	5.61	98.82	6.04	98.79	6.48
100	99.96	3.49	99.91	3.93	99.92	4.36	99.89	4.80	5.23	99.84	5.67	99.81	6.11	99.79	6.54
101	100.96	3.53	100.93	3.97	100.91	4.41	100.90	4.85	5.29	100.85	5.73	100.83	6.17	100.80	6.61
102	101.96	3.56	101.94	4.01	101.91	4.45	101.90	4.89	5.34	101.85	5.78	101.83	6.23	101.80	6.67
103	102.96	3.59	102.93	4.04	102.91	4.49	102.90	4.94	5.39	102.85	5.84	102.83	6.29	102.80	6.74
104	103.96	3.63	103.94	4.08	103.91	4.54	103.90	4.99	5.44	103.85	5.90	103.83	6.35	103.80	6.80
105	104.96	3.67	104.94	4.12	104.91	4.58	104.90	5.04	5.50	104.85	5.95	104.83	6.41	104.80	6.87
106	105.96	3.70	105.94	4.16	105.91	4.62	105.90	5.09	5.55	105.85	6.01	105.83	6.47	105.80	6.93
107	106.96	3.73	106.93	4.20	106.91	4.67	106.90	5.13	5.60	106.85	6.07	106.83	6.53	106.80	7.00
108	107.96	3.77	107.94	4.24	107.91	4.71	107.90	5.18	5.65	107.85	6.12	107.83	6.59	107.80	7.06
109	108.96	3.80	108.94	4.28	108.91	4.76	108.90	5.23	5.71	108.85	6.18	108.83	6.66	108.80	7.13
110	109.96	3.84	109.93	4.32	109.91	4.80	109.90	5.28	5.76	109.85	6.24	109.83	6.72	109.80	7.19
111	110.96	3.87	110.94	4.36	110.91	4.84	110.90	5.33	5.82	110.85	6.29	110.83	6.78	110.80	7.26
112	111.96	3.91	111.93	4.40	111.91	4.89	111.90	5.37	5.86	111.85	6.35	111.83	6.84	111.80	7.33
113	112.96	3.94	112.94	4.44	112.91	4.93	112.90	5.42	5.92	112.85	6.41	112.83	6.90	112.80	7.39
114	113.96	3.98	113.94	4.48	113.91	4.97	113.90	5.47	5.97	113.85	6.47	113.83	6.96	113.80	7.46
115	114.96	4.01	114.94	4.51	114.91	5.02	114.90	5.52	6.02	114.85	6.52	114.83	7.02	114.80	7.52
116	115.96	4.05	115.94	4.55	115.91	5.06	115.90	5.57	6.07	115.85	6.58	115.83	7.08	115.80	7.59
117	116.96	4.08	116.94	4.59	116.91	5.10	116.90	5.62	6.12	116.85	6.63	116.83	7.14	116.80	7.65
118	117.96	4.12	117.94	4.63	117.91	5.15	117.90	5.66	6.18	117.85	6.69	117.83	7.20	117.80	7.72
119	118.96	4.15	118.94	4.67	118.91	5.19	118.90	5.71	6.23	118.85	6.75	118.83	7.27	118.80	7.78
120	119.96	4.19	119.94	4.71	119.91	5.23	119.90	5.76	6.28	119.85	6.80	119.83	7.33	119.80	7.85

Vol. II.

H

Distance			0 Min.			15 Min.			30 Min.			45 Min.			1 Hour		
Lat.	Dep.	Dist.	Lat.	Dep.	Dist.	Lat.	Dep.	Dist.	Lat.	Dep.	Dist.	Lat.	Dep.	Dist.	Lat.	Dep.	Dist.
1	1.00	6.07	1.00	0.07	1.00	0.08	1.00	0.09	1.00	0.09	1.00	0.10	1.00	0.10	1.00	0.10	1.00
2	2.06	6.14	2.00	0.15	1.99	0.17	1.99	0.17	1.99	0.17	1.99	0.18	1.99	0.19	1.99	0.20	1.99
3	3.09	6.21	3.00	0.22	2.99	0.25	2.99	0.26	2.99	0.26	2.99	0.28	2.99	0.29	2.99	0.30	2.99
4	4.09	6.28	4.00	0.30	3.99	0.33	3.99	0.35	3.99	0.35	3.99	0.38	3.98	0.38	3.98	0.40	3.98
5	5.09	6.35	5.00	0.37	4.99	0.41	4.98	0.44	4.98	0.44	4.98	0.46	4.98	0.48	4.98	0.50	4.98
6	6.08	6.42	6.00	0.45	5.98	0.49	5.98	0.52	5.98	0.52	5.98	0.55	5.97	0.58	5.97	0.60	5.97
7	7.08	6.49	7.00	0.52	6.98	0.55	6.98	0.58	6.97	0.61	6.97	0.64	6.97	0.67	6.97	0.70	6.97
8	8.08	6.56	8.00	0.59	7.98	0.63	7.97	0.66	7.97	0.70	7.97	0.73	7.96	0.78	7.96	0.80	7.96
9	9.08	6.63	9.00	0.67	8.97	0.71	8.97	0.75	8.97	0.78	8.96	0.82	8.96	0.86	8.96	0.90	8.96
10	10.08	6.70	10.00	0.74	9.97	0.79	9.97	0.83	9.96	0.87	9.96	0.92	9.95	0.96	9.95	1.00	9.95
11	11.07	6.77	11.00	0.82	10.97	0.86	10.96	0.91	10.96	0.96	10.95	1.01	10.95	1.05	10.95	1.10	10.95
12	12.07	6.84	12.00	0.89	11.96	0.94	11.96	0.99	11.95	1.05	11.95	1.10	11.95	1.15	11.94	1.20	11.94
13	13.07	6.91	13.00	0.96	12.96	1.02	12.96	1.08	12.95	1.13	12.95	1.19	12.94	1.25	12.94	1.30	12.94
14	14.07	6.98	14.00	1.04	13.96	1.10	13.95	1.16	13.95	1.22	13.94	1.28	13.94	1.34	13.93	1.40	13.93
15	15.06	7.05	15.00	1.11	14.95	1.18	14.95	1.24	14.94	1.31	14.94	1.37	14.93	1.44	14.93	1.50	14.93
16	16.06	7.12	16.00	1.19	15.95	1.26	15.95	1.33	15.94	1.40	15.93	1.46	15.93	1.53	15.92	1.60	15.92
17	17.06	7.19	17.00	1.26	16.95	1.33	16.94	1.41	16.94	1.48	16.93	1.56	16.92	1.63	16.91	1.70	16.91
18	18.06	7.26	18.00	1.33	17.94	1.41	17.94	1.49	17.93	1.57	17.92	1.65	17.92	1.73	17.91	1.80	17.91
19	19.05	7.33	19.00	1.40	18.94	1.49	18.93	1.57	18.93	1.66	18.92	1.74	18.91	1.82	18.91	1.90	18.91
20	20.05	7.40	20.00	1.48	19.94	1.57	19.93	1.66	19.92	1.74	19.91	1.83	19.91	1.92	19.90	2.00	19.90
21	21.05	7.47	21.00	1.56	20.94	1.65	20.93	1.74	20.92	1.83	20.91	1.92	20.90	2.01	20.89	2.10	20.89
22	22.05	7.54	22.00	1.63	21.93	1.73	21.93	1.82	21.92	1.92	21.91	2.01	21.90	2.11	21.89	2.20	21.89
23	23.04	7.61	23.00	1.70	22.93	1.80	22.92	1.90	22.91	2.00	22.90	2.10	22.89	2.21	22.88	2.30	22.88
24	24.04	7.68	24.00	1.78	23.93	1.88	23.92	1.99	23.91	2.09	23.90	2.20	23.89	2.30	23.88	2.40	23.88
25	25.04	7.74	25.00	1.85	24.92	1.96	24.91	2.07	24.91	2.18	24.90	2.29	24.89	2.40	24.87	2.50	24.87
26	26.04	7.81	26.00	1.93	25.92	2.04	25.91	2.15	25.90	2.27	25.89	2.38	25.88	2.49	25.87	2.60	25.87
27	27.03	7.88	27.00	2.00	26.91	2.12	26.91	2.24	26.90	2.35	26.89	2.47	26.88	2.59	26.86	2.70	26.86
28	28.03	7.95	28.00	2.08	27.92	2.20	27.91	2.32	27.89	2.44	27.88	2.56	27.87	2.68	27.86	2.80	27.86
29	29.03	8.02	29.00	2.15	28.91	2.28	28.90	2.40	28.89	2.53	28.88	2.65	28.87	2.78	28.85	2.90	28.85
30	30.03	8.09	30.00	2.22	29.91	2.35	29.90	2.48	29.89	2.62	29.87	2.75	29.86	2.88	29.85	3.00	29.85
31	31.03	8.16	31.00	2.30	30.91	2.43	30.89	2.57	30.88	2.70	30.87	2.84	30.86	2.97	30.84	3.10	30.84
32	32.03	8.23	32.00	2.37	31.90	2.51	31.89	2.65	31.88	2.79	31.87	2.93	31.85	3.07	31.84	3.20	31.84
33	33.02	8.30	33.00	2.45	32.90	2.59	32.89	2.73	32.88	2.88	32.86	3.02	32.85	3.17	32.83	3.30	32.83
34	34.02	8.37	34.00	2.52	33.90	2.67	33.88	2.82	33.87	2.96	33.86	3.11	33.85	3.26	33.83	3.40	33.83
35	35.01	8.44	35.00	2.60	34.89	2.75	34.88	2.90	34.87	3.05	34.86	3.20	34.84	3.35	34.82	3.50	34.82
36	36.01	8.51	36.00	2.67	35.89	2.83	35.88	2.98	35.86	3.14	35.85	3.29	35.84	3.45	35.82	3.60	35.82
37	37.01	8.58	37.00	2.74	36.89	2.90	36.87	3.06	36.86	3.23	36.85	3.39	36.83	3.55	36.81	3.70	36.81
38	38.01	8.65	38.00	2.82	37.88	2.98	37.87	3.15	37.86	3.33	37.84	3.48	37.83	3.64	37.81	3.80	37.81
39	39.01	8.72	39.00	2.89	38.88	3.06	38.87	3.23	38.85	3.40	38.84	3.57	38.82	3.74	38.80	3.90	38.80
40	40.00	8.79	40.00	2.96	39.88	3.14	39.86	3.31	39.85	3.49	39.83	3.66	39.82	3.84	39.80	4.00	39.80
41	41.00	8.86	41.00	3.04	40.87	3.22	40.86	3.40	40.84	3.57	40.83	3.75	40.81	3.93	40.79	4.10	40.79
42	42.00	8.93	42.00	3.11	41.87	3.30	41.86	3.48	41.84	3.66	41.82	3.84	41.81	4.03	41.79	4.20	41.79
43	43.00	9.00	43.00	3.19	42.87	3.37	42.85	3.56	42.84	3.75	42.82	3.94	42.80	4.12	42.78	4.30	42.78
44	44.00	9.07	44.00	3.26	43.86	3.45	43.85	3.64	43.83	3.84	43.82	4.03	43.80	4.22	43.78	4.40	43.78
45	45.00	9.14	45.00	3.34	44.86	3.53	44.85	3.73	44.83	3.92	44.81	4.12	44.79	4.31	44.77	4.50	44.77
46	46.00	9.21	46.00	3.41	45.87	3.61	45.86	3.81	45.83	4.00	45.81	4.21	45.79	4.41	45.77	4.60	45.77
47	47.00	9.28	47.00	3.48	46.87	3.69	46.84	3.89	46.82	4.10	46.80	4.30	46.78	4.51	46.76	4.70	46.76
48	48.00	9.35	48.00	3.56	47.87	3.77	47.84	3.97	47.82	4.18	47.80	4.39	47.78	4.60	47.76	4.80	47.76
49	49.00	9.42	49.00	3.63	48.87	3.84	48.85	4.06	48.81	4.27	48.79	4.48	48.78	4.70	48.75	4.90	48.75
50	50.00	9.49	50.00	3.71	49.86	3.92	49.83	4.14	49.81	4.36	49.79	4.58	49.77	4.79	49.75	5.00	49.75
51	51.00	9.56	51.00	3.78	50.84	4.00	50.83	4.22	50.81	4.45	50.79	4.67	50.77	4.89	50.74	5.10	50.74
52	52.00	9.63	52.00	3.85	51.84	4.08	51.82	4.31	51.80	4.54	51.78	4.76	51.76	4.99	51.74	5.20	51.74
53	53.00	9.70	53.00	3.93	52.84	4.16	52.82	4.39	52.80	4.62	52.78	4.85	52.76	5.08	52.73	5.30	52.73
54	54.00	9.77	54.00	4.00	53.83	4.24	53.82	4.47	53.80	4.71	53.77	4.94	53.75	5.18	53.73	5.40	53.73
55	55.00	9.84	55.00	4.08	54.83	4.32	54.81	4.56	54.79	4.79	54.77	5.03	54.75	5.27	54.72	5.50	54.72
56	56.00	9.91	56.00	4.15	55.83	4.39	55.81	4.64	55.79	4.88	55.77	5.12	55.74	5.37	55.72	5.60	55.72
57	57.00	9.98	57.00	4.23	56.82	4.47	56.81	4.72	56.78	4.97	56.76	5.22	56.74	5.46	56.71	5.70	56.71
58	58.00	10.05	58.00	4.30	57.82	4.55	57.80	4.80	57.78	5.06	57.76	5.31	57.73	5.56	57.71	5.80	57.71
59	59.00	10.12	59.00	4.38	58.82	4.63	58.80	4.89	58.78	5.15	58.75	5.40	58.73	5.66	58.70	5.90	58.70
60	60.00	10.19	60.00	4.45	59.82	4.71	59.79	4.97	59.77	5.23	59.75	5.49	59.72	5.75	59.70	6.00	59.70

4 Degrees

5 Degrees

84 Degrees

85 Degrees

86 Deg.

TRAVERSE TABLE. 67.1

F 91

Distance			4 Degrees				5 Degrees				84 Degrees				85 Degrees				86 Deg.				Distance	
0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		Distance		
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.		
60.85	4.26	60.83	4.52	60.81	4.79	60.79	5.05	60.77	5.32	60.74	5.58	60.72	5.85	60.69	6.11	60.67	6.38	60.64	6.64	60.61	6.91	60.58	7.18	
61.15	4.33	61.13	4.60	61.11	4.87	61.09	5.14	61.06	5.40	61.04	5.67	61.02	5.94	60.99	6.21	60.97	6.48	60.94	6.74	60.91	7.01	60.88	7.28	
62.45	4.40	62.43	4.67	62.41	4.94	62.38	5.22	62.36	5.49	62.34	5.77	62.32	6.04	62.29	6.36	62.27	6.63	62.24	6.89	62.21	7.16	62.18	7.43	
63.84	4.47	63.82	4.74	63.80	5.02	63.78	5.30	63.76	5.58	63.73	5.86	63.71	6.13	63.68	6.40	63.66	6.68	63.63	6.94	63.60	7.21	63.57	7.48	
64.84	4.54	64.82	4.81	64.80	5.10	64.78	5.38	64.76	5.67	64.73	5.95	64.70	6.23	64.67	6.50	64.65	6.78	64.62	7.04	64.59	7.31	64.56	7.58	
65.84	4.61	65.82	4.89	65.80	5.18	65.77	5.47	65.75	5.75	65.72	6.04	65.70	6.33	65.67	6.56	65.65	6.84	65.62	7.10	65.59	7.37	65.56	7.64	
66.84	4.67	66.82	4.97	66.79	5.26	66.77	5.55	66.74	5.84	66.72	6.13	66.69	6.42	66.66	6.71	66.64	6.99	66.61	7.27	66.58	7.54	66.55	7.82	
67.84	4.74	67.81	5.04	67.79	5.34	67.77	5.63	67.74	5.93	67.71	6.22	67.69	6.51	67.66	6.80	67.64	7.08	67.61	7.36	67.58	7.64	67.55	7.92	
68.84	4.81	68.81	5.11	68.79	5.41	68.77	5.71	68.74	6.02	68.71	6.31	68.68	6.60	68.65	6.89	68.63	7.17	68.60	7.45	68.57	7.73	68.54	8.01	
69.84	4.88	69.81	5.19	69.78	5.49	69.76	5.80	69.73	6.10	69.70	6.41	69.68	6.71	69.65	7.00	69.63	7.28	69.60	7.56	69.57	7.84	69.54	8.12	
70.84	4.95	70.81	5.26	70.78	5.57	70.76	5.88	70.73	6.19	70.70	6.50	70.67	6.81	70.64	7.11	70.62	7.39	70.59	7.67	70.56	7.95	70.53	8.23	
71.84	5.02	71.80	5.34	71.78	5.65	71.75	5.96	71.73	6.28	71.70	6.59	71.67	6.90	71.64	7.21	71.62	7.49	71.59	7.77	71.56	8.05	71.53	8.33	
72.84	5.09	72.80	5.41	72.78	5.73	72.75	6.04	72.72	6.36	72.69	6.68	72.66	7.00	72.63	7.31	72.61	7.59	72.58	7.87	72.55	8.15	72.52	8.43	
73.84	5.16	73.80	5.48	73.77	5.81	73.75	6.13	73.72	6.45	73.69	6.77	73.66	7.09	73.63	7.41	73.61	7.69	73.58	7.97	73.55	8.25	73.52	8.53	
74.84	5.23	74.79	5.56	74.77	5.88	74.74	6.21	74.71	6.53	74.68	6.86	74.66	7.19	74.63	7.51	74.61	7.79	74.58	8.07	74.55	8.35	74.52	8.63	
75.84	5.30	75.79	5.63	75.77	5.96	75.74	6.29	75.71	6.62	75.68	6.95	75.65	7.29	75.62	7.61	75.60	7.89	75.57	8.17	75.54	8.45	75.51	8.73	
76.84	5.37	76.79	5.71	76.76	6.04	76.74	6.38	76.71	6.71	76.68	7.04	76.65	7.38	76.62	7.70	76.60	7.98	76.57	8.26	76.54	8.54	76.51	8.82	
77.84	5.44	77.79	5.78	77.76	6.12	77.73	6.46	77.70	6.80	77.67	7.14	77.64	7.48	77.61	7.82	77.59	8.10	77.56	8.38	77.53	8.66	77.50	8.94	
78.84	5.51	78.78	5.86	78.76	6.20	78.73	6.54	78.70	6.89	78.67	7.23	78.64	7.58	78.61	7.92	78.59	8.15	78.56	8.43	78.53	8.71	78.50	8.99	
79.84	5.58	79.78	5.93	79.75	6.28	79.73	6.63	79.70	6.97	79.66	7.32	79.63	7.67	79.60	8.01	79.58	8.24	79.55	8.52	79.52	8.80	79.49	9.08	
80.84	5.65	80.78	6.00	80.75	6.36	80.72	6.71	80.69	7.06	80.66	7.41	80.63	7.76	80.60	8.11	80.58	8.34	80.55	8.62	80.52	8.90	80.49	9.18	
81.84	5.72	81.77	6.08	81.75	6.43	81.72	6.79	81.69	7.15	81.66	7.50	81.62	7.86	81.59	8.21	81.57	8.44	81.54	8.72	81.51	9.00	81.48	9.28	
82.84	5.79	82.77	6.15	82.75	6.51	82.72	6.87	82.68	7.24	82.65	7.60	82.61	7.96	82.58	8.31	82.56	8.54	82.53	8.82	82.50	9.10	82.47	9.38	
83.84	5.86	83.77	6.23	83.74	6.59	83.71	6.96	83.68	7.32	83.65	7.69	83.61	8.05	83.58	8.41	83.56	8.64	83.53	8.92	83.50	9.20	83.47	9.48	
84.84	5.93	84.77	6.30	84.74	6.67	84.71	7.04	84.68	7.41	84.64	7.78	84.61	8.15	84.57	8.51	84.55	8.74	84.52	9.02	84.49	9.30	84.46	9.58	
85.84	6.00	85.76	6.37	85.73	6.75	85.70	7.12	85.67	7.50	85.64	7.87	85.61	8.24	85.57	8.61	85.55	8.84	85.52	9.12	85.49	9.40	85.46	9.68	
86.84	6.07	86.76	6.45	86.73	6.83	86.70	7.21	86.67	7.59	86.63	7.96	86.60	8.34	86.56	8.71	86.54	8.94	86.51	9.22	86.48	9.50	86.45	9.78	
87.84	6.14	87.76	6.52	87.73	6.91	87.70	7.29	87.67	7.67	87.63	8.05	87.60	8.43	87.56	8.81	87.54	9.04	87.51	9.32	87.48	9.60	87.45	9.88	
88.84	6.21	88.76	6.60	88.73	6.98	88.70	7.37	88.66	7.76	88.63	8.14	88.59	8.53	88.55	8.91	88.53	9.14	88.50	9.42	88.47	9.70	88.44	9.98	
89.84	6.28	89.75	6.67	89.72	7.06	89.69	7.45	89.66	7.84	89.62	8.24	89.59	8.63	89.55	9.01	89.53	9.24	89.50	9.52	89.47	9.80	89.44	10.08	
90.84	6.35	90.75	6.74	90.72	7.14	90.69	7.54	90.65	7.93	90.62	8.33	90.58	8.72	90.54	9.11	90.52	9.34	90.49	9.62	90.46	9.90	90.43	10.18	
91.84	6.42	91.75	6.82	91.72	7.22	91.68	7.62	91.65	8.02	91.61	8.42	91.58	8.82	91.54	9.21	91.52	9.44	91.49	9.72	91.46	10.00	91.43	10.28	
92.84	6.49	92.75	6.89	92.71	7.30	92.68	7.70	92.65	8.11	92.61	8.51	92.57	8.92	92.53	9.31	92.51	9.54	92.48	9.82	92.45	10.10	92.42	10.38	
93.84	6.56	93.74	6.97	93.71	7.38	93.68	7.78	93.64	8.19	93.61	8.60	93.57	9.01	93.53	9.41	93.51	9.64	93.48	9.92	93.45	10.20	93.42	10.48	
94.84	6.63	94.74	7.04	94.71	7.45	94.67	7.87	94.64	8.28	94.60	8.69	94.56	9.11	94.52	9.51	94.50	9.74	94.47	10.02	94.44	10.30	94.41	10.58	
95.84	6.70	95.74	7.12	95.71	7.53	95.67	7.95	95.63	8.37	95.60	8.78	95.56	9.20	95.52	9.61	95.50	9.84	95.47	10.12	95.44	10.40	95.41	10.68	
96.84	6.77	96.73	7.19	96.70	7.61	96.67	8.03	96.63	8.45	96.59	8.88	96.55	9.30	96.51	9.71	96.49	9.94	96.46	10.22	96.43	10.50	96.40	10.78	
97.84	6.84	97.73	7.26	97.70	7.69	97.66	8.12	97.63	8.54	97.59	8.97	97.55	9.39	97.51	9.81	97.49	10.04	97.46	10.32	97.43	10.60	97.40	10.88	
98.84	6.91	98.73	7.34	98.70	7.77	98.66	8.20	98.62	8.63	98.58	9.06	98.55	9.49	98.50	9.91	98.48	10.14	98.45	10.42	98.42	10.70	98.39	10.98	
99.84	6.98	99.73	7.41	99.69	7.85	99.66	8.28	99.62	8.72	99.59	9.15	99.54	9.59	99.50	10.01	99.48	10.24	99.45	10.52	99.42	10.80	99.39	11.08	
100.84	7.05	100.7	7.49	100.67	7.92	100.6	8.36	100.6	8.80	100.6	9.24	100.5	9.68	100.5	10.11	100.48	10.34	100.45	10.62	100.42	10.90	100.39	11.18	
101.84	7.12	101.7	7.56	101.67	8.00	101.6	8.45	101.6	8.89	101.6	9.33	101.5	9.78	101.5	10.21	101.48	10.44	101.45	10.72	101.42	11.00	101.39	11.28	
102.84	7.19	102.7	7.63	102.7	8.08	102.6	8.53	102.6	8.98	102.6	9.43	102.5	9.87	102.5	10.31	102.48	10.48	102.45	10.76	102.42	11.04	102.39	11.32	
103.84	7.26	103.7	7.71	103.67	8.16	103.6	8.61	103.6	9.07	103.6	9.52	103.5	9.97	103.5	10.41	103.48	10.58	103.45	10.86	103.42	11.14	103.39	11.42	
104.84	7.33	104.7	7.78	104.67	8.24	104.6	8.70	104.6	9.15	104.6	9.60	104.5	10.06	104.5	10.50	104.48	10.67	104.45	10.95	104.42	11.23	104.39	11.51	
105.84	7.40	105.7	7.86	105.67	8.32	105.6	8.78	105.5	9.24	105.6	9.69	105.5	10.10	105.5	10.54	105.48	10.71	105.45	11.00	105.42	11.28	105.39	11.56	
106.84	7.46	106.7	7.93	106.67	8.40	106.6	8.86	106.5	9.33	106.6	9.79	106.5	10.20	106.5	10.64	106.48	10.81	106.45	11.10	106.42	11.38	106.39	11.66	
107.84	7.53	107.7	8.00	107.67	8.47	107.6	8.94	107.5	9.41	107.6	9.88	107.5	10.29	107.5	10.73	107.48	10.96	107.45	11.25	107.42	11.53	107.39	11.81	
108.84	7.60	108.7	8.08	108.67	8.55	108.6	9.03	108.5	9.50	108.6	9.97	108.5	10.38	108.5	10.82	108.48	11.05	108.45	11.34	108.42	11.62	108.39	11.90	
109.84	7.67	109.7	8.15	109.67	8.63	109.6	9.11	109.5	9.59	109.6	10.07	109.5	10.48	109.5	10.92	109.48	11.15	109.45	11.44	109.42	11.72	109.39	12.00	
110.84	7.74	110.7	8.23	110.67	8.71	110.6	9.19	110.5	9.68	110.6	10.16	110.5	10.59	110.5	11.03	110.48	11.20	110.45	11.49	110.42	11.77	110.39	12.05	
111.84	7.81	111.7	8.30																					

TRAVERSE TABLE. 67.

6 Degrees										7 Degrees										8 Degrees												
0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	0,00	0,11	0,00	0,11	0,09	0,22	0,11	0,09	0,12	0,24	0,37	0,50	0,63	0,76	0,89	0,99	0,12	0,24	0,37	0,50	0,63	0,76	0,89	0,99	0,12	0,24	0,37	0,50	0,63	0,76	0,89	
2	1,99	0,21	1,99	0,22	1,99	0,23	1,99	0,23	1,99	0,24	2,98	0,35	1,99	0,24	2,98	0,35	1,99	0,24	2,98	0,35	1,99	0,24	2,98	0,35	1,99	0,24	2,98	0,35	1,99	0,24	2,98	0,35
3	2,99	0,32	2,98	0,33	2,98	0,34	2,98	0,34	2,98	0,35	3,97	0,41	2,98	0,35	3,97	0,41	2,98	0,35	3,97	0,41	2,98	0,35	3,97	0,41	2,98	0,35	3,97	0,41	2,98	0,35	3,97	0,41
4	3,98	0,41	3,98	0,44	3,97	0,45	3,97	0,45	3,97	0,46	4,96	0,53	3,97	0,45	4,96	0,53	3,97	0,46	4,96	0,53	3,97	0,45	4,96	0,53	3,97	0,46	4,96	0,53	3,97	0,45	4,96	0,53
5	4,97	0,52	4,97	0,54	4,97	0,55	4,97	0,55	4,97	0,56	5,96	0,63	4,97	0,55	5,96	0,63	4,97	0,56	5,96	0,63	4,97	0,55	5,96	0,63	4,97	0,56	5,96	0,63	4,97	0,55	5,96	0,63
6	5,97	0,63	5,95	0,65	5,96	0,68	5,96	0,68	5,96	0,69	6,95	0,75	5,96	0,68	6,95	0,75	5,96	0,69	6,95	0,75	5,96	0,68	6,95	0,75	5,96	0,69	6,95	0,75	5,96	0,68	6,95	0,75
7	6,96	0,73	6,96	0,76	6,96	0,77	6,96	0,77	6,96	0,78	7,95	0,86	6,96	0,77	7,95	0,86	6,96	0,78	7,95	0,86	6,96	0,77	7,95	0,86	6,96	0,78	7,95	0,86	6,96	0,77	7,95	0,86
8	7,96	0,84	7,95	0,87	7,95	0,88	7,95	0,88	7,95	0,89	8,94	0,97	7,95	0,88	8,94	0,97	7,95	0,89	8,94	0,97	7,95	0,88	8,94	0,97	7,95	0,89	8,94	0,97	7,95	0,88	8,94	0,97
9	8,95	0,94	8,95	0,98	8,94	0,99	8,94	0,99	8,94	1,00	9,93	1,08	8,94	0,99	9,93	1,08	8,94	1,00	9,93	1,08	8,94	0,99	9,93	1,08	8,94	1,00	9,93	1,08	8,94	0,99	9,93	1,08
10	9,95	1,05	9,94	1,09	9,94	1,10	9,94	1,10	9,94	1,11	10,92	1,19	9,94	1,10	10,92	1,19	9,94	1,11	10,92	1,19	9,94	1,10	10,92	1,19	9,94	1,11	10,92	1,19	9,94	1,10	10,92	1,19
11	10,94	1,15	10,94	1,20	10,93	1,21	10,93	1,21	10,93	1,22	11,92	1,29	10,93	1,22	11,92	1,29	10,93	1,23	11,92	1,29	10,93	1,22	11,92	1,29	10,93	1,23	11,92	1,29	10,93	1,22	11,92	1,29
12	11,93	1,25	11,93	1,31	11,92	1,32	11,92	1,32	11,92	1,33	12,91	1,40	11,92	1,33	12,91	1,40	11,92	1,34	12,91	1,40	11,92	1,33	12,91	1,40	11,92	1,34	12,91	1,40	11,92	1,33	12,91	1,40
13	12,93	1,36	12,92	1,42	12,92	1,43	12,92	1,43	12,92	1,44	13,90	1,51	12,92	1,44	13,90	1,51	12,92	1,45	13,90	1,51	12,92	1,44	13,90	1,51	12,92	1,45	13,90	1,51	12,92	1,44	13,90	1,51
14	13,92	1,46	13,92	1,52	13,91	1,53	13,91	1,53	13,91	1,54	14,89	1,61	13,91	1,54	14,89	1,61	13,91	1,55	14,89	1,61	13,91	1,54	14,89	1,61	13,91	1,55	14,89	1,61	13,91	1,54	14,89	1,61
15	14,92	1,57	14,91	1,63	14,90	1,64	14,90	1,64	14,90	1,65	15,88	1,70	14,90	1,65	15,88	1,70	14,90	1,66	15,88	1,70	14,90	1,65	15,88	1,70	14,90	1,66	15,88	1,70	14,90	1,65	15,88	1,70
16	15,91	1,67	15,91	1,74	15,90	1,75	15,90	1,75	15,90	1,76	16,87	1,81	15,90	1,76	16,87	1,81	15,90	1,77	16,87	1,81	15,90	1,76	16,87	1,81	15,90	1,77	16,87	1,81	15,90	1,76	16,87	1,81
17	16,91	1,78	16,90	1,85	16,89	1,86	16,89	1,86	16,89	1,87	17,86	1,91	16,89	1,87	17,86	1,91	16,89	1,88	17,86	1,91	16,89	1,87	17,86	1,91	16,89	1,88	17,86	1,91	16,89	1,87	17,86	1,91
18	17,90	1,88	17,89	1,96	17,88	1,97	17,88	1,97	17,88	1,98	18,85	2,02	17,88	1,98	18,85	2,02	17,88	1,99	18,85	2,02	17,88	1,98	18,85	2,02	17,88	1,99	18,85	2,02	17,88	1,98	18,85	2,02
19	18,90	1,98	18,89	2,07	18,88	2,08	18,88	2,08	18,88	2,09	19,84	2,12	18,88	2,09	19,84	2,12	18,88	2,10	19,84	2,12	18,88	2,09	19,84	2,12	18,88	2,10	19,84	2,12	18,88	2,09	19,84	2,12
20	19,89	2,09	19,88	2,18	19,87	2,19	19,87	2,19	19,87	2,20	20,83	2,25	19,87	2,20	20,83	2,25	19,87	2,21	20,83	2,25	19,87	2,20	20,83	2,25	19,87	2,21	20,83	2,25	19,87	2,20	20,83	2,25
21	20,89	2,20	20,88	2,29	20,86	2,30	20,86	2,30	20,86	2,31	21,82	2,36	20,86	2,31	21,82	2,36	20,86	2,32	21,82	2,36	20,86	2,31	21,82	2,36	20,86	2,32	21,82	2,36	20,86	2,31	21,82	2,36
22	21,88	2,30	21,87	2,39	21,85	2,40	21,85	2,40	21,85	2,41	22,81	2,46	21,85	2,41	22,81	2,46	21,85	2,42	22,81	2,46	21,85	2,41	22,81	2,46	21,85	2,42	22,81	2,46	21,85	2,41	22,81	2,46
23	22,87	2,41	22,86	2,50	22,84	2,51	22,84	2,51	22,84	2,52	23,78	2,57	22,84	2,52	23,78	2,57	22,84	2,53	23,78	2,57	22,84	2,52	23,78	2,57	22,84	2,53	23,78	2,57	22,84	2,52	23,78	2,57
24	23,87	2,51	23,86	2,61	23,83	2,62	23,83	2,62	23,83	2,63	24,78	2,68	23,83	2,63	24,78	2,68	23,83	2,64	24,78	2,68	23,83	2,63	24,78	2,68	23,83	2,64	24,78	2,68	23,83	2,63	24,78	2,68
25	24,86	2,61	24,85	2,72	24,82	2,73	24,82	2,73	24,82	2,74	25,78	2,79	24,82	2,74	25,78	2,79	24,82	2,75	25,78	2,79	24,82	2,74	25,78	2,79	24,82	2,75	25,78	2,79	24,82	2,74	25,78	2,79
26	25,86	2,72	25,85	2,83	25,83	2,84	25,83	2,84	25,83	2,85	26,78	2,90	25,83	2,85	26,78	2,90	25,83	2,86	26,78	2,90	25,83	2,85	26,78	2,90	25,83	2,86	26,78	2,90	25,83	2,85	26,78	2,90
27	26,85	2,82	26,84	2,94	26,82	2,95	26,82	2,95	26,82	2,96	27,78	2,99	26,82	2,96	27,78	2,99	26,82	2,97	27,78	2,99	26,82	2,96	27,78	2,99	26,82	2,97	27,78	2,99	26,82	2,96	27,78	2,99
28	27,85	2,93	27,83	3,05	27,82	3,06	27,82	3,06	27,82	3,07	28,78	3,10	27,82	3,07	28,78	3,10	27,82	3,08	28,78	3,10	27,82	3,07	28,78	3,10	27,82	3,08	28,78	3,10	27,82	3,07	28,78	3,10
29	28,84	3,03	28,83	3,16	28,81	3,17	28,81	3,17	28,81	3,18	29,78	3,21	28,81	3,18	29,78	3,21	28,81	3,19	29,78	3,21	28,81	3,18	29,78	3,21	28,81	3,19	29,78	3,21	28,81	3,18	29,78	3,21
30	29,84	3,14	29,82	3,27	29,81	3,28	29,81	3,28	29,81	3,29	30,78	3,32	29,81	3,29	30,78	3,32	29,81	3,30	30,78	3,32	29,81	3,29	30,78	3,32	29,81	3,30	30,78	3,32	29,81	3,29	30,78	3,32
31	30,83	3,24	30,82	3,38	30,80	3,39	30,80	3,39	30,80	3,40	31,78	3,43	30,80	3,40	31,78	3,43	30,80	3,41	31,78	3,43	30,80	3,40	31,78	3,43	30,80	3,41	31,78	3,43	30,80	3,40	31,78	3,43
32	31,83	3,35	31,81	3,49	31,79	3,50	31,79	3,50	31,79	3,51	32,78	3,54	31,79	3,51	32,78	3,54	31,79	3,52	32,78	3,54	31,79	3,51	32,78	3,54	31,79	3,52	32,78	3,54	31,79	3,51	32,78	3,54
33	32,82	3,45	32,80	3,59	32,79	3,60	32,79	3,60	32,79	3,61	33,78	3,64	32,79	3,61	33,78	3,64	32,79	3,62	33,78	3,64	32,79	3,61	33,78	3,64	32,79	3,62	33,78	3,64	32,79	3,61	33,78	3,64
34	33,81	3,56	33,80	3,70	33,78	3,71	33,78	3,71	33,78	3,72	34,78	3,75	33,78	3,72	34,78	3,75	33,78	3,73	34,78	3,75	33,78	3,72	34,78	3								

TRAVERSE TABLE. 67.

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Distance		6 Degrees						7 Degrees						Distance			
		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.				30 Min.	
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
61	60,67	6,38	60,64	6,64	60,61	6,91	60,58	7,17	60,55	7,43	60,51	7,70	60,48	7,96	60,44	8,23	60,41
62	61,66	6,48	61,63	6,78	61,60	7,02	61,57	7,29	61,54	7,56	61,50	7,82	61,47	8,09	61,43	8,36	61,40
63	62,66	6,59	62,43	6,86	62,60	7,13	62,56	7,41	62,53	7,68	62,50	7,95	62,46	8,22	62,42	8,50	62,39
64	63,65	6,69	63,62	6,97	63,59	7,24	63,56	7,52	63,52	7,80	63,49	8,08	63,45	8,35	63,42	8,63	63,39
65	64,64	6,80	64,61	7,08	64,58	7,36	64,55	7,64	64,52	7,92	64,48	8,20	64,44	8,48	64,41	8,77	64,38
66	65,64	6,90	65,61	7,19	65,58	7,47	65,55	7,76	65,51	8,04	65,47	8,33	65,44	8,62	65,40	8,90	65,37
67	66,63	7,01	66,60	7,29	66,57	7,58	66,54	7,88	66,50	8,17	66,46	8,45	66,43	8,75	66,39	9,04	66,36
68	67,63	7,11	67,60	7,41	67,56	7,70	67,53	7,99	67,49	8,29	67,46	8,58	67,42	8,88	67,38	9,17	67,35
69	68,62	7,21	68,59	7,51	68,56	7,81	68,52	8,11	68,48	8,41	68,45	8,71	68,41	9,01	68,37	9,31	68,34
70	69,62	7,32	69,59	7,62	69,55	7,92	69,52	8,23	69,48	8,53	69,44	8,83	69,40	9,14	69,36	9,44	69,33
71	70,61	7,42	70,58	7,73	70,54	8,04	70,51	8,35	70,47	8,65	70,43	8,96	70,39	9,27	70,35	9,58	70,32
72	71,61	7,53	71,57	7,84	71,54	8,15	71,50	8,46	71,46	8,77	71,42	9,09	71,38	9,40	71,34	9,71	71,31
73	72,60	7,63	72,57	7,95	72,53	8,26	72,49	8,58	72,46	8,90	72,42	9,21	72,37	9,53	72,33	9,84	72,30
74	73,59	7,74	73,56	8,06	73,52	8,38	73,49	8,70	73,45	9,02	73,41	9,34	73,37	9,66	73,32	9,98	73,29
75	74,58	7,84	74,56	8,17	74,52	8,49	74,48	8,82	74,44	9,14	74,40	9,46	74,36	9,79	74,32	10,11	74,29
76	75,58	7,94	75,55	8,27	75,51	8,60	75,47	8,93	75,44	9,26	75,39	9,59	75,35	9,92	75,31	10,25	75,28
77	76,58	8,05	76,54	8,38	76,51	8,72	76,47	9,05	76,43	9,38	76,38	9,72	76,34	10,05	76,30	10,38	76,27
78	77,57	8,15	77,54	8,49	77,49	8,83	77,46	9,17	77,42	9,51	77,38	9,84	77,33	10,18	77,29	10,52	77,26
79	78,57	8,26	78,53	8,61	78,49	8,94	78,45	9,29	78,41	9,63	78,37	9,97	78,32	10,31	78,28	10,65	78,25
80	79,56	8,36	79,53	8,71	79,49	9,06	79,45	9,40	79,40	9,75	79,36	10,09	79,32	10,44	79,27	10,79	79,24
81	80,56	8,47	80,52	8,82	80,48	9,17	80,44	9,52	80,39	9,87	80,35	10,22	80,31	10,57	80,26	10,92	80,23
82	81,55	8,57	81,51	8,93	81,47	9,28	81,43	9,64	81,38	9,99	81,34	10,35	81,30	10,70	81,25	11,06	81,22
83	82,55	8,68	82,51	9,04	82,47	9,40	82,43	9,76	82,38	10,12	82,34	10,48	82,29	10,83	82,24	11,19	82,21
84	83,54	8,78	83,50	9,15	83,46	9,51	83,42	9,87	83,37	10,24	83,33	10,60	83,28	10,96	83,23	11,33	83,20
85	84,54	8,88	84,50	9,26	84,45	9,62	84,41	9,99	84,37	10,36	84,32	10,73	84,27	11,10	84,22	11,46	84,19
86	85,53	8,99	85,49	9,36	85,45	9,73	85,40	10,11	85,36	10,48	85,31	10,85	85,26	11,23	85,22	11,60	85,19
87	86,52	9,09	86,48	9,47	86,44	9,85	86,40	10,23	86,35	10,60	86,30	10,98	86,26	11,36	86,21	11,73	86,18
88	87,52	9,20	87,48	9,58	87,43	9,96	87,39	10,34	87,34	10,73	87,30	11,11	87,25	11,49	87,20	11,87	87,17
89	88,51	9,30	88,47	9,69	88,43	10,07	88,38	10,46	88,34	10,85	88,29	11,23	88,24	11,62	88,19	12,00	88,16
90	89,51	9,41	89,47	9,80	89,42	10,19	89,38	10,58	89,33	10,97	89,28	11,36	89,23	11,75	89,18	12,14	89,15
91	90,50	9,51	90,46	9,91	90,41	10,30	90,37	10,70	90,32	11,09	90,27	11,49	90,22	11,88	90,17	12,27	90,14
92	91,50	9,62	91,45	10,02	91,41	10,41	91,36	10,81	91,31	11,21	91,26	11,61	91,21	12,01	91,16	12,41	91,12
93	92,49	9,72	92,45	10,13	92,40	10,53	92,36	10,93	92,31	11,33	92,26	11,74	92,20	12,14	92,15	12,54	92,11
94	93,48	9,83	93,44	10,23	93,40	10,64	93,35	11,05	93,30	11,46	93,25	11,86	93,20	12,27	93,14	12,68	93,10
95	94,48	9,93	94,44	10,34	94,39	10,75	94,34	11,17	94,29	11,58	94,24	11,99	94,19	12,40	94,13	12,81	94,09
96	95,47	10,04	95,43	10,45	95,38	10,87	95,34	11,28	95,29	11,70	95,23	12,12	95,18	12,53	95,12	12,95	95,08
97	96,46	10,14	96,42	10,56	96,38	10,98	96,33	11,40	96,28	11,82	96,22	12,24	96,17	12,66	96,11	13,08	96,07
98	97,46	10,24	97,42	10,67	97,37	11,09	97,32	11,52	97,27	11,94	97,22	12,37	97,16	12,79	97,11	13,22	97,07
99	98,46	10,35	98,41	10,78	98,36	11,21	98,31	11,64	98,26	12,07	98,21	12,49	98,15	12,92	98,10	13,35	98,06
100	99,45	10,45	99,41	10,89	99,36	11,32	99,31	11,75	99,26	12,19	99,20	12,62	99,14	13,05	99,09	13,49	99,05
101	100,45	10,56	100,41	11,00	100,36	11,43	100,31	11,87	100,26	12,31	100,21	12,75	100,15	13,18	100,10	13,62	100,06
102	101,45	10,66	101,41	11,11	101,36	11,55	101,31	11,99	101,26	12,43	101,21	12,87	101,15	13,31	101,10	13,75	101,06
103	102,45	10,77	102,41	11,21	102,36	11,66	102,31	12,11	102,26	12,55	102,21	13,00	102,15	13,45	102,10	13,89	102,06
104	103,44	10,87	103,41	11,32	103,36	11,77	103,31	12,22	103,26	12,68	103,21	13,13	103,15	13,58	103,10	14,02	103,06
105	104,44	10,98	104,41	11,43	104,36	11,89	104,31	12,34	104,26	12,80	104,21	13,26	104,15	13,71	104,10	14,16	104,06
106	105,44	11,08	105,41	11,54	105,36	12,00	105,31	12,46	105,26	12,92	105,21	13,38	105,15	13,84	105,10	14,30	105,06
107	106,44	11,18	106,41	11,65	106,36	12,11	106,31	12,58	106,26	13,04	106,21	13,50	106,15	13,97	106,10	14,43	106,06
108	107,44	11,29	107,41	11,76	107,36	12,23	107,31	12,69	107,26	13,16	107,21	13,63	107,15	14,10	107,10	14,56	107,06
109	108,44	11,39	108,41	11,87	108,36	12,34	108,31	12,81	108,26	13,28	108,21	13,76	108,15	14,23	108,10	14,70	108,06
110	109,44	11,50	109,41	11,98	109,36	12,45	109,31	12,93	109,26	13,41	109,21	13,88	109,15	14,36	109,10	14,83	109,06
111	110,44	11,60	110,41	12,09	110,36	12,57	110,31	13,05	110,26	13,53	110,21	14,01	110,15	14,49	110,10	14,97	110,06
112	111,44	11,71	111,41	12,19	111,36	12,68	111,31	13,16	111,26	13,65	111,21	14,14	111,15	14,62	111,10	15,10	111,06
113	112,44	11,81	112,41	12,30	112,36	12,79	112,31	13,28	112,26	13,77	112,21	14,26	112,15	14,75	112,10	15,24	112,06
114	113,44	11,92	113,41	12,41	113,36	12,90	113,31	13,40	113,26	13,89	113,21	14,39	113,15	14,88	113,10	15,37	113,06
115	114,44	12,02	114,41	12,52	114,36	13,02	114,31	13,52	114,26	14,01	114,21	14,51	114,15	15,01	114,10	15,51	114,06
116	115,44	12,13	115,41	12,63	115,36	13,13	115,31	13,63	115,26	14,14	115,21	14,64	115,15	15,14	115,10	16,04	115,06
117	116,44	12,23	116,41	12,74	116,36	13,24	116,31	13,75	116,26	14,26	116,21	14,76	116,15	15,26	116,10	16,14	116,06
118	117,44	12,33	117,41	12,85	117,36	13,36	117,31	13,87	117,26	14,38	117,21	14,89	117,15	15,39	117,10	16,24	117,06
119	118,44	12,44	118,41	12,95	118,36	13,47	118,31	13,99	118,26	14,50	118,21	15,02	118,15	15,53	118,10	16,34	118,06
120	119,44	12,54	119,41	13,06	119,36	13,58	119,31	14,11	119,26	14,62	119,21	15,17	119,15	16,06	119,10	16,44	119,06
121	120,44	13,05	120,41	13,17	120,36	13,69	120,31										

84 Degrees

83 Degrees

82 Degrees

TRAVERSE TABLE. 67.

8 Degrees			9 Degrees			80 Degrees		
0 Min.			15 Min.			30 Min.		
Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.
1	0.00	0.14	0.09	0.15	0.99	0.16	0.99	0.16
2	1.98	0.29	1.98	0.30	1.98	0.31	1.97	0.32
3	3.97	0.43	3.97	0.44	3.97	0.46	3.96	0.48
4	5.95	0.57	5.95	0.59	5.95	0.61	5.95	0.63
5	7.93	0.72	7.93	0.74	7.93	0.76	7.93	0.78
6	9.91	0.86	9.91	0.88	9.91	0.90	9.91	0.92
7	11.88	1.00	11.88	1.02	11.88	1.04	11.88	1.06
8	13.86	1.15	13.86	1.17	13.86	1.19	13.86	1.21
9	15.84	1.29	15.84	1.31	15.84	1.33	15.84	1.35
10	17.82	1.44	17.82	1.46	17.82	1.48	17.82	1.50
11	19.80	1.58	19.80	1.60	19.80	1.62	19.80	1.64
12	21.78	1.72	21.78	1.74	21.78	1.76	21.78	1.78
13	23.76	1.87	23.76	1.89	23.76	1.91	23.76	1.93
14	25.74	2.01	25.74	2.03	25.74	2.05	25.74	2.07
15	27.72	2.15	27.72	2.17	27.72	2.19	27.72	2.21
16	29.70	2.29	29.70	2.31	29.70	2.33	29.70	2.35
17	31.68	2.44	31.68	2.46	31.68	2.48	31.68	2.50
18	33.66	2.58	33.66	2.60	33.66	2.62	33.66	2.64
19	35.64	2.72	35.64	2.74	35.64	2.76	35.64	2.78
20	37.62	2.87	37.62	2.89	37.62	2.91	37.62	2.93
21	39.60	3.01	39.60	3.03	39.60	3.05	39.60	3.07
22	41.58	3.15	41.58	3.17	41.58	3.19	41.58	3.21
23	43.56	3.29	43.56	3.31	43.56	3.33	43.56	3.35
24	45.54	3.44	45.54	3.46	45.54	3.48	45.54	3.50
25	47.52	3.58	47.52	3.60	47.52	3.62	47.52	3.64
26	49.50	3.72	49.50	3.74	49.50	3.76	49.50	3.78
27	51.48	3.87	51.48	3.89	51.48	3.91	51.48	3.93
28	53.46	4.01	53.46	4.03	53.46	4.05	53.46	4.07
29	55.44	4.15	55.44	4.17	55.44	4.19	55.44	4.21
30	57.42	4.29	57.42	4.31	57.42	4.33	57.42	4.35
31	59.40	4.44	59.40	4.46	59.40	4.48	59.40	4.50
32	61.38	4.58	61.38	4.60	61.38	4.62	61.38	4.64
33	63.36	4.72	63.36	4.74	63.36	4.76	63.36	4.78
34	65.34	4.87	65.34	4.89	65.34	4.91	65.34	4.93
35	67.32	5.01	67.32	5.03	67.32	5.05	67.32	5.07
36	69.30	5.15	69.30	5.17	69.30	5.19	69.30	5.21
37	71.28	5.29	71.28	5.31	71.28	5.33	71.28	5.35
38	73.26	5.44	73.26	5.46	73.26	5.48	73.26	5.50
39	75.24	5.58	75.24	5.60	75.24	5.62	75.24	5.64
40	77.22	5.72	77.22	5.74	77.22	5.76	77.22	5.78
41	79.20	5.87	79.20	5.89	79.20	5.91	79.20	5.93
42	81.18	6.01	81.18	6.03	81.18	6.05	81.18	6.07
43	83.16	6.15	83.16	6.17	83.16	6.19	83.16	6.21
44	85.14	6.29	85.14	6.31	85.14	6.33	85.14	6.35
45	87.12	6.44	87.12	6.46	87.12	6.48	87.12	6.50
46	89.10	6.58	89.10	6.60	89.10	6.62	89.10	6.64
47	91.08	6.72	91.08	6.74	91.08	6.76	91.08	6.78
48	93.06	6.87	93.06	6.89	93.06	6.91	93.06	6.93
49	95.04	7.01	95.04	7.03	95.04	7.05	95.04	7.07
50	97.02	7.15	97.02	7.17	97.02	7.19	97.02	7.21
51	99.00	7.29	99.00	7.31	99.00	7.33	99.00	7.35
52	100.98	7.44	100.98	7.46	100.98	7.48	100.98	7.50
53	102.96	7.58	102.96	7.60	102.96	7.62	102.96	7.64
54	104.94	7.72	104.94	7.74	104.94	7.76	104.94	7.78
55	106.92	7.87	106.92	7.89	106.92	7.91	106.92	7.93
56	108.90	8.01	108.90	8.03	108.90	8.05	108.90	8.07
57	110.88	8.15	110.88	8.17	110.88	8.19	110.88	8.21
58	112.86	8.29	112.86	8.31	112.86	8.33	112.86	8.35
59	114.84	8.44	114.84	8.46	114.84	8.48	114.84	8.50
60	116.82	8.58	116.82	8.60	116.82	8.62	116.82	8.64
61	118.80	8.72	118.80	8.74	118.80	8.76	118.80	8.78
62	120.78	8.87	120.78	8.89	120.78	8.91	120.78	8.93
63	122.76	9.01	122.76	9.03	122.76	9.05	122.76	9.07
64	124.74	9.15	124.74	9.17	124.74	9.19	124.74	9.21
65	126.72	9.29	126.72	9.31	126.72	9.33	126.72	9.35
66	128.70	9.44	128.70	9.46	128.70	9.48	128.70	9.50
67	130.68	9.58	130.68	9.60	130.68	9.62	130.68	9.64
68	132.66	9.72	132.66	9.74	132.66	9.76	132.66	9.78
69	134.64	9.87	134.64	9.89	134.64	9.91	134.64	9.93
70	136.62	10.01	136.62	10.03	136.62	10.05	136.62	10.07
71	138.60	10.15	138.60	10.17	138.60	10.19	138.60	10.21
72	140.58	10.29	140.58	10.31	140.58	10.33	140.58	10.35
73	142.56	10.44	142.56	10.46	142.56	10.48	142.56	10.50
74	144.54	10.58	144.54	10.60	144.54	10.62	144.54	10.64
75	146.52	10.72	146.52	10.74	146.52	10.76	146.52	10.78
76	148.50	10.87	148.50	10.89	148.50	10.91	148.50	10.93
77	150.48	11.01	150.48	11.03	150.48	11.05	150.48	11.07
78	152.46	11.15	152.46	11.17	152.46	11.19	152.46	11.21
79	154.44	11.29	154.44	11.31	154.44	11.33	154.44	11.35
80	156.42	11.44	156.42	11.46	156.42	11.48	156.42	11.50
81	158.40	11.58	158.40	11.60	158.40	11.62	158.40	11.64
82	160.38	11.72	160.38	11.74	160.38	11.76	160.38	11.78
83	162.36	11.87	162.36	11.89	162.36	11.91	162.36	11.93
84	164.34	12.01	164.34	12.03	164.34	12.05	164.34	12.07
85	166.32	12.15	166.32	12.17	166.32	12.19	166.32	12.21
86	168.30	12.29	168.30	12.31	168.30	12.33	168.30	12.35
87	170.28	12.44	170.28	12.46	170.28	12.48	170.28	12.50
88	172.26	12.58	172.26	12.60	172.26	12.62	172.26	12.64
89	174.24	12.72	174.24	12.74	174.24	12.76	174.24	12.78
90	176.22	12.87	176.22	12.89	176.22	12.91	176.22	12.93
91	178.20	13.01	178.20	13.03	178.20	13.05	178.20	13.07
92	180.18	13.15	180.18	13.17	180.18	13.19	180.18	13.21
93	182.16	13.29	182.16	13.31	182.16	13.33	182.16	13.35
94	184.14	13.44	184.14	13.46	184.14	13.48	184.14	13.50
95	186.12	13.58	186.12	13.60	186.12	13.62	186.12	13.64
96	188.10	13.72	188.10	13.74	188.10	13.76	188.10	13.78
97	190.08	13.87	190.08	13.89	190.08	13.91	190.08	13.93
98	192.06	14.01	192.06	14.03	192.06	14.05	192.06	14.07
99	194.04	14.15	194.04	14.17	194.04	14.19	194.04	14.21
100	196.02	14.29	196.02	14.31	196.02	14.33	196.02	14.35
101	198.00	14.44	198.00	14.46	198.00	14.48	198.00	14.50
102	200.00	14.58	200.00	14.60	200.00	14.62	200.00	14.64
103	202.00	14.72	202.00	14.74	202.00	14.76	202.00	14.78
104	204.00	14.87	204.00	14.89	204.00	14.91	204.00	14.93
105	206.00	15.01	206.00	15.03	206.00	15.05	206.00	15.07
106	208.00	15.15	208.00	15.17	208.00	15.19	208.00	15.21
107	210.00	15.29	210.00	15.31	210.00	15.33	210.00	15.35
108	212.00	15.44	212.00	15.46	212.00	15.48	212.00	15.50
109	214.00	15.58	214.00	15.60	214.00	15.62	214.00	15.64
110	216.00	15.72	216.00	15.74	216.00	15.76	216.00	15.78
111	218.00	15.87	218.00	15.89	218.00	15.91	218.00	15.93
112	220.00	16.01	220.00	16.03	220.00	16.05	220.00	16.07
113	222.00	16.15	222.00	16.17	222.00	16.19	222.00	16.21
114	224.00	16.29	224.00	16.31	224.00	16.33	224.00	16.35
115	226.00	16.44	226.00	16.46	226.00	16.48	226.00	16.50
116	228.00	16.58	228.00	16.60	228.00	16.62	228.00	16.64
117	230.00	16.72	230.00	16.74	230.00	16.76	230.00	16.78
118	232.00	16.87	232.00	16.89	232.00	16.91	232.00	16.93
119	234.00	17.01	234.00	17.03	234.00	17.05	234.00	17.07
120	236.00	17.15	236.00	17.17	236.00	17.19	236.00	17.21
121	238.00	17.29	238.00	17.31	238.00	17.33	238.00	17.35
122	240.00	17.44	240.00	17.46	240.00	17.48	240.00	17.50
123	242.00	17.58	242.00	17.60	242.00	17.62	242.00	17.64
124	244.00	17.72	244.00	17.74	244.00	17.76	244.00	17.78
125	246.00	17.87	246.00	17.89	246.00	17.91	246.00	17.93
126	248.00	18.01	248.00	18.03	248.00	18.05	248.00	18.07
127	250.00	18.15	250.00	18.17	250.00	18.19	250.00	18.21
128	252.00	18.29	252.00	18.31	252.00	18.33	252.00	18.35
129	254.00	18.44	254.00	18.46	254.00	18.48	25	

TRAVERSE TABLE. 67.

95

8 Degrees										9 Degrees										80 Degrees										81 Degrees										82 Deg.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
Distance		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		Distance																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
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80 Degrees

81 Degrees

82 Deg.

[illegible]

TRAVERSE TABLE. 67.

97

10 Degrees			11 Degrees			78 Degrees			79 Degrees			80 Degrees		
o Min.		15 Min.	30 Min.		45 Min.	o Min.		15 Min.	30 Min.		45 Min.	o Min.		15 Min.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance
61	60,07	60,03	10,85	59,98	11,12	59,93	11,38	59,88	11,56	59,83	11,64	59,78	12,16	59,72
62	61,06	61,01	11,03	60,96	11,30	60,91	11,56	60,86	11,83	60,81	12,09	60,76	12,36	60,70
63	62,04	61,99	11,21	61,94	11,48	61,89	11,75	61,84	12,02	61,79	12,29	61,74	12,56	61,68
64	63,03	62,98	11,39	62,93	11,66	62,88	11,94	62,83	12,24	62,77	12,49	62,72	12,76	62,66
65	64,01	63,96	11,57	63,91	11,85	63,86	12,12	63,81	12,40	63,75	12,68	63,70	12,96	63,64
66	65,00	64,95	11,74	64,89	12,03	64,84	12,31	64,79	12,59	64,73	12,88	64,68	13,16	64,62
67	65,98	64,93	11,92	64,88	12,21	64,83	12,50	64,77	12,79	64,71	13,07	64,66	13,36	64,60
68	66,97	65,92	12,10	65,86	12,39	65,81	12,68	65,75	12,98	65,69	13,27	65,64	13,56	65,58
69	67,95	66,90	12,28	66,84	12,57	66,79	12,87	66,73	13,17	66,67	13,46	66,61	13,76	66,55
70	68,94	67,88	12,46	67,82	12,76	67,77	13,06	67,71	13,36	67,65	13,66	67,59	13,96	67,53
71	69,92	68,87	12,63	68,81	12,94	68,75	13,24	68,69	13,55	68,63	13,85	68,57	14,16	68,51
72	70,91	69,85	12,81	69,79	13,12	69,74	13,43	69,68	13,74	69,62	14,05	69,56	14,36	69,50
73	71,89	70,83	12,99	70,77	13,30	70,72	13,61	70,66	13,93	70,60	14,24	70,54	14,55	70,48
74	72,88	71,82	13,17	71,76	13,49	71,71	13,80	71,65	14,12	71,59	14,44	71,53	14,75	71,47
75	73,86	72,80	13,35	72,74	13,67	72,69	13,99	72,63	14,31	72,57	14,63	72,51	14,95	72,45
76	74,85	73,79	13,52	73,73	13,85	73,68	14,18	73,62	14,50	73,56	14,83	73,50	15,15	73,44
77	75,83	74,77	13,70	74,71	14,03	74,66	14,36	74,60	14,69	74,54	15,02	74,48	15,35	74,42
78	76,82	75,76	13,88	75,71	14,21	75,66	14,55	75,60	14,88	75,54	15,20	75,48	15,55	75,42
79	77,80	76,74	14,06	76,69	14,39	76,64	14,73	76,58	15,07	76,52	15,41	76,46	15,75	76,40
80	78,79	77,73	14,24	77,68	14,58	77,63	14,92	77,57	15,27	77,51	15,61	77,45	15,95	77,39
81	79,77	78,71	14,41	78,66	14,76	78,61	15,11	78,55	15,47	78,49	15,80	78,43	16,15	78,37
82	80,76	79,70	14,59	79,65	14,94	79,60	15,30	79,54	15,65	79,48	16,00	79,42	16,35	79,36
83	81,74	80,68	14,77	80,63	15,13	80,58	15,48	80,52	15,84	80,46	16,19	80,40	16,55	80,34
84	82,73	81,67	14,95	81,62	15,31	81,57	15,67	81,51	16,03	81,45	16,39	81,39	16,75	81,33
85	83,72	82,66	15,13	82,59	15,49	82,54	15,86	82,48	16,22	82,42	16,58	82,36	16,95	82,30
86	84,71	83,65	15,30	83,56	15,67	83,51	16,04	83,45	16,41	83,39	16,78	83,33	17,15	83,27
87	85,69	84,63	15,48	84,54	15,85	84,49	16,23	84,43	16,60	84,37	17,00	84,31	17,37	84,25
88	86,68	85,62	15,66	85,51	16,04	85,46	16,43	85,40	16,80	85,34	17,17	85,28	17,55	85,22
89	87,67	86,61	15,84	86,53	16,22	86,48	16,60	86,42	17,00	86,36	17,37	86,30	17,75	86,24
90	88,65	87,59	16,02	87,49	16,40	87,44	16,79	87,38	17,17	87,32	17,55	87,26	17,94	87,20
91	89,62	88,56	16,19	88,49	16,58	88,44	17,00	88,38	17,37	88,32	17,75	88,26	18,15	88,20
92	90,60	89,54	16,37	89,46	16,77	89,40	17,16	89,34	17,55	89,28	17,93	89,22	18,33	89,16
93	91,59	90,53	16,55	90,46	16,95	90,39	17,35	90,33	17,73	90,27	18,11	90,21	18,51	90,15
94	92,57	91,51	16,73	91,44	17,13	91,37	17,55	91,31	17,93	91,25	18,31	91,19	18,71	91,13
95	93,56	92,50	16,91	92,43	17,31	92,35	17,73	92,29	18,11	92,23	18,49	92,17	18,89	92,11
96	94,54	93,48	17,09	93,41	17,50	93,35	17,93	93,29	18,31	93,23	18,69	93,17	19,09	93,11
97	95,53	94,47	17,27	94,39	17,68	94,32	18,11	94,26	18,49	94,20	18,87	94,14	19,27	94,08
98	96,51	95,45	17,45	95,38	17,86	95,30	18,29	95,24	18,67	95,18	19,05	95,12	19,45	95,06
99	97,50	96,44	17,63	96,36	18,04	96,28	18,47	96,22	18,85	96,16	19,23	96,10	19,65	96,04
100	98,48	97,42	17,81	97,34	18,22	97,26	18,65	97,20	19,03	97,14	19,41	97,08	19,89	97,02
101	99,47	98,41	17,99	98,33	18,41	98,25	18,84	98,19	19,22	98,13	19,60	98,07	20,01	98,01
102	100,44	99,38	18,17	99,31	18,59	99,23	19,03	99,17	19,41	99,11	19,79	99,05	20,19	99,01
103	101,42	100,36	18,35	100,33	18,77	100,21	19,21	100,15	19,59	100,09	19,97	100,01	20,37	99,95
104	102,40	101,34	18,53	101,31	18,95	101,22	19,40	101,16	19,79	101,10	20,17	101,04	20,55	100,98
105	103,38	102,32	18,71	102,33	19,14	102,22	19,59	102,16	19,97	102,10	20,35	102,04	20,73	101,97
106	104,36	103,30	18,89	103,29	19,32	103,22	19,77	103,16	20,17	103,10	20,55	103,04	20,93	102,97
107	105,34	104,28	19,07	104,24	19,50	104,17	19,96	104,11	20,35	104,05	20,73	103,99	21,11	103,93
108	106,32	105,26	19,25	105,22	19,68	105,15	20,14	105,09	20,55	105,03	20,93	104,97	21,31	104,91
109	107,30	106,24	19,43	106,20	19,86	106,13	20,33	106,07	20,73	106,01	21,11	105,95	21,51	105,89
110	108,28	107,22	19,61	107,18	20,05	107,11	20,52	107,05	20,93	106,99	21,31	106,93	21,71	106,87
111	109,26	108,20	19,79	108,16	20,23	108,09	20,70	108,03	21,11	107,97	21,51	107,91	21,91	107,85
112	110,24	109,18	19,97	109,14	20,41	109,07	20,89	109,01	21,29	108,95	21,69	108,89	22,11	108,83
113	111,22	110,16	20,15	110,12	20,59	110,05	21,08	109,99	21,47	109,93	21,87	109,87	22,31	109,81
114	112,20	111,14	20,33	111,10	20,78	111,03	21,26	110,97	21,65	110,91	22,05	110,85	22,45	110,79
115	113,18	112,12	20,51	112,08	20,96	112,01	21,45	111,95	21,83	111,89	22,24	111,83	22,65	111,77
116	114,16	113,10	20,69	113,06	21,14	113,00	21,64	112,93	22,03	112,87	22,44	112,81	22,85	112,75
117	115,14	114,08	20,87	114,04	21,32	114,00	21,82	113,93	22,23	113,87	22,63	113,81	23,05	113,75
118	116,12	115,06	21,05	115,02	21,50	115,00	22,01	114,91	22,41	114,85	22,83	114,79	23,25	114,73
119	117,10	116,04	21,23	116,00	21,69	116,00	22,20	115,92	22,60	115,86	23,02	115,80	23,45	115,74
120	118,08	117,02	21,41	117,00	21,87	117,00	22,38	116,90	22,79	116,84	23,22	116,78	23,65	116,72
121	119,06	118,00	21,59	118,00	22,05	118,00	22,56	117,92	23,00	117,86	23,41	117,80	23,85	117,74
122	120,04	119,00	21,77	119,00	22,23	119,00	22,74	118,94	23,18	118,88	23,59	118,82	24,05	118,76

VOL. II.

I

TRAVERSE TABLE. 67.

Distance		12 Degrees				13 Degrees			
		0 Min.		15 Min.		30 Min.		45 Min.	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0.98	0.21	0.98	0.22	0.98	0.23	0.97	0.23	0.97
2	1.96	0.42	1.95	0.43	1.95	0.45	1.95	0.47	1.94
3	2.93	0.62	2.93	0.63	2.92	0.68	2.92	0.69	2.91
4	3.91	0.83	3.91	0.85	3.90	0.90	3.89	0.92	3.89
5	4.89	1.04	4.89	1.06	4.88	1.10	4.87	1.13	4.86
6	5.87	1.25	5.86	1.27	5.85	1.32	5.85	1.35	5.84
7	6.85	1.46	6.84	1.49	6.83	1.55	6.82	1.58	6.81
8	7.83	1.66	7.82	1.70	7.81	1.73	7.80	1.77	7.79
9	8.80	1.87	8.80	1.91	8.79	1.95	8.78	1.99	8.77
10	9.78	2.08	9.77	2.12	9.76	2.16	9.75	2.21	9.74
11	10.76	2.29	10.75	2.33	10.74	2.38	10.73	2.43	10.72
12	11.74	2.50	11.73	2.55	11.72	2.60	11.70	2.65	11.69
13	12.72	2.70	12.71	2.76	12.69	2.81	12.68	2.87	12.67
14	13.70	2.91	13.68	2.97	13.67	3.03	13.66	3.09	13.64
15	14.67	3.12	14.66	3.18	14.65	3.25	14.63	3.31	14.62
16	15.65	3.33	15.64	3.40	15.62	3.46	15.61	3.53	15.59
17	16.63	3.53	16.61	3.61	16.60	3.68	16.58	3.75	16.57
18	17.61	3.74	17.59	3.82	17.57	3.90	17.56	3.97	17.54
19	18.59	3.95	18.57	4.03	18.55	4.11	18.53	4.19	18.51
20	19.56	4.16	19.55	4.24	19.53	4.33	19.51	4.41	19.49
21	20.54	4.37	20.52	4.46	20.50	4.55	20.48	4.64	20.46
22	21.52	4.57	21.50	4.67	21.48	4.76	21.46	4.86	21.44
23	22.50	4.78	22.48	4.88	22.46	4.98	22.43	5.08	22.41
24	23.48	4.99	23.45	5.09	23.43	5.20	23.41	5.30	23.38
25	24.45	5.20	24.43	5.31	24.41	5.41	24.39	5.51	24.36
26	25.43	5.41	25.41	5.52	25.38	5.63	25.36	5.74	25.33
27	26.41	5.61	26.39	5.73	26.36	5.85	26.34	5.96	26.31
28	27.39	5.82	27.36	5.94	27.34	6.06	27.31	6.18	27.28
29	28.37	6.03	28.34	6.15	28.31	6.28	28.29	6.40	28.26
30	29.35	6.24	29.32	6.37	29.29	6.49	29.26	6.62	29.23
31	30.32	6.45	30.29	6.58	30.27	6.71	30.24	6.84	30.21
32	31.30	6.65	31.27	6.79	31.24	6.93	31.21	7.06	31.18
33	32.28	6.86	32.25	7.00	32.22	7.14	32.19	7.28	32.15
34	33.26	7.07	33.23	7.21	33.19	7.36	33.16	7.50	33.13
35	34.24	7.28	34.20	7.43	34.17	7.58	34.14	7.73	34.10
36	35.21	7.48	35.18	7.64	35.15	7.79	35.11	7.95	35.08
37	36.19	7.69	36.16	7.85	36.12	8.01	36.09	8.17	36.05
38	37.17	7.90	37.14	8.06	37.10	8.23	37.06	8.39	37.03
39	38.15	8.11	38.11	8.28	38.08	8.44	38.04	8.61	38.00
40	39.13	8.32	39.09	8.48	39.05	8.66	39.01	8.83	38.98
41	40.10	8.52	40.07	8.70	40.03	8.88	39.99	9.05	39.95
42	41.08	8.73	41.04	8.91	41.01	9.09	40.97	9.27	40.92
43	42.06	8.94	42.02	9.12	41.98	9.31	41.94	9.49	41.90
44	43.04	9.15	43.00	9.34	42.96	9.52	42.92	9.71	42.87
45	44.02	9.36	43.98	9.55	43.93	9.74	43.89	9.93	43.85
46	45.00	9.56	44.95	9.76	44.91	9.96	44.87	10.15	44.82
47	45.97	9.77	45.93	9.97	45.89	10.17	45.84	10.37	45.80
48	46.95	9.98	46.91	10.19	46.86	10.39	46.82	10.59	46.77
49	47.93	10.19	47.88	10.40	47.84	10.61	47.79	10.81	47.74
50	48.91	10.40	48.86	10.61	48.82	10.82	48.77	11.04	48.72
51	49.89	10.60	49.84	10.82	49.79	11.03	49.74	11.26	49.69
52	50.86	10.81	50.82	11.03	50.77	11.26	50.72	11.48	50.67
53	51.84	11.02	51.79	11.25	51.74	11.47	51.69	11.70	51.64
54	52.82	11.23	52.77	11.46	52.72	11.69	52.67	11.92	52.62
55	53.80	11.44	53.75	11.67	53.70	11.91	53.65	12.14	53.59
56	54.78	11.64	54.73	11.88	54.67	12.12	54.62	12.36	54.57
57	55.76	11.85	55.70	12.09	55.65	12.34	55.60	12.58	55.54
58	56.73	12.06	56.68	12.31	56.63	12.55	56.57	12.80	56.52
59	57.71	12.27	57.66	12.52	57.60	12.77	57.55	13.02	57.49
60	58.69	12.48	58.63	12.73	58.58	12.99	58.52	13.24	58.46
Distance		0 Min.		15 Min.		30 Min.		45 Min.	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0.97	0.21	0.97	0.22	0.97	0.23	0.97	0.23	0.97
2	1.94	0.48	1.95	0.49	1.95	0.50	1.95	0.51	1.94
3	2.91	0.71	2.92	0.72	2.92	0.73	2.92	0.74	2.91
4	3.88	0.95	3.89	0.96	3.89	0.97	3.89	0.98	3.88
5	4.85	1.19	4.86	1.20	4.86	1.21	4.86	1.22	4.85
6	5.82	1.43	5.83	1.44	5.83	1.45	5.83	1.46	5.82
7	6.79	1.66	6.80	1.67	6.80	1.68	6.80	1.69	6.79
8	7.76	1.90	7.77	1.91	7.77	1.92	7.77	1.93	7.76
9	8.73	2.14	8.74	2.15	8.74	2.16	8.74	2.17	8.73
10	9.70	2.38	9.71	2.39	9.71	2.40	9.71	2.41	9.70
11	10.67	2.62	10.68	2.63	10.68	2.64	10.68	2.65	10.67
12	11.64	2.85	11.65	2.86	11.65	2.87	11.65	2.88	11.64
13	12.61	3.09	12.62	3.10	12.62	3.11	12.62	3.12	12.61
14	13.58	3.33	13.59	3.34	13.59	3.35	13.59	3.36	13.58
15	14.55	3.57	14.56	3.58	14.56	3.59	14.56	3.60	14.55
16	15.52	3.80	15.53	3.81	15.53	3.82	15.53	3.83	15.52
17	16.49	4.04	16.50	4.05	16.50	4.06	16.50	4.07	16.49
18	17.46	4.28	17.47	4.29	17.47	4.30	17.47	4.31	17.46
19	18.43	4.52	18.44	4.53	18.44	4.54	18.44	4.55	18.43
20	19.40	4.75	19.41	4.76	19.41	4.77	19.41	4.78	19.40
21	20.37	4.99	20.38	5.00	20.38	5.01	20.38	5.02	20.37
22	21.34	5.23	21.35	5.24	21.35	5.25	21.35	5.26	21.34
23	22.31	5.47	22.32	5.48	22.32	5.49	22.32	5.50	22.31
24	23.28	5.71	23.29	5.72	23.29	5.73	23.29	5.74	23.28
25	24.25	5.95	24.26	5.96	24.26	5.97	24.26	5.98	24.25
26	25.22	6.19	25.23	6.20	25.23	6.21	25.23	6.22	25.22
27	26.19	6.43	26.20	6.44	26.20	6.45	26.20	6.46	26.19
28	27.16	6.67	27.17	6.68	27.17	6.69	27.17	6.70	27.16
29	28.13	6.91	28.14	6.92	28.14	6.93	28.14	6.94	28.13
30	29.10	7.15	29.11	7.16	29.11	7.17	29.11	7.18	29.10
31	30.07	7.39	30.08	7.40	30.08	7.41	30.08	7.42	30.07
32	31.04	7.63	31.05	7.64	31.05	7.65	31.05	7.66	31.04
33	32.01	7.87	32.02	7.88	32.02	7.89	32.02	7.90	32.01
34	32.98	8.11	32.99	8.12	32.99	8.13	32.99	8.14	32.98
35	33.95	8.35	33.96	8.36	33.96	8.37	33.96	8.38	33.95
36	34.92	8.59	34.93	8.60	34.93	8.61	34.93	8.62	34.92
37	35.89	8.83	35.90	8.84	35.90	8.85	35.90	8.86	35.89
38	36.86	9.07	36.87	9.08	36.87	9.09	36.87	9.10	36.86
39	37.83	9.31	37.84	9.32	37.84	9.33	37.84	9.34	37.83
40	38.80	9.55	38.81	9.56	38.81	9.57	38.81	9.58	38.80
41	39.77	9.79	39.78	9.80	39.78	9.81	39.78	9.82	39.77
42	40.74	10.03	40.75	10.04	40.75	10.05	40.75	10.06	40.74
43	41.71	10.27	41.72	10.28	41.72	10.29	41.72	10.30	41.71
44	42.68	10.51	42.69	10.52	42.69	10.53	42.69	10.54	42.68
45	43.65	10.75	43.66	10.76	43.66	10.77	43.66	10.78	43.65
46	44.62	10.99	44.63	11.00	44.63	11.01	44.63	11.02	44.62
47	45.59	11.23	45.60	11.24	45.60	11.25	45.60	11.26	45.59
48	46.56	11.47	46.57	11.48	46.57	11.49	46.57	11.50	46.56
49	47.53	11.71	47.54	11.72	47.54	11.73	47.54	11.74	47.53
50	48.50	11.95	48.51	11.96	48.51	11.97	48.51	11.98	48.50
51	49.47	12.19	49.48	12.20	49.48	12.21	49.48	12.22	49.47
52	50.44	12.43	50.45	12.44	50.45	12.45	50.45	12.46	50.44
53	51.41	12.67	51.42	12.68	51.42	12.69	51.42	12.70	51.41
54	52.38	12.91	52.39	12.92	52.39	12.93	52.39	12.94	52.38
55	53.35	13.15	53.36	13.16	53.36	13.17	53.36	13.18	53.35
56	54.32	13.39	54.33	13.40	54.33	13.41	54.33	13.42	54.32
57	55.29	13.63	55.30	13.64	55.30	13.65	55.30	13.66	55.29
58	56.26	13.87	56.27	13.88	56.27	13.89	56.27	13.90	56.26
59	57.23	14.11	57.24	14.12	57.24	14.13	57.24	14.14	57.23
60	58.20	14.35	58.21	14.36	58.21	14.37	58.21	14.38	58.20

TRAVERSE TABLE. 67.

[illegible]

[illegible]

TRAVERSE TABLE. 67.

Distance	16 Degrees						17 Degrees						72 Degrees					
	0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.		
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.
1	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28	0 96	0 28
2	1 92	0 51	1 92	0 56	1 92	0 57	1 92	0 57	1 92	0 58	1 91	0 58	1 91	0 58	1 91	0 59	1 91	0 60
3	2 88	0 83	2 88	0 84	2 88	0 85	2 87	0 87	2 87	0 88	2 87	0 88	2 87	0 88	2 87	0 89	2 86	0 90
4	3 84	1 10	3 84	1 12	3 84	1 14	3 83	1 15	3 83	1 15	3 82	1 17	3 82	1 19	3 82	1 20	3 81	1 22
5	4 80	1 38	4 80	1 40	4 79	1 42	4 79	1 44	4 77	1 46	4 78	1 48	4 77	1 50	4 77	1 50	4 76	1 52
6	5 77	1 65	5 76	1 68	5 75	1 70	5 75	1 73	5 74	1 75	5 73	1 78	5 72	1 80	5 72	1 80	5 71	1 83
7	6 73	1 93	6 72	1 96	6 71	1 99	6 70	2 02	6 69	2 05	6 69	2 08	6 68	2 11	6 67	2 11	6 67	2 13
8	7 69	2 21	7 68	2 24	7 67	2 27	7 66	2 31	7 65	2 34	7 64	2 37	7 63	2 41	7 62	2 41	7 62	2 44
9	8 65	2 48	8 64	2 52	8 63	2 56	8 62	2 59	8 61	2 63	8 60	2 67	8 58	2 71	8 57	2 71	8 57	2 74
10	9 61	2 76	9 60	2 80	9 59	2 84	9 58	2 86	9 56	2 92	9 55	2 97	9 54	3 01	9 52	3 01	9 52	3 05
11	10 57	3 03	10 56	3 08	10 55	3 12	10 53	3 17	10 52	3 22	10 51	3 26	10 49	3 31	10 48	3 31	10 48	3 35
12	11 54	3 31	11 52	3 36	11 51	3 41	11 49	3 46	11 48	3 51	11 46	3 56	11 45	3 61	11 43	3 61	11 43	3 66
13	12 50	3 58	12 48	3 64	12 46	3 69	12 45	3 75	12 43	3 81	12 42	3 86	12 40	3 86	12 38	3 86	12 38	3 93
14	13 46	3 86	13 44	3 92	13 42	3 98	13 41	4 04	13 39	4 09	13 37	4 15	13 35	4 21	13 33	4 21	13 33	4 27
15	14 42	4 13	14 40	4 20	14 38	4 26	14 36	4 33	14 35	4 39	14 33	4 45	14 31	4 51	14 29	4 51	14 29	4 57
16	15 38	4 41	15 36	4 48	15 34	4 54	15 32	4 61	15 30	4 68	15 28	4 74	15 26	4 81	15 24	4 81	15 24	4 88
17	16 34	4 69	16 32	4 76	16 30	4 83	16 28	4 90	16 26	4 97	16 24	5 04	16 21	5 11	16 19	5 11	16 19	5 18
18	17 30	4 96	17 28	5 04	17 26	5 11	17 23	5 18	17 21	5 26	17 19	5 34	17 17	5 41	17 14	5 41	17 14	5 49
19	18 26	5 24	18 24	5 32	18 22	5 40	18 19	5 48	18 17	5 56	18 15	5 63	18 12	5 71	18 10	5 71	18 10	5 79
20	19 23	5 51	19 20	5 60	19 18	5 68	19 15	5 76	19 13	5 85	19 10	5 93	19 07	6 01	19 05	6 01	19 05	6 09
21	20 19	5 79	20 16	5 88	20 14	5 96	20 11	6 05	20 08	6 14	20 06	6 23	20 03	6 32	20 00	6 32	20 00	6 40
22	21 15	6 06	21 12	6 16	21 09	6 25	21 07	6 34	21 04	6 43	21 01	6 52	20 98	6 59	20 95	6 59	20 95	7 07
23	22 11	6 34	22 08	6 44	22 05	6 53	22 02	6 62	21 99	6 72	21 97	6 82	21 94	6 92	21 91	6 92	21 91	7 01
24	23 07	6 62	23 04	6 72	23 01	6 82	22 98	6 92	22 95	7 01	22 92	7 12	22 89	7 22	22 86	7 22	22 86	7 34
25	24 03	6 89	24 00	7 00	23 97	7 10	23 94	7 21	23 91	7 31	23 88	7 41	23 84	7 52	23 81	7 52	23 81	7 62
26	24 99	7 17	24 96	7 28	24 93	7 38	24 90	7 49	24 86	7 60	24 83	7 70	24 80	7 82	24 76	7 82	24 76	7 93
27	25 95	7 44	25 92	7 56	25 89	7 67	25 85	7 78	25 82	7 89	25 79	8 00	25 75	8 12	25 72	8 12	25 72	8 23
28	26 91	7 72	26 88	7 84	26 85	7 95	26 81	8 07	26 78	8 19	26 74	8 30	26 70	8 42	26 67	8 42	26 67	8 54
29	27 88	7 99	27 84	8 12	27 81	8 24	27 77	8 36	27 73	8 48	27 70	8 60	27 66	8 72	27 62	8 72	27 62	8 84
30	28 84	8 27	28 80	8 40	28 77	8 52	28 73	8 65	28 69	8 77	28 65	8 90	28 61	9 02	28 57	9 02	28 57	9 15
31	29 80	8 55	29 76	8 68	29 72	8 81	29 68	8 93	29 65	9 06	29 61	9 19	29 57	9 29	29 52	9 29	29 52	9 45
32	30 76	8 82	30 72	8 96	30 69	9 09	30 64	9 23	30 60	9 36	30 56	9 49	30 52	9 62	30 48	9 62	30 48	9 76
33	31 72	9 10	31 68	9 23	31 64	9 37	31 60	9 51	31 56	9 64	31 52	9 79	31 47	9 92	31 43	9 92	31 43	10 06
34	32 68	9 37	32 64	9 51	32 60	9 66	32 56	9 80	32 51	9 94	32 47	10 08	32 42	10 22	32 38	10 22	32 38	10 37
35	33 64	9 65	33 60	9 79	33 56	9 94	33 51	10 09	33 47	10 23	33 43	10 38	33 38	10 53	33 33	10 53	33 33	10 67
36	34 61	9 92	34 56	10 07	34 52	10 23	34 47	10 38	34 43	10 53	34 38	10 68	34 33	10 83	34 29	10 83	34 29	10 97
37	35 57	10 20	35 52	10 35	35 48	10 51	35 43	10 66	35 38	10 82	35 34	10 97	35 29	11 13	35 24	11 13	35 24	11 28
38	36 53	10 47	36 48	10 63	36 44	10 79	36 39	10 95	36 34	11 11	36 29	11 27	36 24	11 43	36 19	11 43	36 19	11 58
39	37 49	10 75	37 44	10 91	37 39	11 08	37 35	11 24	37 30	11 40	37 25	11 56	37 20	12 03	37 14	12 03	37 14	12 19
40	38 45	11 03	38 40	11 19	38 35	11 36	38 30	11 53	38 25	12 17	38 20	12 34	38 15	12 51	38 10	12 51	38 10	13 07
41	39 41	11 30	39 36	11 47	39 31	11 65	39 26	11 82	39 21	12 09	39 16	12 26	39 11	12 43	39 05	12 43	39 05	13 20
42	40 37	11 58	40 32	12 15	40 27	12 33	40 22	12 50	40 17	13 06	40 12	13 23	40 06	13 40	40 00	13 40	40 00	14 06
43	41 33	12 25	41 28	12 43	41 23	12 61	41 18	12 78	41 13	12 95	41 08	13 12	41 03	13 29	40 57	13 29	40 57	14 33
44	42 30	12 53	42 24	13 11	42 19	13 28	42 13	13 45	42 08	13 62	42 03	13 79	41 57	13 95	41 51	13 95	41 51	15 00
45	43 26	13 21	43 20	13 39	43 15	13 56	43 09	14 13	43 03	14 30	42 57	14 46	42 51	15 03	42 45	15 03	42 45	15 30
46	44 22	13 49	44 16	13 67	44 11	13 84	44 05	14 00	43 99	14 15	43 93	14 30	43 87	14 45	43 81	14 45	43 81	15 57
47	45 18	14 17	45 12	14 35	45 07	14 52	45 01	15 07	44 95	15 22	44 89	15 39	44 83	15 54	44 76	15 54	44 76	16 24
48	46 14	14 45	46 08	15 03	46 02	15 20	45 96	15 37	45 90	15 54	45 84	16 11	45 78	16 28	45 72	16 28	45 72	16 55
49	47 10	15 13	47 04	15 31	46 98	15 48	46 92	16 05	46 86	16 22	46 80	16 39	46 74	16 56	46 68	16 56	46 68	17 23
50	48 06	15 41	48 00	15 59	47 94	16 16	47 88	16 33	47 82	16 50	47 76	17 07	47 70	17 24	47 64	17 24	47 64	17 51
51	49 02	16 09	48 96	16 27	48 90	16 44	48 84	16 61	48 77	16 78	48 71	16 95	48 65	17 15	48 59	17 15	48 59	18 00
52	49 99	16 37	49 92	16 55	49 86	17 12	49 79	17 29	49 73	17 46	49 66	17 63	49 60	17 80	49 54	17 80	49 54	18 27
53	50 95	17 05	50 88	17 23	50 82	17 40	50 75	17 57	50 68	18 14	50 62	18 31	50 56	18 48	50 50	18 48	50 50	19 15
54	51 91	17 33	51 84	17 51	51 78	18 08	51 71	18 25	51 64	18 42	51 57	19 00	51 50	19 17	51 44	19 17	51 44	19 44
55	52 87	18 01	52 80	18 19	52 74	18 36	52 67	18 53	52 60	19 10	52 53	19 27	52 46	19 44	52 40	19 44	52 40	20 11
56	53 83	18 29	53 76	18 47	53 69	19 04	53 62	19 21	53 55	19 38	53 48	19 55	53 41	20 05	53 34	20 05	53 34	20 32
57	54 79	18 57	54 72	19 15	54 65	19 32	54 58	19 49	54 51	20 06	54 44	20 23	54 37	20 40	54 30	20 40	54 30	21 09
58	55 75	19 25	55 68	19 43	55 61	20 00	55 54	20 17	55 47	20 34	55 40	20 51	55 33	21 08	55 26	21 08	55 26	21 35
59	56 71	19 53	56 64	20 11	56 57	20 28	56 50	20 45	56 43	21 02	56 36	21 19	56 29	21 36	56 22	21 36	56 22	22 03
60	57 68	20 21	57 60	20 39	57 53	21 06	57 45	21 23	57 38	21 40	57 31	21 57	57 24	22 04	57 17	22 04	57 17	22 31

TRAVERSE TABLE. 67.

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16 Degrees				17 Degrees				72 Degrees				74 Deg.			
o Min.		15 Min.		30 Min.		45 Min.		o Min.		15 Min.		30 Min.		45 Min.	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
58,64	16,81	58,56	17,07	58,49	17,33	58,41	17,58	58,33	17,85	58,26	18,09	58,18	18,34	58,10	18,60
61	59,60	59,52	17,35	59,45	17,61	59,37	17,87	59,29	18,13	59,21	18,39	59,13	18,64	59,05	18,90
62	60,56	60,48	17,63	60,41	17,89	60,33	18,16	60,25	18,42	60,17	18,68	60,08	18,95	60,00	19,21
63	61,52	61,44	17,91	61,36	18,17	61,28	18,45	61,20	18,71	61,12	18,98	61,04	19,25	60,95	19,51
64	62,48	62,40	18,19	62,32	18,46	62,24	18,73	62,16	19,00	62,08	19,27	61,99	19,54	61,91	19,82
65	63,44	63,36	18,47	63,28	18,75	63,20	19,02	63,12	19,30	63,03	19,57	62,95	19,85	62,86	20,12
66	64,40	64,32	18,75	64,24	19,03	64,16	19,31	64,07	19,59	63,99	19,87	63,90	20,15	63,81	20,43
67	65,36	65,28	19,03	65,20	19,31	65,11	19,60	65,03	19,88	64,94	20,16	64,85	20,45	64,76	20,75
68	66,32	66,24	19,31	66,16	19,60	66,07	19,89	65,99	20,17	65,91	20,46	65,81	20,75	65,72	21,04
69	67,28	67,20	19,59	67,12	19,88	67,03	20,17	66,94	20,47	66,85	20,76	66,76	21,05	66,69	21,34
70	68,24	68,16	19,87	68,08	20,17	67,99	20,46	67,90	20,76	67,81	21,05	67,71	21,35	67,62	21,65
71	69,20	69,12	20,15	69,04	20,45	68,95	20,75	68,85	21,05	68,76	21,35	68,67	21,65	68,57	21,95
72	70,16	70,08	20,43	70,00	20,73	69,90	21,04	69,81	21,34	69,72	21,65	69,62	21,95	69,53	22,26
73	71,12	71,04	20,71	70,95	21,02	70,86	21,33	70,77	21,64	70,67	21,94	70,58	22,25	70,48	22,56
74	72,08	72,00	20,99	71,91	21,30	71,82	21,62	71,72	21,93	71,63	22,24	71,53	22,55	71,43	22,86
75	73,04	72,96	21,27	72,87	21,59	72,78	21,90	72,68	22,22	72,58	22,54	72,48	22,85	72,38	23,17
76	74,00	73,92	21,55	73,83	21,87	73,73	22,19	73,64	22,51	73,54	22,83	73,44	23,16	73,34	23,48
77	74,96	74,88	21,83	74,79	22,15	74,69	22,48	74,59	22,81	74,49	23,16	74,39	23,46	74,29	23,78
78	75,92	75,84	22,11	75,75	22,44	75,65	22,77	75,55	23,10	75,45	23,43	75,34	23,76	75,24	24,08
79	76,88	76,80	22,39	76,71	22,72	76,61	23,06	76,50	23,39	76,40	23,72	76,30	24,06	76,19	24,39
80	77,84	77,76	22,67	77,67	23,01	77,56	23,34	77,46	23,68	77,36	24,02	77,25	24,36	77,14	24,69
81	78,80	78,72	22,95	78,62	23,29	78,52	23,62	78,42	23,98	78,32	24,36	78,21	24,66	78,10	25,00
82	79,76	79,68	23,23	79,58	23,57	79,47	24,27	79,37	24,27	79,27	24,61	79,16	24,96	79,05	25,30
83	80,72	80,64	23,51	80,54	23,86	80,44	24,21	80,33	24,56	80,22	24,91	80,11	25,26	80,00	25,61
84	81,68	81,60	23,79	81,50	24,14	81,40	24,50	81,29	24,88	81,18	25,21	81,07	25,56	80,95	25,91
85	82,64	82,56	24,07	82,46	24,43	82,35	24,79	82,24	25,14	82,13	25,50	82,02	25,86	81,91	26,22
86	83,60	83,52	24,35	83,42	24,71	83,31	25,07	83,20	25,44	83,09	25,86	82,97	26,16	82,86	26,52
87	84,56	84,48	24,63	84,38	24,99	84,27	25,36	84,15	25,73	84,04	26,10	83,93	26,46	83,81	26,83
88	85,52	85,44	24,91	85,34	25,28	85,22	25,65	85,11	26,02	85,00	26,39	84,88	26,76	84,76	27,13
89	86,48	86,40	25,19	86,29	25,56	86,18	25,94	86,07	26,31	85,95	26,69	85,84	27,06	85,72	27,44
90	87,44	87,37	25,47	87,32	25,85	87,14	26,23	87,02	26,61	86,91	26,99	86,79	27,36	86,67	27,74
91	88,40	88,32	25,75	88,21	26,13	88,10	26,51	87,98	26,90	87,86	27,28	87,74	27,67	87,62	28,05
92	89,36	89,28	26,03	89,17	26,41	89,05	26,80	88,94	27,19	88,82	27,58	88,70	27,97	88,57	28,35
93	90,32	90,25	26,30	90,13	26,70	90,01	27,09	89,89	27,48	89,77	27,88	89,65	28,27	89,53	28,66
94	91,28	91,21	26,58	91,09	26,98	90,97	27,38	90,85	27,76	90,73	28,17	90,60	28,57	90,48	28,96
95	92,24	92,17	26,86	92,05	27,27	91,93	27,67	91,81	28,07	91,68	28,47	91,56	28,87	91,43	29,27
96	93,20	93,14	27,14	93,01	27,55	92,88	27,96	92,76	28,36	92,64	28,76	92,51	29,17	92,38	29,57
97	94,16	94,09	27,42	93,97	27,83	93,84	28,24	93,72	28,65	93,59	29,06	93,47	29,47	93,34	29,88
98	95,12	95,05	27,70	94,92	28,12	94,80	28,53	94,67	28,94	94,55	29,36	94,42	29,77	94,29	30,18
99	96,08	96,01	27,98	95,88	28,40	95,76	28,82	95,63	29,24	95,50	29,65	95,37	30,07	95,24	30,49
100	97,04	96,97	28,26	96,92	28,60	96,77	29,01	96,65	29,53	96,46	29,95	96,33	30,37	96,19	30,79
101	98,00	97,93	28,54	97,80	28,97	97,67	29,40	97,54	29,82	97,41	30,25	97,28	30,67	97,15	31,10
102	99,00	98,93	29,22	98,78	29,55	98,65	29,69	98,50	30,11	98,37	30,54	98,23	30,97	98,10	31,40
103	100,00	99,92	29,54	99,72	30,24	99,59	29,97	99,46	30,41	99,32	30,84	99,19	31,27	99,05	31,71
104	101,00	100,92	29,86	100,70	30,56	100,57	30,26	100,44	30,70	100,31	31,14	100,18	31,58	100,05	32,01
105	102,00	101,92	30,18	101,66	30,88	101,53	30,55	101,40	30,99	101,27	31,43	101,14	31,88	100,99	32,32
106	103,00	102,92	30,50	102,68	31,20	102,55	30,84	102,42	31,28	102,29	31,73	102,16	32,18	101,99	32,62
107	104,00	103,92	30,82	103,60	31,52	103,47	31,13	103,33	31,58	103,19	32,03	103,06	32,48	102,93	32,93
108	105,00	104,92	31,14	104,70	31,84	104,57	31,47	104,43	31,87	104,29	32,32	104,16	32,78	103,98	33,23
109	106,00	105,92	31,46	105,64	32,16	105,51	31,79	105,37	32,16	105,24	32,62	105,11	33,08	104,88	33,54
110	107,00	106,92	31,78	106,66	32,48	106,53	32,11	106,39	32,55	106,26	32,92	106,13	33,38	105,97	33,84
111	108,00	107,92	32,10	107,68	32,80	107,55	32,43	107,42	32,87	107,29	33,21	107,16	33,68	107,03	34,14
112	109,00	108,92	32,42	108,68	33,12	108,55	32,75	108,42	33,19	108,29	33,58	108,16	33,98	108,03	34,45
113	110,00	109,92	32,74	109,68	33,44	109,55	33,07	109,42	33,53	109,29	33,92	109,16	34,28	109,03	34,75
114	111,00	110,92	33,06	110,68	33,76	110,55	33,39	110,42	33,87	110,29	34,21	110,16	34,58	110,03	35,06
115	112,00	111,92	33,38	111,68	34,08	111,55	33,71	111,42	34,19	111,29	34,58	111,16	34,95	111,03	35,36
116	113,00	112,92	33,70	112,68	34,40	112,55	34,03	112,42	34,37	112,29	34,76	112,16	35,18	112,03	35,67
117	114,00	113,92	34,02	113,68	34,72	113,55	34,35	113,42	34,67	113,29	35,09	113,16	35,48	113,03	35,97
118	115,00	114,92	34,34	114,68	35,04	114,55	34,67	114,42	35,03	114,29	35,42	114,16	35,83	114,03	36,28
119	116,00	115,92	34,66	115,68	35,36	115,55	34,99	115,42	35,38	115,29	35,77	115,16	36,15	115,03	36,58
120	117,00	116,92	34,98	116,68	35,68	116,55	35,31	116,42	35,69	116,29	36,09	116,16	36,50	116,03	36,89

[illegible]

TRAVERSE TABLE. 67.

18 Degrees													19 Degrees													20 Degrees												
0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.					
Distance	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.						
1	0,95	0,31	0,95	0,32	0,95	0,33	0,95	0,34	0,95	0,35	0,95	0,36	0,95	0,37	0,95	0,38	0,95	0,39	0,95	0,40	0,95	0,41	0,95	0,42	0,95	0,43	0,95	0,44	0,95	0,45	0,95							
2	1,90	0,62	1,90	0,63	1,90	0,64	1,90	0,65	1,90	0,66	1,90	0,67	1,90	0,68	1,90	0,69	1,90	0,70	1,90	0,71	1,90	0,72	1,90	0,73	1,90	0,74	1,90	0,75	1,90	0,76	1,90							
3	2,85	0,93	2,85	0,94	2,84	0,95	2,84	0,96	2,84	0,97	2,84	0,98	2,84	0,99	2,84	1,00	2,84	1,01	2,84	1,02	2,84	1,03	2,84	1,04	2,84	1,05	2,84	1,06	2,84	1,07	2,84							
4	3,80	1,24	3,80	1,25	3,79	1,26	3,79	1,27	3,79	1,28	3,79	1,29	3,78	1,30	3,78	1,31	3,78	1,32	3,78	1,33	3,77	1,34	3,77	1,35	3,77	1,36	3,77	1,37	3,76	1,38	3,76							
5	4,75	1,54	4,75	1,55	4,74	1,56	4,73	1,57	4,73	1,58	4,73	1,59	4,72	1,60	4,72	1,61	4,72	1,62	4,72	1,63	4,71	1,64	4,71	1,65	4,71	1,66	4,71	1,67	4,70	1,68	4,70							
6	5,71	1,85	5,70	1,86	5,69	1,87	5,68	1,88	5,68	1,89	5,67	1,90	5,67	1,91	5,66	1,92	5,66	1,93	5,66	1,94	5,65	1,95	5,65	1,96	5,65	1,97	5,64	1,98	5,64	1,99	5,63							
7	6,66	2,16	6,65	2,17	6,64	2,18	6,63	2,19	6,63	2,20	6,62	2,21	6,62	2,22	6,61	2,23	6,61	2,24	6,61	2,25	6,60	2,26	6,60	2,27	6,60	2,28	6,59	2,29	6,59	2,30	6,58							
8	7,61	2,47	7,60	2,48	7,59	2,49	7,58	2,50	7,58	2,51	7,57	2,52	7,56	2,53	7,56	2,54	7,55	2,55	7,55	2,56	7,54	2,57	7,54	2,58	7,54	2,59	7,53	2,60	7,53	2,61	7,52							
9	8,56	2,78	8,55	2,79	8,53	2,80	8,52	2,81	8,52	2,82	8,51	2,83	8,51	2,84	8,50	2,85	8,50	2,86	8,50	2,87	8,49	2,88	8,49	2,89	8,49	2,90	8,48	2,91	8,48	2,92	8,47							
10	9,51	3,09	9,50	3,10	9,48	3,11	9,47	3,12	9,47	3,13	9,46	3,14	9,46	3,15	9,45	3,16	9,45	3,17	9,45	3,18	9,44	3,19	9,44	3,20	9,44	3,21	9,43	3,22	9,43	3,23	9,42							
11	10,46	3,40	10,45	3,41	10,43	3,42	10,42	3,43	10,42	3,44	10,41	3,45	10,41	3,46	10,40	3,47	10,40	3,48	10,40	3,49	10,39	3,50	10,39	3,51	10,39	3,52	10,38	3,53	10,38	3,54	10,37							
12	11,41	3,71	11,40	3,72	11,38	3,73	11,37	3,74	11,37	3,75	11,36	3,76	11,36	3,77	11,35	3,78	11,35	3,79	11,35	3,80	11,34	3,81	11,34	3,82	11,34	3,83	11,33	3,84	11,33	3,85	11,32							
13	12,36	4,02	12,35	4,03	12,33	4,04	12,32	4,05	12,32	4,06	12,31	4,07	12,31	4,08	12,30	4,09	12,30	4,10	12,30	4,11	12,29	4,12	12,29	4,13	12,29	4,14	12,28	4,15	12,28	4,16	12,27							
14	13,31	4,33	13,30	4,34	13,28	4,35	13,27	4,36	13,27	4,37	13,26	4,38	13,26	4,39	13,25	4,40	13,25	4,41	13,25	4,42	13,24	4,43	13,24	4,44	13,24	4,45	13,23	4,46	13,23	4,47	13,22							
15	14,27	4,63	14,24	4,64	14,22	4,65	14,21	4,66	14,21	4,67	14,20	4,68	14,20	4,69	14,19	4,70	14,19	4,71	14,19	4,72	14,18	4,73	14,18	4,74	14,18	4,75	14,17	4,76	14,17	4,77	14,16							
16	15,22	4,94	15,19	4,95	15,17	4,96	15,16	4,97	15,16	4,98	15,15	4,99	15,15	5,00	15,14	5,01	15,14	5,02	15,14	5,03	15,13	5,04	15,13	5,05	15,13	5,06	15,12	5,07	15,12	5,08	15,11							
17	16,17	5,25	16,14	5,26	16,12	5,27	16,11	5,28	16,11	5,29	16,10	5,30	16,10	5,31	16,09	5,32	16,09	5,33	16,09	5,34	16,08	5,35	16,08	5,36	16,08	5,37	16,07	5,38	16,07	5,39	16,06							
18	17,12	5,56	17,09	5,57	17,07	5,58	17,06	5,59	17,06	5,60	17,05	5,61	17,05	5,62	17,04	5,63	17,04	5,64	17,04	5,65	17,03	5,66	17,03	5,67	17,03	5,68	17,02	5,69	17,02	5,70	17,01							
19	18,07	5,87	18,04	5,88	18,02	5,89	18,01	5,90	18,01	5,91	18,00	5,92	18,00	5,93	17,99	5,94	17,99	5,95	17,99	5,96	17,98	5,97	17,98	5,98	17,98	5,99	17,97	6,00	17,97	6,01	17,96							
20	19,02	6,18	18,99	6,19	18,97	6,20	18,96	6,21	18,96	6,22	18,95	6,23	18,95	6,24	18,94	6,25	18,94	6,26	18,94	6,27	18,93	6,28	18,93	6,29	18,93	6,30	18,92	6,31	18,92	6,32	18,91							
21	19,97	6,49	19,94	6,50	19,91	6,52	19,90	6,53	19,90	6,54	19,89	6,55	19,89	6,56	19,88	6,57	19,88	6,58	19,88	6,59	19,87	6,60	19,87	6,61	19,87	6,62	19,86	6,63	19,86	6,64	19,85							
22	20,92	6,80	20,89	6,81	20,86	6,83	20,85	6,84	20,85	6,85	20,84	6,86	20,84	6,87	20,83	6,88	20,83	6,89	20,83	6,90	20,82	6,91	20,82	6,92	20,82	6,93	20,81	6,94	20,81	6,95	20,80							
23	21,87	7,11	21,84	7,12	21,81	7,14	21,80	7,15	21,80	7,16	21,79	7,17	21,79	7,18	21,78	7,19	21,78	7,20	21,78	7,21	21,77	7,22	21,77	7,23	21,77	7,24	21,76	7,25	21,76	7,26	21,75							
24	22,82	7,42	22,79	7,43	22,76	7,46	22,75	7,47	22,75	7,48	22,74	7,49	22,74	7,50	22,73	7,51	22,73	7,52	22,73	7,53	22,72	7,54	22,72	7,55	22,72	7,56	22,71	7,57	22,71	7,58	22,70							
25	23,78	7,72	23,74	7,73	23,71	7,75	23,70	7,76	23,70	7,77	23,69	7,78	23,69	7,79	23,68	7,80	23,68	7,81	23,68	7,82	23,67	7,83	23,67	7,84	23,67	7,85	23,66	7,86	23,66	7,87	23,65							
26	24,73	8,03	24,69	8,04	24,66	8,07	24,65	8,08	24,65	8,09	24,64	8,10	24,64	8,11	24,63	8,12	24,63	8,13	24,63	8,14	24,62	8,15	24,62	8,16	24,62	8,17	24,61	8,18	24,61	8,19	24,60							
27	25,68	8,34	25,64	8,35	25,60	8,38	25,59	8,39	25,59	8,40	25,58	8,41	25,58	8,42	25,57	8,43	25,57	8,44	25,57	8,45	25,56	8,46	25,56	8,47	25,56	8,48	25,55	8,49	25,55	8,50	25,54							
28	26,63	8,65	26,59	8,66	26,55	8,69	26,54	8,70	26,54	8,71	26,53	8,72	26,53	8,73	26,52	8,74	26,52	8,75	26,52	8,76	26,51	8,77	26,51	8,78	26,51	8,79	26,50	8,80	26,50	8,81	26,49							
29	27,58	8,96	27,54	8,97	27,50	8,99	27,49	9,00	27,49	9,01	27,48	9,02	27,48	9,03	27,47	9,04	27,47	9,05	27,47	9,06	27,46	9,07	27,46	9,08	27,46	9,09	27,45	9,10	27,45	9,11	27,44							
30	28,53	9,27	28,49	9,28	28,45	9,30	28,44	9,31	28,44	9,32	28,43	9,33	28,43	9,34	28,42	9,35	28,42	9,36	28,42	9,37	28,41	9,38	28,41	9,39	28,41	9,40	28,40	9,41	28,40	9,42	28,39							
31	29,48	9,58	29,44	9,59	29,40	9,61	29,39	9,62	29,39	9,63	29,38	9,64	29,38	9,65	29,37	9,66	29,37	9,67	29,37	9,68	29,36	9,69	29,36	9,70	29,36	9,71	29,35	9,72	29,35	9,73	29,34							
32	30,43	9,89	30,39	9,90	30,35	9,92	30,34	9,93	30,34	9,94	30,33	9,95	30,33	9,96	30,32	9,97	30,32	9,98	30,32	9,99	30,31	10,00	30,31	10,01	30,31	10,02	30,30	10,03	30,30	10,04	30,29							
33	31,38	10,20	31,34	10,21	31,29	10,23	31,28	10,24	31,28	10,25	31,27	10,26	31,27	10,27	31,26	10,28	31,26	10,29	31,26	10,30	31,25	10,31	31,25	10,32	31,25	10,33	31,24	10,34	31,24	10,35	31,23							
34	32,34	10,51	32,30	10,52	32,24	10,56	32,23	10,57	32,23	10,58	32,22	10,59	32,22	10,60	32,21	10,61	32,21	10,62	32,21	10,63	32,20	10,64	32,20	10,65	32,20	10,66	32,19	10,67	32,19	10,68	32,18							
35	33,29	10,82	33,25	10,83	33,19	10,88	33,18	10,89	33,18	10,90	33,17	10,91	33,17	10,92	33,16	10,93	33,16	10,94	33,16	10,95	33,15	10,96	33,15	10,97	33,15	10,98	33,14	10,99	33,14	10,10	33,13							
36	34,24	11,13	34,20	11,14	34,14	11,19	34,13	11,20	34,13	11,21	34,12	11,22	34,12	11,23	34,11	11,24	34,11	11,25	34,11	11,26	34,10	11,27	34,10	11,28	34,10	11,29	34,09	11,30	34,09	11,31	34,08							
37	35,19	11,44	35,15	11,45	35,09	11,50	35,08	11,51	35,08	11,52	35,07	11,53	35,07	11,54	35,06	11,55	35,06	11,56	35,06	11,57	35,05	11,58	35,05	11,59	35,05	11,60	35,04	11,61	35,04	11,62	35,03							
38	36,14	11,74	36,10	11,75	36,04	11,80	36,03	11,81	36,03	11,82	36,02	11,83	36,02	11,84	36,01	11,85	36,01	11,86	36,01	11,87	36,00	11,88	36,00	11,89	36,00	11,90	35,99	11,91	35,99	11,92	35,98							
39	37,09	12,05	37,05	12,06	36,98	12,11	36,97	12,12	36,97	12,13	36,96	12,14	36,96	12,15	36,95	12,16	36,95	12,17	36,95	12,18	36,94	12,19	36,94	12,20	36,94	12,21	36,93	12,22	36,93	12,23	36,92							
40	38,04	12,36	37,99	12,37	37,93	12,42	37,92	12,43	37,92	12,44	37,91	12,45	3																									

TRAVERSE TABLE. 67.

[illegible]

[illegible]

TRAVERSE TABLE 67.

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Distance			20 Degrees						21 Degrees						Distance						
			0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.				
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
61	57,32	20,86	57,22	21,11	57,14	21,36	57,04	21,61	56,95	21,86	56,85	22,11	56,76	22,36	56,66	22,60	61	57,32	20,86	57,22	21,11
62	58,26	21,21	58,17	21,46	58,07	21,71	57,98	21,97	57,88	22,22	57,79	22,47	57,69	22,73	57,59	22,98	62	58,26	21,21	58,17	21,46
63	59,20	21,55	59,11	21,81	59,01	22,06	58,91	22,32	58,82	22,58	58,72	22,83	58,62	23,09	58,52	23,35	63	59,20	21,55	59,11	21,81
64	60,14	21,89	60,04	22,15	59,95	22,41	59,85	22,67	59,75	22,94	59,65	23,20	59,55	23,46	59,44	23,72	64	60,14	21,89	60,04	22,15
65	61,08	22,23	60,98	22,50	60,88	22,76	60,79	23,03	60,68	23,30	60,58	23,56	60,48	23,82	60,37	24,09	65	61,08	22,23	60,98	22,50
66	62,02	22,57	61,92	22,84	61,82	23,11	61,72	23,38	61,62	23,65	61,51	23,92	61,41	24,19	61,30	24,46	66	62,02	22,57	61,92	22,84
67	62,96	22,92	62,86	23,19	62,76	23,46	62,66	23,74	62,55	24,01	62,45	24,28	62,34	24,56	62,23	24,83	67	62,96	22,92	62,86	23,19
68	63,90	23,26	63,80	23,54	63,69	23,82	63,59	24,09	63,48	24,37	63,38	24,65	63,27	24,92	63,16	25,20	68	63,90	23,26	63,80	23,54
69	64,84	23,60	64,74	23,88	64,63	24,17	64,53	24,45	64,42	24,73	64,31	25,01	64,20	25,29	64,09	25,57	69	64,84	23,60	64,74	23,88
70	65,78	23,94	65,67	24,23	65,57	24,52	65,46	24,80	65,35	25,09	65,24	25,37	65,13	25,66	64,99	25,94	70	65,78	23,94	65,67	24,23
71	66,72	24,28	66,61	24,57	66,50	24,87	66,40	25,15	66,28	25,44	66,17	25,73	66,06	26,02	65,95	26,31	71	66,72	24,28	66,61	24,57
72	67,66	24,63	67,55	24,92	67,44	25,22	67,33	25,51	67,22	25,80	67,11	26,10	66,99	26,39	66,88	26,68	72	67,66	24,63	67,55	24,92
73	68,60	24,97	68,49	25,27	68,38	25,57	68,27	25,86	68,15	26,16	68,04	26,46	67,92	26,76	67,80	27,05	73	68,60	24,97	68,49	25,27
74	69,54	25,31	69,43	25,61	69,31	25,92	69,20	26,22	69,09	26,52	68,97	26,82	68,85	27,12	68,73	27,42	74	69,54	25,31	69,43	25,61
75	70,48	25,65	70,36	25,96	70,25	26,27	70,14	26,57	70,02	26,88	69,90	27,18	69,78	27,49	69,66	27,79	75	70,48	25,65	70,36	25,96
76	71,42	25,99	71,30	26,31	71,19	26,62	71,07	26,93	70,95	27,24	70,83	27,55	70,71	27,85	70,59	28,16	76	71,42	25,99	71,30	26,31
77	72,36	26,34	72,24	26,65	72,12	26,97	72,01	27,28	71,89	27,59	71,77	27,91	71,64	28,22	71,52	28,52	77	72,36	26,34	72,24	26,65
78	73,30	26,68	73,18	27,00	73,06	27,31	72,94	27,63	72,82	27,95	72,70	28,27	72,57	28,59	72,45	28,90	78	73,30	26,68	73,18	27,00
79	74,24	27,02	74,12	27,34	74,00	27,67	73,88	27,99	73,75	28,31	73,63	28,62	73,50	28,95	73,38	29,27	79	74,24	27,02	74,12	27,34
80	75,18	27,36	75,06	27,69	74,94	28,02	74,81	28,34	74,69	28,67	74,56	29,00	74,43	29,32	74,31	29,65	80	75,18	27,36	75,06	27,69
81	76,12	27,70	75,99	28,04	75,87	28,37	75,75	28,70	75,62	29,03	75,49	29,36	75,37	29,69	75,23	30,02	81	76,12	27,70	75,99	28,04
82	77,05	28,05	76,93	28,38	76,81	28,72	76,68	29,05	76,55	29,39	76,43	29,72	76,30	30,05	76,16	30,39	82	77,05	28,05	76,93	28,38
83	77,99	28,39	77,87	28,73	77,74	29,07	77,62	29,41	77,49	29,75	77,36	30,08	77,23	30,42	77,09	30,76	83	77,99	28,39	77,87	28,73
84	78,93	28,73	78,81	29,07	78,68	29,42	78,55	29,76	78,42	30,10	78,29	30,45	78,16	30,79	78,02	31,13	84	78,93	28,73	78,81	29,07
85	79,87	29,07	79,75	29,42	79,62	29,77	79,49	30,11	79,35	30,46	79,22	30,81	79,09	31,15	78,95	31,50	85	79,87	29,07	79,75	29,42
86	80,81	29,41	80,68	29,77	80,55	30,12	80,42	30,47	80,29	30,82	80,15	31,17	80,02	31,52	79,88	31,87	86	80,81	29,41	80,68	29,77
87	81,75	29,76	81,62	30,11	81,49	30,47	81,36	30,82	81,22	31,16	81,09	31,51	80,95	31,89	80,81	32,24	87	81,75	29,76	81,62	30,11
88	82,69	30,10	82,56	30,46	82,43	30,82	82,29	31,18	82,15	31,54	82,02	31,89	81,88	32,25	81,74	32,61	88	82,69	30,10	82,56	30,46
89	83,63	30,44	83,50	30,81	83,36	31,17	83,23	31,53	83,09	31,90	82,95	32,26	82,81	32,62	82,66	32,98	89	83,63	30,44	83,50	30,81
90	84,57	30,78	84,44	31,15	84,30	31,52	84,16	31,89	84,02	32,25	83,88	32,62	83,74	32,99	83,59	33,35	90	84,57	30,78	84,44	31,15
91	85,51	31,12	85,38	31,50	85,24	31,87	85,10	32,24	84,96	32,61	84,81	32,98	84,67	33,35	84,52	33,72	91	85,51	31,12	85,38	31,50
92	86,45	31,47	86,31	31,84	86,17	32,22	86,03	32,60	85,89	32,97	85,75	33,35	85,60	33,72	85,45	34,09	92	86,45	31,47	86,31	31,84
93	87,39	31,81	87,25	32,19	87,11	32,57	86,97	32,95	86,82	33,33	86,68	33,71	86,53	34,09	86,38	34,46	93	87,39	31,81	87,25	32,19
94	88,33	32,15	88,19	32,54	88,05	32,92	87,90	33,30	87,76	33,69	87,61	34,07	87,46	34,45	87,31	34,83	94	88,33	32,15	88,19	32,54
95	89,27	32,49	89,13	32,88	88,98	33,27	88,84	33,66	88,69	34,04	88,54	34,43	88,39	34,82	88,24	35,20	95	89,27	32,49	89,13	32,88
96	90,21	32,83	90,07	33,23	89,92	33,62	89,77	34,01	89,62	34,40	89,47	34,80	89,32	35,18	89,17	35,57	96	90,21	32,83	90,07	33,23
97	91,15	33,18	91,00	33,57	90,86	33,97	90,71	34,37	90,56	34,76	90,41	35,16	90,25	35,55	90,10	35,94	97	91,15	33,18	91,00	33,57
98	92,09	33,52	91,94	33,92	91,79	34,32	91,64	34,72	91,49	35,11	91,34	35,52	91,18	35,92	91,02	36,31	98	92,09	33,52	91,94	33,92
99	93,03	33,86	92,88	34,27	92,73	34,67	92,58	35,07	92,42	35,48	92,27	35,88	92,11	36,28	91,95	36,69	99	93,03	33,86	92,88	34,27
100	93,97	34,20	93,82	34,61	93,67	35,02	93,51	35,43	93,36	35,84	93,20	36,22	93,04	36,65	92,88	37,06	100	93,97	34,20	93,82	34,61
101	94,91	34,54	94,76	34,96	94,60	35,37	94,45	35,78	94,29	36,20	94,13	36,61	93,97	37,02	93,81	37,43	101	94,91	34,54	94,76	34,96
102	95,85	34,89	95,70	35,30	95,54	35,72	95,39	36,14	95,23	36,55	95,06	36,97	94,90	37,38	94,74	37,80	102	95,85	34,89	95,70	35,30
103	96,79	35,23	96,63	35,65	96,48	36,07	96,32	36,49	96,16	36,92	96,00	37,33	95,83	37,75	95,67	38,17	103	96,79	35,23	96,63	35,65
104	97,73	35,57	97,57	36,00	97,41	36,42	97,26	36,85	97,09	37,27	96,93	37,70	96,76	38,12	96,60	38,54	104	97,73	35,57	97,57	36,00
105	98,67	35,91	98,51	36,34	98,35	36,77	98,19	37,20	98,03	37,63	97,86	38,06	97,69	38,48	97,53	38,91	105	98,67	35,91	98,51	36,34
106	99,61	36,25	99,45	36,69	99,29	37,12	99,12	37,56	98,96	37,99	98,79	38,42	98,63	38,85	98,45	39,28	106	99,61	36,25	99,45	36,69
107	100,55	36,60	100,39	37,04	100,22	37,47	100,05	37,91	99,89	38,35	99,73	38,78	99,56	39,22	99,38	39,65	107	100,55	36,60	100,39	37,04
108	101,51	36,94	101,33	37,38	101,16	37,82	100,99	38,26	100,82	38,70	100,64	39,14	100,46	39,58	100,28	40,02	108	101,51	36,94	101,33	37,38
109	102,44	37,28																			

TRAVERSE TABLE. 67.

Distance	22 Degrees						23 Degrees																									
	0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.																	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.																
1	0,93	0,38	0,93	0,38	0,92	0,39	0,92	0,39	0,92	0,39	0,92	0,40	0,92	0,40	0,92	0,40																
2	1,85	0,75	1,85	0,76	1,85	0,77	1,84	0,77	1,84	0,78	1,84	0,79	1,84	0,79	1,83	0,80																
3	2,78	1,12	2,78	1,14	2,77	1,15	2,77	1,16	2,76	1,17	2,76	1,18	2,75	1,20	2,75	1,21																
4	3,71	1,50	3,70	1,52	3,70	1,53	3,69	1,55	3,68	1,56	3,68	1,58	3,67	1,60	3,66	1,61																
5	4,64	1,87	4,63	1,89	4,62	1,91	4,61	1,93	4,6	1,95	4,59	1,97	4,59	1,99	4,58	2,01																
6	5,56	2,25	5,55	2,27	5,54	2,30	5,53	2,32	5,52	2,34	5,51	2,37	5,50	2,39	5,49	2,42																
7	6,49	2,62	6,48	2,65	6,47	2,68	6,46	2,71	6,44	2,74	6,43	2,76	6,42	2,79	6,41	2,82																
8	7,42	3,00	7,40	3,03	7,39	3,06	7,38	3,09	7,36	3,13	7,35	3,16	7,34	3,19	7,32	3,22																
9	8,35	3,37	8,33	3,41	8,32	3,44	8,30	3,48	8,29	3,52	8,27	3,55	8,25	3,59	8,24	3,63																
10	9,27	3,75	9,26	3,79	9,24	3,83	9,22	3,87	9,21	3,91	9,19	3,95	9,17	3,99	9,15	4,03																
11	10,20	4,12	10,18	4,17	10,16	4,21	10,14	4,25	10,13	4,30	10,11	4,34	10,09	4,39	10,07	4,43																
12	11,13	4,50	11,11	4,54	11,09	4,59	11,07	4,64	11,05	4,70	11,03	4,74	11,01	4,79	10,98	4,83																
13	12,05	4,87	12,03	4,92	12,01	4,97	11,99	5,03	11,97	5,08	11,94	5,13	11,92	5,18	11,90	5,24																
14	12,98	5,24	12,96	5,30	12,94	5,36	12,91	5,41	12,89	5,47	12,86	5,53	12,84	5,58	12,81	5,64																
15	13,91	5,62	13,88	5,68	13,86	5,74	13,83	5,80	13,81	5,86	13,78	5,92	13,76	5,98	13,73	6,04																
16	14,84	5,99	14,81	6,06	14,78	6,12	14,76	6,19	14,73	6,25	14,70	6,32	14,67	6,38	14,65	6,44																
17	15,76	6,37	15,73	6,44	15,71	6,51	15,68	6,57	15,65	6,64	15,62	6,71	15,59	6,78	15,55	6,85																
18	16,69	6,74	16,66	6,82	16,63	6,89	16,60	6,96	16,57	7,03	16,54	7,11	16,51	7,18	16,48	7,25																
19	17,62	7,12	17,59	7,19	17,55	7,27	17,52	7,35	17,49	7,42	17,46	7,50	17,42	7,58	17,39	7,65																
20	18,54	7,49	18,51	7,57	18,48	7,65	18,44	7,73	18,41	7,82	18,38	7,90	18,35	7,98	18,31	8,06																
21	19,47	7,87	19,44	7,95	19,40	8,04	19,37	8,12	19,33	8,21	19,30	8,29	19,26	8,37	19,22	8,46																
22	20,40	8,24	20,36	8,33	20,33	8,42	20,29	8,51	20,25	8,60	20,21	8,69	20,18	8,77	20,14	8,87																
23	21,33	8,62	21,29	8,71	21,25	8,80	21,21	8,89	21,17	8,99	21,13	9,08	21,09	9,17	21,05	9,28																
24	22,25	8,99	22,21	9,09	22,17	9,18	22,13	9,28	22,09	9,38	22,05	9,47	22,01	9,57	21,97	9,69																
25	23,18	9,37	23,14	9,47	23,10	9,57	23,06	9,67	23,01	9,77	22,97	9,87	22,93	9,97	22,88	10,10																
26	24,11	9,74	24,06	9,84	24,02	9,95	23,98	10,05	23,93	10,16	23,89	10,26	23,84	10,37	23,80	10,47																
27	25,03	10,11	24,99	10,22	24,95	10,33	24,90	10,44	24,85	10,55	24,81	10,66	24,76	10,77	24,71	10,87																
28	25,96	10,49	25,92	10,60	25,87	10,72	25,82	10,83	25,77	10,94	25,73	11,05	25,68	11,16	25,63	11,28																
29	26,89	10,86	26,84	10,98	26,79	11,10	26,74	11,21	26,70	11,33	26,65	11,45	26,60	11,56	26,54	11,68																
30	27,82	11,24	27,77	11,36	27,72	11,48	27,67	11,60	27,62	11,72	27,56	11,84	27,51	11,96	27,46	12,08																
31	28,74	11,61	28,69	11,74	28,64	11,86	28,59	11,99	28,54	12,11	28,48	12,24	28,43	12,36	28,37	12,49																
32	29,67	11,99	29,62	12,12	29,56	12,25	29,51	12,38	29,46	12,50	29,40	12,63	29,35	12,76	29,29	12,89																
33	30,60	12,36	30,55	12,50	30,49	12,63	30,43	12,76	30,38	12,89	30,32	13,03	30,26	13,16	30,21	13,29																
34	31,52	12,74	31,47	12,87	31,41	13,01	31,36	13,15	31,31	13,28	31,24	13,42	31,18	13,56	31,12	13,69																
35	32,45	13,11	32,39	13,25	32,34	13,39	32,28	13,53	32,23	13,68	32,16	13,82	32,10	13,96	32,04	14,10																
36	33,38	13,49	33,32	13,63	33,26	13,78	33,20	13,92	33,14	14,07	33,08	14,21	33,01	14,36	32,95	14,50																
37	34,31	13,86	34,25	14,01	34,18	14,16	34,12	14,31	34,06	14,46	34,00	14,61	33,93	14,75	33,87	14,99																
38	35,23	14,24	35,17	14,39	35,11	14,54	35,04	14,69	34,98	14,85	34,91	15,00	34,85	15,15	34,78	15,30																
39	36,16	14,61	36,10	14,77	36,03	14,92	35,97	15,08	35,90	15,24	35,83	15,40	35,77	15,55	35,70	15,71																
40	37,09	14,98	37,02	15,15	36,96	15,30	36,89	15,47	36,82	15,63	36,75	15,79	36,68	15,95	36,61	16,11																
41	38,01	15,36	37,95	15,52	37,88	15,69	37,81	15,86	37,74	16,02	37,67	16,19	37,60	16,35	37,53	16,51																
42	38,94	15,73	38,87	15,90	38,80	16,07	38,73	16,24	38,66	16,41	38,59	16,59	38,52	16,75	38,44	16,92																
43	39,87	16,10	39,80	16,28	39,73	16,45	39,66	16,63	39,58	16,80	39,51	16,97	39,43	17,15	39,36	17,32																
44	40,80	16,48	40,72	16,66	40,65	16,84	40,58	17,02	40,50	17,19	40,43	17,37	40,35	17,54	40,27	17,72																
45	41,73	16,86	41,65	17,04	41,57	17,22	41,50	17,40	41,42	17,58	41,35	17,76	41,27	17,94	41,19	18,12																
46	42,65	17,23	42,58	17,42	42,50	17,60	42,42	17,79	42,34	17,97	42,27	18,16	42,18	18,34	42,10	18,53																
47	43,58	17,61	43,50	17,80	43,42	17,99	43,34	18,18	43,26	18,37	43,18	18,55	43,10	18,74	43,02	18,93																
48	44,51	17,98	44,43	18,18	44,35	18,37	44,27	18,56	44,18	18,76	44,10	18,95	44,02	19,14	43,93	19,33																
49	45,43	18,36	45,35	18,55	45,27	18,75	45,19	18,95	45,11	19,15	45,02	19,34	44,94	19,54	44,85	19,73																
50	46,36	18,73	46,28	18,93	46,19	19,13	46,11	19,34	46,03	19,54	45,94	19,74	45,85	19,94	44,75	20,14																
51	47,29	19,11	47,20	19,31	47,12	19,52	47,03	19,72	46,95	19,93	46,86	20,13	46,77	20,34	46,68	20,54																
52	48,21	19,48	48,13	19,69	48,04	19,90	47,95	20,11	47,87	20,32	47,78	20,53	47,69	20,74	47,60	20,94																
53	49,14	19,86	49,05	20,07	48,97	20,18	48,88	20,39	48,79	20,71	48,70	20,92	48,60	21,13	48,51	21,35																
54	50,07	20,23	49,98	20,45	49,89	20,66	49,80	20,88	49,71	21,10	49,62	21,32	49,52	21,53	49,43	21,75																
55	51,00	20,60	50,90	20,83	50,81	21,05	50,72	21,27	50,63	21,49	50,53	21,71	50,44	21,93	50,34	22,15																
56	51,92	20,98	51,82	21,20	51,74	21,43	51,64	21,66	51,55	21,88	51,45	22,11	51,36	22,33	51,25	22,55																
57	52,85	21,35	52,76	21,58	52,65	21,81	52,57	22,04	52,46	22,27	52,37	22,50	52,27	22,73	52,17	22,96																
58	53,78	21,73	53,68	21,96	53,59	22,20	53,49	22,43	53,39	22,66	53,29	22,90	53,19	23,13	53,00	23,36																
59	54,70	22,10	54,61	22,34	54,51	22,58	54,41	22,82	54,31	23,05	54,21	23,29	54,11	23,53	54,00	23,76																
60	55,63	22,48	55,53	22,72	55,43	22,96	55,32	23,20	55,23	23,44	55,13	23,68	55,02	23,95	54,92	24,17																
Distance	0 Min.			45 Min.			30 Min.			15 Min.			45 Min.			30 Min.			15 Min.													
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.												
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.												
68 Deg.																					67 Degrees						66 Degrees					

TRAVERSE TABLE. 67.

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Distance		22 Degrees				23 Degrees				66 Degrees				Distance				
		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.				30 Min.		45 Min.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
61	56,56	22,85	56,46	23,10	56,36	23,34	56,25	23,59	56,15	23,83	56,05	24,08	55,94	24,32	55,83	24,57	61	
62	57,49	23,23	57,38	23,48	57,28	23,73	57,18	23,98	57,07	24,22	56,97	24,47	56,86	24,72	56,75	24,97	62	
63	58,41	23,60	58,31	23,86	58,21	24,11	58,10	24,37	57,99	24,62	57,88	24,87	57,78	25,12	57,66	25,37	63	
64	59,34	23,98	59,23	24,23	59,13	24,49	59,02	24,75	58,91	25,01	58,80	25,26	58,69	25,52	58,58	25,78	64	
65	60,27	24,35	60,16	24,61	60,05	24,87	59,94	25,14	59,83	25,40	59,72	25,66	59,61	25,92	59,49	26,18	65	
66	61,19	24,72	61,08	24,98	60,98	25,26	60,87	25,52	60,75	25,79	60,64	26,05	60,53	26,32	60,41	26,58	66	
67	62,12	25,10	62,01	25,37	61,90	25,63	61,79	25,91	61,67	26,18	61,56	26,45	61,44	26,72	61,32	26,99	67	
68	63,05	25,47	62,94	25,75	62,82	26,02	62,71	26,30	62,59	26,57	62,48	26,84	62,36	27,12	62,24	27,39	68	
69	63,98	25,85	63,86	26,13	63,75	26,41	63,63	26,68	63,52	26,96	63,40	27,23	63,28	27,52	63,15	27,79	69	
70	64,90	26,22	64,79	26,51	64,67	26,79	64,55	27,07	64,44	27,35	64,32	27,63	64,19	27,91	64,07	28,19	70	
71	65,83	26,60	65,71	26,89	65,60	27,17	65,48	27,46	65,36	27,74	65,23	28,03	65,11	28,31	64,99	28,60	71	
72	66,76	26,97	66,64	27,26	66,52	27,55	66,40	27,84	66,28	28,13	66,15	28,42	66,03	28,71	65,90	29,00	72	
73	67,69	27,35	67,57	27,64	67,44	27,94	67,32	28,22	67,20	28,52	67,07	28,82	66,95	29,11	66,82	29,40	73	
74	68,61	27,72	68,49	28,02	68,37	28,32	68,24	28,62	68,12	28,91	67,99	29,21	67,86	29,50	67,73	29,81	74	
75	69,54	28,10	69,42	28,40	69,29	28,70	69,17	29,00	69,04	29,30	68,91	29,61	68,78	29,91	68,65	30,21	75	
76	70,47	28,47	70,34	28,78	70,21	29,08	70,09	29,39	69,96	29,69	69,83	30,00	69,70	30,31	69,56	30,61	76	
77	71,39	28,85	71,27	29,16	71,14	29,47	71,01	29,78	70,88	30,08	70,75	30,40	70,61	30,71	70,48	31,01	77	
78	72,32	29,22	72,19	29,54	72,06	29,85	71,93	30,16	71,80	30,48	71,67	30,79	71,53	31,11	71,39	31,42	78	
79	73,25	29,59	73,12	29,91	72,99	30,23	72,85	30,55	72,72	30,87	72,59	31,19	72,45	31,50	72,31	31,82	79	
80	74,17	29,97	74,04	30,29	73,91	30,61	73,78	30,94	73,64	31,26	73,50	31,58	73,37	31,90	73,23	32,22	80	
81	75,10	30,34	74,97	30,67	74,83	31,00	74,70	31,32	74,56	31,65	74,42	31,97	74,28	32,30	74,14	32,62	81	
82	76,03	30,72	75,89	31,05	75,76	31,38	75,62	31,71	75,48	32,04	75,34	32,37	75,20	32,70	75,06	33,03	82	
83	76,96	31,09	76,82	31,43	76,68	31,76	76,54	32,09	76,40	32,43	76,26	32,76	76,12	33,10	75,97	33,43	83	
84	77,88	31,47	77,75	31,81	77,61	32,15	77,47	32,48	77,32	32,82	77,18	33,15	77,03	33,50	76,89	33,83	84	
85	78,81	31,84	78,67	32,18	78,53	32,53	78,39	32,87	78,24	33,21	78,10	33,55	77,95	33,90	77,80	34,24	85	
86	79,74	32,22	79,60	32,56	79,45	32,91	79,31	33,26	79,16	33,60	79,02	33,95	78,87	34,29	78,71	34,64	86	
87	80,66	32,59	80,52	32,94	80,38	33,29	80,23	33,64	80,08	34,00	79,94	34,34	79,78	34,69	79,63	35,04	87	
88	81,59	32,97	81,45	33,32	81,30	33,68	81,16	34,03	81,00	34,39	80,86	34,73	80,71	35,09	80,55	35,44	88	
89	82,52	33,34	82,37	33,70	82,23	34,06	82,08	34,42	81,93	34,78	81,77	35,13	81,62	35,49	81,46	35,85	89	
90	83,45	33,72	83,30	34,08	83,15	34,44	83,00	34,81	82,85	35,17	82,69	35,53	82,54	35,89	82,38	36,25	90	
91	84,37	34,09	84,22	34,46	84,07	34,82	83,92	35,19	83,77	35,56	83,61	35,92	83,46	36,29	83,29	36,65	91	
92	85,30	34,46	85,15	34,84	85,00	35,21	84,84	35,57	84,69	35,95	84,53	36,32	84,37	36,69	84,21	37,05	92	
93	86,23	34,84	86,08	35,22	85,92	35,59	85,77	35,97	85,61	36,34	85,45	36,71	85,29	37,09	85,12	37,46	93	
94	87,16	35,21	87,00	35,59	86,85	35,97	86,69	36,35	86,53	36,73	86,37	37,11	86,20	37,48	86,04	37,86	94	
95	88,08	35,59	87,92	35,97	87,77	36,35	87,61	36,74	87,45	37,12	87,29	37,50	87,12	37,88	86,95	38,26	95	
96	89,01	35,96	88,85	36,35	88,69	36,74	88,53	37,12	88,37	37,51	88,21	37,90	88,04	38,28	87,87	38,67	96	
97	89,94	36,34	89,78	36,73	89,62	37,12	89,45	37,51	89,29	37,90	89,12	38,29	88,95	38,68	88,78	39,07	97	
98	90,86	36,71	90,70	37,11	90,54	37,50	90,38	37,90	90,21	38,29	90,04	38,69	89,87	39,08	89,70	39,47	98	
99	91,79	37,09	91,63	37,49	91,46	37,89	91,30	38,28	91,13	38,69	90,96	39,08	90,79	39,48	90,61	39,88	99	
100	92,72	37,46	92,55	37,87	92,39	38,27	92,22	38,67	92,05	39,07	91,88	39,47	91,71	39,88	91,53	40,28	100	
101	93,65	37,84	93,48	38,24	93,31	38,65	93,14	39,00	92,97	39,46	92,80	39,87	92,63	40,28	92,45	40,68	101	
102	94,57	38,21	94,41	38,62	94,24	39,03	94,06	39,45	93,89	39,86	93,72	40,26	93,54	40,67	93,36	41,08	102	
103	95,50	38,59	95,33	39,00	95,16	39,42	94,98	39,83	94,81	40,25	94,63	40,66	94,46	41,07	94,28	41,48	103	
104	96,43	38,96	96,26	39,38	96,08	39,80	95,91	40,22	95,73	40,64	95,56	41,05	95,37	41,47	95,19	41,89	104	
105	97,35	39,33	97,18	39,76	97,01	40,18	96,83	40,61	96,65	41,03	96,47	41,45	96,29	41,87	96,11	42,29	105	
106	98,28	39,71	98,11	40,14	97,93	40,56	97,75	40,99	97,57	41,42	97,39	41,84	97,21	42,27	97,02	42,69	106	
107	99,21	40,08	99,03	40,52	98,86	40,95	98,68	41,38	98,49	41,81	98,31	42,24	98,13	42,67	97,94	43,10	107	
108	100,14	40,46	99,96	40,89	99,78	41,33	99,60	41,76	99,41	42,20	99,23	42,63	99,04	43,07	98,85	43,50	108	
109	101,07	40,83	100,89	41,27	100,71	41,71	100,53	42,15	100,33	42,59	100,14	43,03	99,96	43,47	99,77	43,90	109	
110	102,00	41,21	101,81	41,65	101,64	42,10	101,44	42,54	101,23	42,98	101,03	43,42	100,83	43,86	100,63	44,30	110	
111	102,93	41,58	102,74	42,03	102,56	42,48	102,36	42,93	102,15	43,37	101,94	43,82	101,73	44,26	101,52	44,71	111	
112	103,86	41,96	103,67	42,41	103,48	42,86	103,28	43,31	103,07	43,76	102,86	44,21	102,65	44,66	102,44	45,11	112	
113	104,79	42,33	104,60	42,79	104,41	43,24	104,21	43,70	104,00	44,15	103,79	44,61	103,58	45,06	103,36	45,51	113	
114	105,72	42,71	105,53	43,17	105,33	43,63	105,13	44,09	104,92	44,54	104,71	45,00	104,50	45,46	104,28	45,92	114	
115	106,65	43,08	106,46	43,55	106,26	44,01	106,05	44,47	105,84	44,94	105,63	45,40	105,42	45,86	105,21	46,32	115	
116	107,58	43,45	107,39	43,92	107,20	44,39	107,00	44,86	106,79	45,33	106,58	45,79	106,37	46,26	106,16	46,72	116	
117	108,51	43,83	108,32	44,30	108,13	44,77	107,93	45,25	107,72	45,72	107,51	46,19	107,30	46,66	107,09	47,12	117	
118	109,44	44,21	109,25	44,68	109,06	45,16	108,86	45,63	108,65	46,11	108,44	46,58	108,23	47,06	108,02	47,53	118	
119	110,37	44,58	110,18	45,06	109,99	45,54	109,78	46,01	109,57	46,50	109,36	46,98	109,15	47,45	108,94	47,93	119	
120	111,30	44,95	111,11	45,44	110,91	45,92	110,70	46,41	110,49	46,89	110,28	47,31	110,07	47,85	109,86	48,33	120	

Distance		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	Distance																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
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TRAVERSE TABLE. 67.

Distance	24 Degrees						25 Degrees						64 Degrees											
	15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.			60 Min.		
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.
1	0,91	0,41	0,91	0,42	0,84	0,91	0,42	0,84	0,91	0,41	0,85	0,90	0,43	0,85	0,90	0,43	0,86	0,90	0,43	0,87	0,89	0,43	0,88	0,89
2	1,83	0,81	1,82	0,83	1,66	1,82	0,83	1,66	1,81	0,85	1,72	0,85	1,81	0,85	1,80	0,86	1,72	1,80	0,86	1,73	1,80	0,87	1,81	1,82
3	2,74	1,21	2,73	1,24	2,57	2,72	1,24	2,57	2,71	1,26	2,55	2,70	1,28	2,54	2,69	1,29	2,54	2,67	1,29	2,55	2,68	1,30	2,56	2,69
4	3,65	1,61	3,64	1,66	3,33	3,63	1,68	3,33	3,62	1,69	3,29	3,61	1,71	3,28	3,60	1,72	3,28	3,58	1,72	3,29	3,59	1,73	3,30	3,60
5	4,57	2,03	4,56	2,05	4,15	4,55	2,09	4,15	4,54	2,11	4,12	4,53	2,13	4,11	4,52	2,15	4,10	4,50	2,15	4,11	4,53	2,16	4,54	2,17
6	5,48	2,44	5,47	2,46	5,06	5,46	2,49	5,06	5,45	2,51	5,04	5,44	2,54	5,03	5,43	2,56	5,02	5,40	2,56	5,03	5,44	2,57	5,45	2,58
7	6,40	2,85	6,38	2,88	6,00	6,37	2,90	6,00	6,36	2,93	5,97	6,35	2,96	5,94	6,33	2,99	5,92	6,30	2,99	5,93	6,34	3,01	6,36	3,04
8	7,31	3,25	7,29	3,29	6,92	7,28	3,32	6,92	7,27	3,35	6,89	7,25	3,38	6,87	7,24	3,41	6,84	7,21	3,44	6,85	7,25	3,46	7,27	7,30
9	8,22	3,66	8,21	3,70	7,81	8,19	3,73	7,81	8,17	3,77	7,78	8,15	3,80	7,76	8,13	3,84	7,74	8,11	3,88	7,75	8,14	3,91	7,99	8,02
10	9,14	4,07	9,12	4,11	8,75	9,10	4,15	8,75	9,08	4,19	8,69	9,06	4,23	8,67	9,04	4,27	8,65	9,01	4,31	8,68	9,03	4,35	8,72	8,75
11	10,05	4,47	10,03	4,52	9,66	10,01	4,56	9,66	10,00	4,61	9,59	9,97	4,65	9,57	9,95	4,69	9,54	9,91	4,74	9,57	9,93	4,78	10,01	10,04
12	10,96	4,88	10,94	4,93	10,57	10,92	4,98	10,57	10,90	5,02	10,48	10,88	5,07	10,45	10,85	5,12	10,43	10,81	5,17	10,46	10,84	5,21	10,88	10,91
13	11,88	5,29	11,85	5,34	11,50	11,83	5,39	11,50	11,81	5,44	11,47	11,78	5,49	11,44	11,76	5,55	11,41	11,73	5,60	11,44	11,75	5,65	11,80	11,83
14	12,79	5,69	12,76	5,75	12,44	12,74	5,81	12,44	12,71	5,86	12,69	5,92	12,66	5,97	12,64	6,03	12,61	6,08	12,64	6,03	12,66	6,08	12,71	12,74
15	13,70	6,10	13,68	6,16	13,35	13,65	6,22	13,35	13,62	6,28	13,59	6,34	13,57	6,40	13,54	6,46	13,51	6,48	13,54	6,46	13,57	6,52	13,59	13,62
16	14,62	6,51	14,59	6,57	14,19	14,56	6,64	14,19	14,53	6,70	14,50	6,76	14,47	6,83	14,44	6,89	14,41	6,95	14,44	6,89	14,47	6,95	14,52	14,55
17	15,53	6,91	15,50	6,98	15,09	15,47	7,05	15,09	15,43	7,12	15,41	7,19	15,37	7,25	15,35	7,32	15,31	7,39	15,31	7,32	15,37	7,43	15,40	15,43
18	16,44	7,32	16,41	7,39	16,03	16,38	7,47	16,03	16,35	7,54	16,31	7,61	16,28	7,68	16,25	7,75	16,21	7,82	16,21	7,75	16,28	7,82	16,29	16,32
19	17,36	7,73	17,32	7,80	17,29	17,68	7,88	17,25	17,66	7,96	17,62	8,03	17,58	8,11	17,55	8,18	17,51	8,20	17,51	8,18	17,58	8,24	17,61	17,64
20	18,27	8,14	18,24	8,21	18,20	8,29	8,29	18,16	8,37	18,13	8,45	18,09	8,53	18,05	8,53	18,05	8,61	18,01	8,61	18,05	8,60	18,09	8,66	18,09
21	19,18	8,54	19,15	8,63	19,11	8,71	19,07	19,07	19,03	8,89	18,93	8,88	18,89	8,96	18,86	9,04	18,82	18,82	9,04	18,86	9,04	18,92	18,99	19,02
22	20,10	8,95	20,06	9,04	20,02	9,12	19,98	19,98	19,94	9,30	19,90	9,30	19,90	9,39	19,86	9,47	19,82	9,47	19,86	9,47	19,86	9,53	19,93	19,96
23	21,01	9,36	20,97	9,45	20,94	9,55	20,89	9,55	20,89	9,63	20,84	9,72	20,80	9,81	20,76	9,90	20,72	9,90	20,76	9,90	20,76	9,99	20,81	20,84
24	21,93	9,76	21,88	9,86	21,85	9,96	21,80	10,05	21,75	10,14	21,71	10,24	21,66	10,33	21,62	10,41	21,60	10,33	21,62	10,41	21,60	10,49	21,61	21,64
25	22,84	10,17	22,79	10,37	22,75	10,38	22,70	10,47	22,66	10,57	22,61	10,67	22,57	10,77	22,53	10,86	22,52	10,77	22,53	10,86	22,52	10,95	22,57	22,60
26	23,75	10,58	23,71	10,79	23,66	10,79	23,61	10,89	23,56	10,99	23,51	11,09	23,47	11,20	23,42	11,30	23,42	11,20	23,42	11,30	23,42	11,41	23,46	23,49
27	24,67	10,98	24,62	11,09	24,57	11,21	24,52	11,31	24,47	11,41	24,41	11,52	24,37	11,63	24,32	11,73	24,37	11,63	24,32	11,73	24,37	11,84	24,42	
28	25,58	11,39	25,53	11,50	25,48	11,62	25,43	11,73	25,37	11,84	25,32	11,95	25,28	12,06	25,22	12,17	25,28	12,06	25,22	12,17	25,28	12,29	25,34	25,37
29	26,49	11,80	26,44	11,91	26,39	12,04	26,34	12,14	26,28	12,26	26,23	12,37	26,18	12,49	26,12	12,60	26,12	12,49	26,12	12,60	26,12	12,71	26,16	26,19
30	27,41	12,20	27,35	12,32	27,30	12,44	27,24	12,56	27,19	12,68	27,13	12,80	27,08	12,92	27,02	13,03	27,02	12,92	27,02	13,03	27,02	13,14	13,07	13,10
31	28,32	12,61	28,26	12,73	28,21	12,86	28,15	12,98	28,10	13,10	28,04	13,22	27,98	13,35	27,92	13,47	13,31	27,92	13,47	13,31	27,92	13,52	13,45	13,48
32	29,23	13,02	29,18	13,14	29,12	13,27	29,06	13,40	29,00	13,53	28,94	13,65	28,88	13,78	28,82	13,90	28,76	13,78	28,82	13,90	28,76	14,01	13,94	13,97
33	30,15	13,42	30,09	13,55	30,03	13,69	29,97	13,82	29,91	13,95	29,85	14,08	29,79	14,21	29,72	14,34	29,65	14,21	29,72	14,34	29,65	14,45	14,38	14,41
34	31,06	13,83	31,00	13,97	30,94	14,10	30,88	14,26	30,81	14,37	30,75	14,51	30,69	14,64	30,62	14,77	30,53	14,64	30,62	14,77	30,53	14,88	14,81	14,84
35	31,97	14,24	31,91	14,38	31,85	14,51	31,78	14,66	31,72	14,79	31,65	14,93	31,59	15,07	31,53	15,21	31,53	15,07	31,53	15,21	31,53	15,34	15,27	15,30
36	32,89	14,64	32,82	14,79	32,76	14,93	32,69	15,07	32,63	15,22	32,56	15,36	32,50	15,50	32,43	15,64	32,36	15,50	32,43	15,64	32,36	15,75	15,68	15,71
37	33,80	15,05	33,73	15,20	33,67	15,35	33,60	15,49	33,53	15,64	33,46	15,93	33,40	16,03	33,33	16,28	33,26	16,03	33,33	16,28	33,26	16,43	16,36	16,39
38	34,72	15,46	34,65	15,61	34,58	15,76	34,51	15,91	34,44	16,06	34,37	16,21	34,30	16,36	34,23	16,51	34,16	16,36	34,23	16,51	34,16	16,76	16,69	16,72
39	35,63	15,86	35,56	16,02	35,49	16,18	35,42	16,33	35,34	16,49	35,28	16,64	35,21	16,79	35,14	17,00	35,07	16,79	35,14	17,00	35,07	17,25	17,18	17,21
40	36,54	16,27	36,47	16,43	36,40	16,59	36,33	16,75	36,25	16,91	36,18	17,07	36,10	17,22	36,03	17,38	36,03	17,22	36,03	17,38	36,03	17,63	17,56	17,59
41	37,46	16,68	37,38	16,84	37,31	17,00	37,23	17,17	37,16	17,33	37,08	17,49	37,01	17,65	36,93	17,81	36,93	17,65	36,93	17,81	36,93	18,08	18,01	18,04
42	38,37	17,09	38,29	17,25	38,22	17,42	38,14	17,58	38,06	17,75	37,99	17,92	37,92	18,08	37,85	18,25	37,85	18,08	37,85	18,25	37,85	18,50	18,43	18,46
43	39,28	17,49	39,20	17,66	39,13	17,83	39,05	18,00	38,97	18,18	38,89	18,34	38,82	18,51	38,73	18,68	38,73	18,51	38,73	18,68	38,73	19,01	18,94	18,97
44	40,20	17,90																						

TRAVERSE TABLE. 67.

III

Distance	24 Degrees						25 Degrees						64 Degrees											
	15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.			60 Min.		
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	
61	55°57'	24°41'	55°56'	25°06'	55°51'	25°30'	55°40'	25°54'	55°29'	25°78'	55°17'	26°02'	55°06'	26°26'	54°94'	26°51'	53°18'	26°51'	53°18'	26°51'	53°18'	26°51'	53°18'	
62	56°04'	25°22'	56°03'	25°47'	56°02'	25°51'	56°01'	25°50'	55°59'	25°77'	55°48'	26°01'	55°37'	26°25'	54°82'	26°48'	52°59'	26°48'	52°59'	26°48'	52°59'	26°48'	52°59'	
63	57°50'	25°53'	57°44'	25°58'	57°33'	26°13'	57°21'	26°38'	57°10'	26°63'	56°98'	26°88'	56°86'	27°12'	56°75'	27°37'	56°64'	27°37'	56°64'	27°37'	56°64'	27°37'		
64	58°47'	26°04'	58°35'	26°29'	58°24'	26°54'	58°12'	26°80'	57°58'	27°05'	57°47'	27°30'	57°27'	27°56'	57°15'	27°80'	56°58'	27°80'	56°58'	27°80'	56°58'	27°80'		
65	59°38'	26°44'	59°27'	26°70'	59°15'	26°96'	58°59'	27°22'	58°49'	27°47'	58°39'	27°73'	58°27'	27°99'	57°55'	28°24'	57°43'	28°24'	57°43'	28°24'	57°43'	28°24'		
66	60°30'	26°58'	60°18'	27°11'	60°01'	27°37'	59°49'	27°53'	59°38'	28°19'	59°28'	28°46'	59°18'	28°85'	58°58'	29°16'	58°47'	29°16'	58°47'	29°16'	58°47'	29°16'		
67	61°21'	27°26'	61°09'	27°52'	60°97'	27°79'	60°84'	28°05'	60°72'	28°32'	60°60'	28°58'	60°48'	28°85'	60°35'	29°11'	60°23'	29°11'	60°23'	29°11'	60°23'	29°11'		
68	62°13'	27°66'	62°00'	27°93'	61°88'	28°20'	61°73'	28°41'	61°53'	28°74'	61°40'	29°01'	61°28'	29°28'	61°15'	29°54'	61°03'	29°54'	61°03'	29°54'	61°03'	29°54'		
69	63°04'	28°07'	62°51'	28°34'	62°37'	28°62'	62°22'	28°89'	62°03'	29°16'	61°53'	29°44'	61°40'	29°71'	61°27'	30°04'	61°14'	30°04'	61°14'	30°04'	61°14'	30°04'		
70	63°59'	28°47'	63°46'	28°75'	63°30'	29°03'	63°17'	29°31'	63°04'	29°58'	62°51'	30°26'	62°38'	30°54'	62°25'	31°01'	62°12'	31°01'	62°12'	31°01'	62°12'	31°01'		
71	64°58'	28°88'	64°45'	29°16'	64°28'	29°44'	64°15'	29°73'	64°02'	30°01'	63°45'	30°29'	63°32'	30°57'	63°19'	31°24'	63°06'	31°24'	63°06'	31°24'	63°06'	31°24'		
72	65°58'	29°29'	65°45'	29°57'	65°28'	29°86'	65°15'	29°86'	65°02'	30°13'	64°45'	30°41'	64°28'	31°08'	64°00'	31°33'	63°47'	31°33'	63°47'	31°33'	63°47'	31°33'		
73	66°59'	29°59'	66°46'	30°27'	66°25'	30°56'	66°12'	30°86'	65°59'	31°13'	65°46'	31°41'	65°33'	31°68'	65°05'	32°03'	64°19'	32°03'	64°19'	32°03'	64°19'	32°03'		
74	67°61'	30°10'	67°47'	30°38'	67°34'	30°66'	67°21'	30°96'	67°08'	31°23'	66°55'	31°51'	66°42'	32°18'	66°14'	32°43'	65°30'	32°43'	65°30'	32°43'	65°30'	32°43'		
75	68°52'	30°51'	68°39'	30°81'	68°25'	31°10'	68°11'	31°40'	67°57'	31°70'	67°44'	32°00'	67°31'	32°29'	67°03'	32°54'	66°19'	32°54'	66°19'	32°54'	66°19'	32°54'		
76	69°43'	30°91'	69°30'	31°22'	69°16'	31°52'	69°02'	31°82'	68°88'	32°12'	68°75'	32°42'	68°62'	33°02'	68°34'	33°27'	67°40'	33°27'	67°40'	33°27'	67°40'	33°27'		
77	70°35'	31°32'	70°21'	31°63'	70°07'	31°93'	69°53'	32°24'	69°38'	32°54'	69°25'	33°25'	69°12'	33°55'	68°79'	34°26'	67°45'	34°26'	67°45'	34°26'	67°45'	34°26'		
78	71°26'	31°73'	71°12'	32°04'	70°98'	32°35'	70°83'	32°66'	70°69'	32°97'	70°55'	33°28'	70°41'	33°58'	70°13'	34°32'	68°54'	34°32'	68°54'	34°32'	68°54'	34°32'		
79	72°18'	32°14'	72°03'	32°45'	71°89'	32°76'	71°74'	33°08'	71°60'	33°39'	71°46'	33°70'	71°31'	34°02'	71°03'	34°56'	69°40'	34°56'	69°40'	34°56'	69°40'	34°56'		
80	73°08'	32°54'	72°54'	32°86'	72°80'	33°18'	72°65'	33°49'	72°51'	33°81'	72°36'	34°13'	72	34°44'	72°06'	34°76'	70	34°76'	70	34°76'	70	34°76'		
81	74°00'	32°95'	73°85'	33°27'	73°71'	33°59'	73°56'	33°91'	73°41'	34°23'	73°26'	34°55'	73	34	73	35	73	35	73	35	73	35		
82	74°51'	33°35'	74°77'	33°68'	74°62'	34°00'	74°47'	34°33'	74°32'	34°66'	74	35	74	35	74	36	74	36	74	36	74	36		
83	75°42'	33°76'	75°28'	33°99'	75°13'	34°32'	75°00'	34°65'	74°48'	34°98'	75	36	75	36	75	37	75	37	75	37	75	37		
84	76°33'	34°16'	76°19'	34°49'	76°04'	34°84'	75°49'	35°17'	75°35'	35°50'	75	37	75	37	75	38	75	38	75	38	75	38		
85	77°24'	34°57'	77°10'	34°91'	76°55'	35°25'	76°40'	35°59'	76°25'	36°33'	76	38	76	38	76	39	76	39	76	39	76	39		
86	78°15'	35°38'	78°01'	35°34'	77°50'	35°59'	77°35'	36°33'	77°20'	36°67'	77	39	77	39	77	40	77	40	77	40	77	40		
87	79°06'	35°79'	78°52'	35°74'	78°37'	36°08'	78°22'	36°42'	78°07'	37°16'	78	40	78	40	78	41	78	41	78	41	78	41		
88	80°00'	36°20'	79°44'	36°15'	79°29'	36°50'	79°14'	37°44'	78°59'	37°78'	79	41	79	41	79	42	79	42	79	42	79	42		
89	81°00'	36°20'	80°45'	36°15'	80°30'	36°50'	80°15'	37°44'	80°00'	38°38'	80	42	80	42	80	43	80	43	80	43	80	43		
90	82°00'	36°20'	81°45'	36°15'	81°30'	36°50'	81°15'	37°44'	81°00'	38°38'	81	43	81	43	81	44	81	44	81	44	81	44		
91	83°00'	36°20'	82°45'	36°15'	82°30'	36°50'	82°15'	37°44'	82°00'	38°38'	82	44	82	44	82	45	82	45	82	45	82	45		
92	84°00'	36°20'	83°45'	36°15'	83°30'	36°50'	83°15'	37°44'	83°00'	38°38'	83	45	83	45	83	46	83	46	83	46	83	46		
93	85°00'	36°20'	84°45'	36°15'	84°30'	36°50'	84°15'	37°44'	84°00'	38°38'	84	46	84	46	84	47	84	47	84	47	84	47		
94	86°00'	36°20'	85°45'	36°15'	85°30'	36°50'	85°15'	37°44'	85°00'	38°38'	85	47	85	47	85	48	85	48	85	48	85	48		
95	87°00'	36°20'	86°45'	36°15'	86°30'	36°50'	86°15'	37°44'	86°00'	38°38'	86	48	86	48	86	49	86	49	86	49	86	49		
96	88°00'	36°20'	87°45'	36°15'	87°30'	36°50'	87°15'	37°44'	87°00'	38°38'	87	49	87	49	87	50	87	50	87	50	87	50		
97	89°00'	36°20'	88°45'	36°15'	88°30'	36°50'	88°15'	37°44'	88°00'	38°38'	88	50	88	50	88	51	88	51	88	51	88	51		
98	90°00'	36°20'	89°45'	36°15'	89°30'	36°50'	89°15'	37°44'	89°00'	38°38'	89	51	89	51	89	52	89	52	89	52	89	52		
99	91°00'	36°20'	90°45'	36°15'	90°30'	36°50'	90°15'	37°44'	90°00'	38°38'	90	52	90	52	90	53	90	53	90	53	90	53		
100	92°00'	36°20'	91°45'	36°15'	91°30'	36°50'	91°15'	37°44'	91°00'	38°38'	91	53	91	53	91	54	91	54	91	54	91	54		
101	93°00'	36°20'	92°45'	36°15'	92°30'	36°50'	92°15'	37°44'	92°00'	38°38'	92	54	92	54	92	55	92	55	92	55	92	55		
102	94°00'	36°20'	93°45'	36°15'	93°30'	36°50'	93°15'	37°44'	93°00'	38°38'	93	55	93	55	93	56	93	56	93	56	93	56		
103	95°00'	36°20'	94°45'	36°15'	94°30'	36°50'	94°15'	37°44'	94°00'	38°38'	94	56	94	56	94	57	94	57	94	57	94	57		
104	96°00'	36°20'	95°45'	36°15'	95°30'	36°50'	95°15'	37°44'	95°00'	38°38'	95	57	95	57	95	58	95	58	95	58	95	58		
105	97°00'	36°20'	96°45'	36°15'	96°30'	36°50'	96°15'	37°44'	96°00'	38°38'	96	58	96	58	96	59	96	59	96	59	96	59		
106	98°00'	36°20'	97°45'	36°15'	97°30'	36°50'	97°15'	37°44'	97°00'	38°38'	97	59	97	59	97	60	97	60	97	60	97	60		
107	99°00'	36°20'	98°45'	36°15'	98°30'	36°50'	98°15'	37°44'	98°00'	38°38'	98	60	98	60	98	61	98	61	98	61	98	61		
108	100°00'	36°20'	99°45'	36°15'	99°30'	36°50'	99°15'	37°44'	99°00'	38°38'	99	61	99	61	99	62	99	62	99	62	99	62		
109	101°00'	36°20'	100°45'	36°15'	100°30'	36°50'	100°15'	37°44'	100°00'	38°38'	100	62	100	62	100	63	100	63	100	63	100	63		
110	102°00'	36°20'	101°45'	36°15'	101°30'	36°50'	101°15'	37°44'	101°00'	38°38'	101	63	101	63	101	64	101	64						

[illegible]

TRAVVERSE TABLE. 67.

26 Degrees

27 Degrees

Distance

0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.			Distance																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
61	54.83	26.74	54.71	26.98	54.59	27.22	54.47	27.46	54.35	27.69	54.23	27.93	54.11	28.17	53.98	28.40	53.86	54.35	28.15	55.24	28.36	56.13	28.60	56.90	57.03	29.06	57.79	29.51	58.68	30.22	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	70																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
62	55.73	27.18	55.61	27.42	55.49	27.66	55.37	27.91	55.24	28.15	55.12	28.39	55.00	28.63	54.87	28.87	54.83	56.13	28.60	56.90	29.06	57.03	29.51	58.68	30.22	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	71																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
63	56.62	27.62	56.50	27.86	56.38	28.11	56.25	28.36	56.13	28.60	56.02	28.85	55.88	29.09	55.75	29.34	55.71	57.03	29.06	57.79	29.51	58.68	30.22	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	72																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
64	57.52	28.05	57.40	28.29	57.28	28.56	57.15	28.81	57.03	29.06	56.90	29.30	56.77	29.55	56.64	29.80	56.67	57.92	29.51	58.68	30.22	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	73																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
65	58.42	28.49	58.30	28.73	58.17	29.00	58.05	29.26	57.92	29.51	57.79	29.76	57.66	30.02	57.53	30.27	57.50	58.81	29.96	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	74																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
66	59.32	28.93	59.19	29.17	59.07	29.45	58.94	29.71	58.81	29.96	58.68	30.22	58.54	30.48	58.41	30.73	58.38	59.70	30.42	59.56	30.68	59.43	30.94	59.29	31.20	60.32	31.40	60.18	31.67	61.06	31.86	61.06	32.33	61.95	32.59	75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
67	60.22	29.37	60.09	29.63	59.96	29.89	59.83	30.16	59.70	30.42	59.56	30.68	59.43	30.94	59.29	31.20	60.32	60.59	30.87	60.45	31.14	61.06	31.86	61.06	32.33	61.95	32.59	76																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
68	61.12	29.81	60.99	30.07	60.86	30.34	60.72	30.61	60.59	30.87	60.45	31.14	61.06	31.86	61.06	32.33	61.95	61.48	31.33	61.34	31.59	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	62.23	31.78	6

TRAVERSE TABLE. 67.

[illegible]

TRAVERSE TABLE 67.

115

28 Degrees															29 Degrees															60 Degrees															61 Degrees															62 Degrees																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
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TRAVERSE TABLE. 67.

Distance	30 Degrees						31 Degrees						58 Degrees					
	0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.		
	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Dep.	Lat.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
1	0,87	0,50	0,86	0,50	0,86	0,51	0,86	0,51	0,86	0,51	0,86	0,51	0,86	0,51	0,86	0,51	0,86	0,51
2	1,73	1,00	1,73	1,00	1,72	1,01	1,72	1,01	1,71	1,02	1,71	1,03	1,71	1,03	1,70	1,04	1,70	1,05
3	2,60	1,50	2,59	1,51	2,58	1,52	2,58	1,52	2,57	1,53	2,57	1,54	2,57	1,54	2,56	1,55	2,55	1,56
4	3,46	2,00	3,45	2,01	3,45	2,03	3,44	2,04	3,43	2,06	3,42	2,07	3,41	2,09	3,40	2,10	3,40	2,11
5	4,33	2,50	4,32	2,52	4,31	2,54	4,30	2,56	4,29	2,57	4,27	2,59	4,26	2,61	4,25	2,63	4,25	2,64
6	5,20	3,00	5,18	3,02	5,17	3,04	5,16	3,07	5,14	3,09	5,13	3,11	5,12	3,13	5,10	3,16	5,10	3,17
7	6,06	3,50	6,05	3,53	6,03	3,55	6,02	3,58	6,00	3,60	5,98	3,63	5,97	3,66	5,95	3,68	5,95	3,69
8	6,93	4,00	6,91	4,03	6,89	4,06	6,88	4,09	6,86	4,12	6,84	4,15	6,82	4,18	6,80	4,21	6,80	4,22
9	7,79	4,50	7,77	4,53	7,75	4,57	7,73	4,60	7,71	4,63	7,69	4,67	7,67	4,70	7,65	4,74	7,65	4,75
10	8,66	5,00	8,64	5,04	8,62	5,07	8,59	5,11	8,57	5,15	8,55	5,19	8,53	5,22	8,50	5,26	8,50	5,27
11	9,53	5,50	9,50	5,54	9,48	5,58	9,45	5,62	9,43	5,66	9,40	5,71	9,38	5,75	9,35	5,79	9,35	5,80
12	10,39	6,00	10,35	6,04	10,34	6,09	10,31	6,13	10,29	6,18	10,26	6,22	10,23	6,27	10,20	6,31	10,20	6,32
13	11,26	6,50	11,23	6,55	11,20	6,60	11,17	6,65	11,14	6,69	11,11	6,74	11,08	6,79	11,05	6,84	11,05	6,85
14	12,12	7,00	12,09	7,05	12,06	7,10	12,03	7,16	12,00	7,21	11,97	7,26	11,94	7,32	11,90	7,37	11,90	7,38
15	12,99	7,50	12,96	7,56	12,93	7,61	12,89	7,67	12,86	7,72	12,82	7,78	12,79	7,84	12,75	7,89	12,75	7,90
16	13,86	8,00	13,82	8,06	13,79	8,12	13,75	8,18	13,71	8,24	13,68	8,30	13,64	8,36	13,60	8,42	13,60	8,43
17	14,72	8,50	14,69	8,56	14,65	8,63	14,61	8,69	14,57	8,75	14,53	8,82	14,50	8,88	14,45	8,94	14,45	8,95
18	15,59	9,00	15,55	9,07	15,51	9,14	15,47	9,20	15,43	9,27	15,39	9,34	15,35	9,40	15,30	9,47	15,30	9,48
19	16,45	9,50	16,41	9,57	16,37	9,65	16,33	9,71	16,28	9,78	16,24	9,86	16,20	9,93	16,15	10,00	16,15	10,01
20	17,32	10,00	17,28	10,07	17,23	10,15	17,19	10,23	17,14	10,30	17,10	10,37	17,05	10,45	17,00	10,52	17,00	10,53
21	18,19	10,50	18,14	10,58	18,09	10,66	18,05	10,74	18,01	10,82	17,95	10,89	17,91	10,97	17,86	11,05	17,86	11,06
22	19,05	11,00	19,00	11,08	18,96	11,17	18,91	11,25	18,86	11,33	18,81	11,41	18,76	11,50	18,71	11,58	18,71	11,59
23	19,92	11,50	19,87	11,59	19,52	11,67	19,76	11,76	19,71	11,85	19,66	11,93	19,61	12,02	19,56	12,10	19,56	12,11
24	20,78	12,00	20,73	12,09	20,68	12,18	20,62	12,27	20,57	12,36	20,52	12,45	20,46	12,54	20,41	12,63	20,41	12,64
25	21,65	12,50	21,60	12,59	21,54	12,69	21,48	12,78	21,43	12,88	21,37	12,97	21,32	13,06	21,26	13,15	21,26	13,16
26	22,52	13,00	22,46	13,10	22,40	13,20	22,34	13,29	22,28	13,39	22,23	13,49	22,17	13,59	22,11	13,68	22,11	13,69
27	23,38	13,50	23,32	13,60	23,27	13,71	23,20	13,80	23,14	13,91	23,08	14,01	23,02	14,11	22,96	14,21	22,96	14,22
28	24,24	14,00	24,19	14,11	24,13	14,21	24,06	14,31	24,00	14,42	23,94	14,53	23,88	14,63	23,81	14,73	23,81	14,74
29	25,11	14,50	25,05	14,61	24,99	14,72	24,92	14,82	24,86	14,94	24,79	15,05	24,73	15,16	24,66	15,26	24,66	15,27
30	25,98	15,00	25,91	15,11	25,85	15,22	25,78	15,34	25,71	15,45	25,65	15,56	25,58	15,68	25,51	15,79	25,51	15,80
31	26,85	15,50	26,78	15,62	26,71	15,73	26,64	15,85	26,57	15,97	26,50	16,08	26,43	16,20	26,30	16,31	26,30	16,32
32	27,72	16,00	27,64	16,12	27,57	16,24	27,50	16,36	27,43	16,48	27,36	16,60	27,22	16,72	27,21	16,84	27,21	16,85
33	28,58	16,50	28,51	16,62	28,43	16,75	28,36	16,87	28,29	17,00	28,21	17,12	28,14	17,24	28,06	17,36	28,06	17,37
34	29,44	17,00	29,37	17,13	29,30	17,26	29,22	17,38	29,14	17,51	29,07	17,64	28,99	17,77	28,91	17,89	28,91	17,90
35	30,31	17,50	30,23	17,63	30,16	17,77	30,08	17,89	30,00	18,03	29,92	18,16	29,84	18,29	29,76	18,41	29,76	18,42
36	31,18	18,00	31,10	18,14	31,02	18,27	30,94	18,40	30,86	18,54	30,78	18,68	30,70	18,81	30,61	18,94	30,61	18,95
37	32,04	18,50	31,96	18,64	31,88	18,78	31,79	18,92	31,71	19,06	31,63	19,20	31,55	19,34	31,45	19,47	31,45	19,48
38	32,91	19,00	32,83	19,14	32,74	19,29	32,65	19,43	32,57	19,59	32,49	19,71	32,40	19,86	32,31	19,99	32,31	20,00
39	33,77	19,50	33,69	19,65	33,60	19,80	33,51	19,94	33,43	20,09	33,34	20,23	33,26	20,38	33,16	20,52	33,16	20,53
40	34,64	20,00	34,55	20,15	34,46	20,30	34,38	20,45	34,29	20,60	34,20	20,75	34,11	20,90	34,01	21,05	34,01	21,06
41	35,51	20,50	35,42	20,65	35,33	20,81	35,23	20,96	35,14	21,12	35,05	21,27	34,96	21,42	34,86	21,57	34,86	21,58
42	36,37	21,00	36,28	21,16	36,19	21,32	36,09	21,47	36,00	21,63	35,92	21,79	35,81	21,95	35,71	22,10	35,71	22,11
43	37,24	21,50	37,15	21,66	37,05	21,83	36,95	21,98	36,86	22,15	36,76	22,31	36,66	22,47	36,56	22,63	36,56	22,64
44	38,10	22,00	38,01	22,17	37,91	22,33	37,81	22,48	37,71	22,66	37,62	22,83	37,52	22,99	37,41	23,15	37,41	23,16
45	38,97	22,50	38,87	22,67	38,77	22,84	38,67	22,99	38,57	23,18	38,47	23,35	38,37	23,51	38,26	23,68	38,26	23,69
46	39,84	23,00	39,74	23,17	39,64	23,35	39,53	23,50	39,43	23,69	39,33	23,86	39,22	24,04	39,11	24,20	39,11	24,21
47	40,70	23,50	40,60	23,68	40,50	23,86	40,39	24,01	40,29	24,21	40,18	24,38	40,08	24,56	39,96	24,73	39,96	24,74
48	41,57	24,00	41,47	24,18	41,36	24,36	41,25	24,52	41,14	24,72	41,04	24,90	40,93	25,08	40,81	25,26	40,81	25,27
49	42,43	24,50	42,33	24,69	42,22	24,86	42,11	25,03	42,00	25,24	41,89	25,42	41,78	25,61	41,66	25,78	41,66	25,79
50	43,30	25,00	43,19	25,19	43,08	25,36	42,97	25,56	42,86	25,75	42,75	25,94	42,64	26,12	42,52	26,31	42,52	26,32
51	44,16	25,50	44,06	25,69	43,94	25,88	43,83	26,08	43,72	26,27	43,60	26,46	43,48	26,65	43,37	26,84	43,37	26,85
52	45,03	26,00	44,92	26,20	44,81	26,39	44,69	26,59	44,57	26,77	44,46	26,98	44,34	27,17	44,22	27,36	44,22	27,37
53	45,90	26,50	45,78	26,70	45,67	26,90	45,55	27,10	45,45	27,29	45,31	27,50	45,19	27,69	45,07	27,89	45,07	27,90
54	46,76	27,00	46,65	27,20	46,53	27,41	46,41	27,61	46,29	27,80	46,17	28,01	46,04	28,22	45,92	28,41	45,92	28,42
55	47,63	27,50	47,51	27,71	47,39	27,92	47,27	28,12	47,14	28,32	47,02	28,53	46,90	28,74	46,77	28,94	46,77	28,95
56	48,50	28,00	48,38	28,21	48,25	28,42	48,13	28,63	48,00	28,85	47,88	29,05	47,75	29,26	47,62	29,47	47,62	29,48
57	49,36	28,50	49,24	28,72	49,12	28,93	48,98	29,14	48,86	29,35	48,73	29,57	48,60	29,79	48,47	29,99	48,47	30,00
58	50,23	29,00	50,10	29,22	49,98	29,44	49,84	29,65	49,71	29,86	49,59	30,09	49,46	30,31	49,32	30,52	49,32	30,53
59	51,10	29,50	50,97	29,72	50,84	29,95	50,70	30,16	50,57	30,38	50,44	30,61	50,31	30,83	50,17	31,04	50,17	31,05
60	51,96	30,00	51,83	30,23	51,76	30,45	51,56	30,68	51,43	30,90	51,29	31,13	51,16	31,35	51,02	31,57	51,02	31,58

[illegible]

TRAVERSE TABLE. 67.

Distance	32 Degrees						33 Degrees										
	0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	0 85	0 53	0 85	0 53	0 84	0 54	0 84	0 54	0 84	0 54	0 84	0 54	0 84	0 55	0 83	0 56	
2	1 70	1 06	1 69	1 07	1 68	1 08	1 68	1 08	1 68	1 09	1 67	1 10	1 67	1 10	1 66	1 67	
3	2 54	1 59	2 54	1 60	2 53	1 61	2 52	1 62	2 52	1 63	2 51	1 64	2 50	1 66	2 49	1 71	
4	3 39	2 12	3 38	2 13	3 37	2 15	3 36	2 16	3 35	2 18	3 34	2 19	3 34	2 21	3 33	2 22	
5	4 24	2 65	4 23	2 67	4 22	2 69	4 20	2 70	4 19	2 72	4 18	2 74	4 17	2 76	4 16	2 78	
6	5 09	3 18	5 07	3 20	5 06	3 22	5 05	3 25	5 03	3 27	5 02	3 29	5 00	3 31	4 99	3 33	
7	5 94	3 71	5 92	3 73	5 90	3 76	5 89	3 78	5 87	3 81	5 85	3 84	5 84	3 86	5 82	3 89	
8	6 78	4 24	6 77	4 27	6 75	4 30	6 73	4 33	6 71	4 36	6 69	4 39	6 67	4 42	6 65	4 45	
9	7 63	4 77	7 61	4 80	7 59	4 84	7 57	4 87	7 55	4 90	7 53	4 93	7 50	4 97	7 48	5 00	
10	8 48	5 30	8 46	5 34	8 43	5 37	8 41	5 41	8 39	5 45	8 36	5 48	8 34	5 52	8 32	5 56	
11	9 33	5 83	9 30	5 87	9 27	5 91	9 25	5 95	9 22	5 99	9 20	6 03	9 17	6 07	9 15	6 11	
12	10 18	6 36	10 15	6 40	10 12	6 45	10 09	6 49	10 06	6 54	10 03	6 58	10 01	6 62	9 98	6 67	
13	11 02	6 89	10 99	6 94	10 96	6 99	10 91	7 03	10 90	7 08	10 87	7 13	10 84	7 17	10 81	7 22	
14	11 87	7 42	11 84	7 47	11 81	7 52	11 77	7 57	11 74	7 63	11 71	7 67	11 67	7 73	11 64	7 78	
15	12 72	7 95	12 69	8 00	12 65	8 06	12 59	8 11	12 58	8 17	12 54	8 22	12 51	8 28	12 47	8 33	
16	13 57	8 48	13 53	8 54	13 49	8 60	13 45	8 65	13 42	8 72	13 38	8 77	13 34	8 83	13 30	8 89	
17	14 42	9 01	14 38	9 07	14 34	9 13	14 28	9 19	14 26	9 26	14 21	9 32	14 18	9 38	14 13	9 45	
18	15 26	9 54	15 22	9 60	15 18	9 67	15 14	9 73	15 10	9 81	15 05	9 87	15 01	9 93	14 96	9 99	
19	16 11	10 07	16 07	10 14	16 02	10 21	16 06	10 27	15 99	10 35	15 89	10 41	15 84	10 49	15 86	10 50	
20	16 96	10 60	16 91	10 67	16 87	10 75	16 82	10 82	16 77	10 89	16 73	10 97	16 68	11 04	16 63	11 10	
21	17 81	11 13	17 76	11 21	17 71	11 28	17 66	11 36	17 61	11 44	17 56	11 51	17 51	11 59	17 46	11 67	
22	18 65	11 66	18 61	11 74	18 55	11 82	18 50	11 90	18 45	11 98	18 40	12 06	18 35	12 14	18 29	12 22	
23	19 50	12 19	19 45	12 27	19 40	12 36	19 34	12 44	19 29	12 53	19 23	12 61	19 18	12 69	19 12	12 78	
24	20 35	12 72	20 30	12 81	20 24	12 90	20 18	12 98	20 13	13 07	20 07	13 16	20 01	13 25	19 59	13 34	
25	21 20	13 25	21 14	13 34	21 08	13 44	21 03	13 52	20 97	13 62	20 91	13 71	20 85	13 80	20 78	13 86	
26	22 05	13 78	21 99	13 88	21 93	13 97	21 87	14 06	21 81	14 16	21 74	14 25	21 68	14 35	21 62	14 45	
27	22 50	14 31	22 44	14 41	22 37	14 51	22 31	14 61	22 25	14 71	22 18	14 80	22 52	14 90	22 45	15 00	
28	23 34	14 84	23 28	14 94	23 22	15 05	23 15	15 15	23 08	15 25	23 01	15 35	23 54	23 28	15 56	23 28	
29	24 19	15 37	24 13	15 48	24 05	15 58	24 00	15 69	24 32	15 80	24 25	15 90	24 18	16 01	24 41	16 12	
30	25 04	15 90	25 37	16 01	25 30	16 12	25 23	16 23	25 16	16 34	25 09	16 45	25 02	16 56	24 94	16 57	
31	25 49	16 43	25 42	16 54	25 36	16 66	25 30	16 77	25 23	16 88	25 16	16 98	25 10	17 00	25 78	17 22	
32	26 34	16 96	26 27	17 08	26 19	17 19	26 12	17 31	26 04	17 43	26 00	17 54	26 68	17 66	26 61	17 78	
33	27 19	17 49	27 11	17 61	27 03	17 73	27 00	17 85	26 52	17 97	26 46	18 09	27 54	27 44	18 34	18 32	
34	28 04	18 02	27 97	18 14	27 89	18 27	27 82	18 39	27 75	18 52	27 68	18 69	28 35	28 27	18 89	18 92	
35	28 49	18 55	28 42	18 68	28 34	18 80	28 27	18 93	28 19	19 06	28 12	19 19	29 19	29 12	19 45	19 48	
36	29 34	19 08	29 27	19 21	29 19	19 34	29 12	19 47	29 05	19 61	28 58	19 74	30 10	29 93	20 00	20 06	
37	30 19	19 61	30 12	19 75	30 06	19 88	30 00	20 02	30 03	20 15	30 04	20 28	30 58	20 42	30 76	20 56	
38	31 04	20 14	30 97	20 28	30 90	20 41	30 83	20 56	30 75	20 70	30 68	20 83	31 69	20 97	31 59	21 38	
39	31 49	20 67	31 42	20 81	31 34	20 95	31 26	21 10	31 18	21 24	32 01	31 38	32 52	32 42	32 67	32 59	
40	32 34	21 20	32 27	21 34	32 19	21 49	32 11	21 64	32 05	21 79	33 45	32 53	33 36	33 26	33 26	33 40	
41	33 19	21 73	33 12	21 88	33 05	22 03	32 57	22 18	32 50	22 33	34 29	32 48	34 19	22 63	34 09	22 78	
42	34 04	22 26	33 57	22 41	33 49	22 57	33 42	23 12	33 35	23 38	35 12	23 03	35 02	23 18	34 92	23 34	
43	34 49	22 79	34 42	22 95	34 34	23 10	34 27	23 26	34 20	23 49	35 06	23 58	35 86	23 73	35 75	23 89	
44	35 34	23 32	35 27	23 48	35 19	23 56	35 12	24 06	35 06	24 34	36 02	24 32	36 69	24 29	36 50	24 43	
45	36 19	23 85	36 12	24 01	36 04	24 13	35 57	24 23	35 50	24 52	36 41	24 67	37 53	24 84	37 41	25 00	
46	37 04	24 38	36 97	24 55	36 89	24 71	36 82	25 01	36 74	25 20	38 47	25 22	38 36	25 39	38 25	25 46	
47	37 49	24 91	37 42	25 08	37 34	25 12	37 27	25 23	37 19	25 32	39 04	25 27	39 19	25 94	39 08	26 12	
48	38 34	25 44	38 27	25 62	38 19	25 70	38 12	25 81	38 04	25 90	40 14	26 32	40 01	26 49	39 51	26 67	
49	39 19	25 97	39 12	26 15	39 04	26 32	38 97	26 43	38 89	26 50	40 98	26 86	40 86	27 05	40 74	27 30	
50	40 04	26 50	40 00	26 58	40 00	27 06	40 00	27 05	40 00	27 05	41 81	27 41	41 69	27 60	41 51	27 78	
51	40 49	27 03	40 42	27 21	40 34	27 40	40 27	27 59	40 19	27 66	42 65	27 96	42 53	28 15	42 41	28 34	
52	41 34	27 56	41 27	27 75	41 20	27 86	41 12	28 05	41 04	28 12	43 40	28 51	43 36	28 70	43 24	28 89	
53	42 19	28 09	42 12	28 28	42 04	28 48	41 57	28 67	41 49	28 89	44 32	29 06	44 20	29 25	44 07	29 45	
54	43 04	28 62	42 57	28 82	42 49	29 01	42 42	29 21	42 34	29 43	45 16	29 61	45 03	29 80	44 90	30 00	
55	43 49	29 15	43 42	29 35	43 34	29 55	43 27	29 75	43 19	29 98	46 09	30 15	45 86	30 36	45 71	30 56	
56	44 34	29 68	44 27	29 88	44 19	30 09	44 12	30 29	44 04	30 49	46 83	30 70	46 70	30 91	46 56	31 12	
57	45 19	30 21	45 12	30 42	45 04	30 62	44 57	30 84	44 49	31 07	47 67	31 25	47 53	31 46	47 39	31 67	
58	46 04	30 74	45 57	30 95	45 49	31 16	45 42	31 38	45 34	31 61	48 50	31 80	48 37	32 01	48 22	32 38	
59	46 49	31 27	46 42	31 49	46 34	31 70	46 27	31 92	46 19	32 16	49 34	32 35	49 20	32 56	49 05	32 78	
60	47 34	31 79	47 27	32 02	47 19	32 24	47 12	32 46	47 04	32 68	50 18	32 90	50 01	33 12	49 86	33 33	
Distance	0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		Distance

TRAVERSE TABLE. 67.

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32 Degrees			33 Degrees			56 Degrees			57 Degrees			58 Degrees		
0 Min.		15 Min.	30 Min.		45 Min.	0 Min.		15 Min.	30 Min.		45 Min.	0 Min.		15 Min.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
61	51,35	32,55	51,45	32,77	51,30	33,00	51,16	33,22	51,01	33,44	50,85	33,57	50,72	33,59
62	51,36	32,56	51,46	32,78	51,31	33,01	51,17	33,23	51,02	33,45	50,86	33,58	50,73	33,60
63	51,37	32,57	51,47	32,79	51,32	33,02	51,18	33,24	51,03	33,46	50,87	33,59	50,74	33,61
64	51,38	32,58	51,48	32,80	51,33	33,03	51,19	33,25	51,04	33,47	50,88	33,60	50,75	33,62
65	51,39	32,59	51,49	32,81	51,34	33,04	51,20	33,26	51,05	33,48	50,89	33,61	50,76	33,63
66	51,40	32,60	51,50	32,82	51,35	33,05	51,21	33,27	51,06	33,49	50,90	33,62	50,77	33,64
67	51,41	32,61	51,51	32,83	51,36	33,06	51,22	33,28	51,07	33,50	50,91	33,63	50,78	33,65
68	51,42	32,62	51,52	32,84	51,37	33,07	51,23	33,29	51,08	33,51	50,92	33,64	50,79	33,66
69	51,43	32,63	51,53	32,85	51,38	33,08	51,24	33,30	51,09	33,52	50,93	33,65	50,80	33,67
70	51,44	32,64	51,54	32,86	51,39	33,09	51,25	33,31	51,10	33,53	50,94	33,66	50,81	33,68
71	51,45	32,65	51,55	32,87	51,40	33,10	51,26	33,32	51,11	33,54	50,95	33,67	50,82	33,69
72	51,46	32,66	51,56	32,88	51,41	33,11	51,27	33,33	51,12	33,55	50,96	33,68	50,83	33,70
73	51,47	32,67	51,57	32,89	51,42	33,12	51,28	33,34	51,13	33,56	50,97	33,69	50,84	33,71
74	51,48	32,68	51,58	32,90	51,43	33,13	51,29	33,35	51,14	33,57	50,98	33,70	50,85	33,72
75	51,49	32,69	51,59	32,91	51,44	33,14	51,30	33,36	51,15	33,58	50,99	33,71	50,86	33,73
76	51,50	32,70	51,60	32,92	51,45	33,15	51,31	33,37	51,16	33,59	51,00	33,72	50,87	33,74
77	51,51	32,71	51,61	32,93	51,46	33,16	51,32	33,38	51,17	33,60	51,01	33,73	50,88	33,75
78	51,52	32,72	51,62	32,94	51,47	33,17	51,33	33,39	51,18	33,61	51,02	33,74	50,89	33,76
79	51,53	32,73	51,63	32,95	51,48	33,18	51,34	33,40	51,19	33,62	51,03	33,75	50,90	33,77
80	51,54	32,74	51,64	32,96	51,49	33,19	51,35	33,41	51,20	33,63	51,04	33,76	50,91	33,78
81	51,55	32,75	51,65	32,97	51,50	33,20	51,36	33,42	51,21	33,64	51,05	33,77	50,92	33,79
82	51,56	32,76	51,66	32,98	51,51	33,21	51,37	33,43	51,22	33,65	51,06	33,78	50,93	33,80
83	51,57	32,77	51,67	32,99	51,52	33,22	51,38	33,44	51,23	33,66	51,07	33,79	50,94	33,81
84	51,58	32,78	51,68	33,00	51,53	33,23	51,39	33,45	51,24	33,67	51,08	33,80	50,95	33,82
85	51,59	32,79	51,69	33,01	51,54	33,24	51,40	33,46	51,25	33,68	51,09	33,81	50,96	33,83
86	51,60	32,80	51,70	33,02	51,55	33,25	51,41	33,47	51,26	33,69	51,10	33,82	50,97	33,84
87	51,61	32,81	51,71	33,03	51,56	33,26	51,42	33,48	51,27	33,70	51,11	33,83	50,98	33,85
88	51,62	32,82	51,72	33,04	51,57	33,27	51,43	33,49	51,28	33,71	51,12	33,84	50,99	33,86
89	51,63	32,83	51,73	33,05	51,58	33,28	51,44	33,50	51,29	33,72	51,13	33,85	51,00	33,87
90	51,64	32,84	51,74	33,06	51,59	33,29	51,45	33,51	51,30	33,73	51,14	33,86	51,01	33,88
91	51,65	32,85	51,75	33,07	51,60	33,30	51,46	33,52	51,31	33,74	51,15	33,87	51,02	33,89
92	51,66	32,86	51,76	33,08	51,61	33,31	51,47	33,53	51,32	33,75	51,16	33,88	51,03	33,90
93	51,67	32,87	51,77	33,09	51,62	33,32	51,48	33,54	51,33	33,76	51,17	33,89	51,04	33,91
94	51,68	32,88	51,78	33,10	51,63	33,33	51,49	33,55	51,34	33,77	51,18	33,90	51,05	33,92
95	51,69	32,89	51,79	33,11	51,64	33,34	51,50	33,56	51,35	33,78	51,19	33,91	51,06	33,93
96	51,70	32,90	51,80	33,12	51,65	33,35	51,51	33,57	51,36	33,79	51,20	33,92	51,07	33,94
97	51,71	32,91	51,81	33,13	51,66	33,36	51,52	33,58	51,37	33,80	51,21	33,93	51,08	33,95
98	51,72	32,92	51,82	33,14	51,67	33,37	51,53	33,59	51,38	33,81	51,22	33,94	51,09	33,96
99	51,73	32,93	51,83	33,15	51,68	33,38	51,54	33,60	51,39	33,82	51,23	33,95	51,10	33,97
100	51,74	32,94	51,84	33,16	51,69	33,39	51,55	33,61	51,40	33,83	51,24	33,96	51,11	33,98
101	51,75	32,95	51,85	33,17	51,70	33,40	51,56	33,62	51,41	33,84	51,25	33,97	51,12	33,99
102	51,76	32,96	51,86	33,18	51,71	33,41	51,57	33,63	51,42	33,85	51,26	33,98	51,13	34,00
103	51,77	32,97	51,87	33,19	51,72	33,42	51,58	33,64	51,43	33,86	51,27	33,99	51,14	34,01
104	51,78	32,98	51,88	33,20	51,73	33,43	51,59	33,65	51,44	33,87	51,28	34,00	51,15	34,02
105	51,79	32,99	51,89	33,21	51,74	33,44	51,60	33,66	51,45	33,88	51,29	34,01	51,16	34,03
106	51,80	33,00	51,90	33,22	51,75	33,45	51,61	33,67	51,46	33,89	51,30	34,02	51,17	34,04
107	51,81	33,01	51,91	33,23	51,76	33,46	51,62	33,68	51,47	33,90	51,31	34,03	51,18	34,05
108	51,82	33,02	51,92	33,24	51,77	33,47	51,63	33,69	51,48	33,91	51,32	34,04	51,19	34,06
109	51,83	33,03	51,93	33,25	51,78	33,48	51,64	33,70	51,49	33,92	51,33	34,05	51,20	34,07
110	51,84	33,04	51,94	33,26	51,79	33,49	51,65	33,71	51,50	33,93	51,34	34,06	51,21	34,08
111	51,85	33,05	51,95	33,27	51,80	33,50	51,66	33,72	51,51	33,94	51,35	34,07	51,22	34,09
112	51,86	33,06	51,96	33,28	51,81	33,51	51,67	33,73	51,52	33,95	51,36	34,08	51,23	34,10
113	51,87	33,07	51,97	33,29	51,82	33,52	51,68	33,74	51,53	33,96	51,37	34,09	51,24	34,11
114	51,88	33,08	51,98	33,30	51,83	33,53	51,69	33,75	51,54	33,97	51,38	34,10	51,25	34,12
115	51,89	33,09	51,99	33,31	51,84	33,54	51,70	33,76	51,55	33,98	51,39	34,11	51,26	34,13
116	51,90	33,10	52,00	33,32	51,85	33,55	51,71	33,77	51,56	33,99	51,40	34,12	51,27	34,14
117	51,91	33,11	52,01	33,33	51,86	33,56	51,72	33,78	51,57	34,00	51,41	34,13	51,28	34,15
118	51,92	33,12	52,02	33,34	51,87	33,57	51,73	33,79	51,58	34,01	51,42	34,14	51,29	34,16
119	51,93	33,13	52,03	33,35	51,88	33,58	51,74	33,80	51,59	34,02	51,43	34,15	51,30	34,17
120	51,94	33,14	52,04	33,36	51,89	33,59	51,75	33,81	51,60	34,03	51,44	34,16	51,31	34,18
121	51,95	33,15	52,05	33,37	51,90	33,60	51,76	33,82	51,61	34,04	51,45	34,17	51,32	34,19
122	51,96	33,16	52,06	33,38	51,91	33,61	51,77	33,83	51,62	34,05	51,46	34,18	51,33	34,20
123	51,97	33,17	52,07	33,39	51,92	33,62	51,78	33,84	51,63	34,06	51,47	34,19	51,34	34,21
124	51,98	33,18	52,08	33,40	51,93	33,63	51,79	33,85	51,64	34,07	51,48	34,20	51,35	34,22
125	51,99	33,19	52,09	33,41	51,94	33,64	51,80	33,86	51,65	34,08	51,49	34,21	51,36	34,23
126	52,00	33,20	52,10	33,42	51,95	33,65	51,81	33,87	51,66	34,09	51,50	34,22	51,37	34,24
127	52,01	33,21	52,11	33,43	51,96	33,66	51,82	33,88	51,67	34,10	51,51	34,23	51,38	34,25
128	52,02	33,22	52,12	33,44	51,97	33,67	51,83	33,89	51,68	34,11	51,52	34,24	51,39	34,26
129	52,03	33,23	52,13	33,45	51,98	33,68	51,84	33,90	51,69	34,12	51,53	34,25	51,40	34,27
130	52,04	33,24	52,14	33,46	51,99	33,69	51,85	33,91	51,70	34,13	51,54	34,26	51,41	34,28
131	52,05	33,25	52,15	33,47	52,00	33,70	51,86	33,92	51,71	34,14	51,55	34,27	51,42	34,29
132	52,06	33,26	52,16	33,48	52,01	33,71	51,87	33,93	51,72	34,15	51,56	34,28	51,43	34,30
133	52,07	33,27	52,17	33,49	52,02	33,72	51,88	33,94	51,73	34,16	51,57	34,29	51,44	34,31
134	52,08	33,28	52,18	33,50	52,03	33,73	51,89	33,95	51,74	34,17	51,58	34,30	51,45	34,32
135	52,09	33,29	52,19	33,51	52,04	33,74	51,90	33,96	51,75	34,18	51,59	34,31	51,46	34,33
136	52,10	33,30	52,20	33,52	52,05	33,75	51,91	33,97	51,76	34,19	51,60	34,32	51,47	34,34
137	52,11	33,31	52,21	33,53	52,06	33,76	51,92	33,98	51,77	34,20	51,61	34,33	51,48	34,35

TRAVERSE TABLE 67.

Distance	34 Degrees						35 Degrees						54 Degrees					
	0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0,83	0,56	0,83	0,57	0,82	0,57	0,82	0,57	0,82	0,57	0,82	0,57	0,82	0,58	0,81	0,58	0,81	0,58
2	1,66	1,12	1,65	1,13	1,65	1,14	1,64	1,15	1,64	1,15	1,63	1,15	1,63	1,16	1,62	1,17	1,62	1,17
3	2,49	1,69	2,47	1,70	2,47	1,71	2,46	1,72	2,46	1,72	2,45	1,73	2,44	1,74	2,43	1,75	2,43	1,75
4	3,32	2,25	3,30	2,27	3,30	2,28	3,28	2,29	3,28	2,29	3,27	2,30	3,26	2,32	3,25	2,34	3,25	2,34
5	4,15	2,81	4,13	2,83	4,12	2,85	4,10	2,87	4,10	2,87	4,08	2,89	4,07	2,90	4,06	2,92	4,06	2,92
6	4,97	3,36	4,96	3,40	4,94	3,42	4,91	3,44	4,91	3,44	4,90	3,46	4,88	3,48	4,87	3,51	4,87	3,51
7	5,80	3,91	5,79	3,96	5,77	3,99	5,75	4,01	5,75	4,01	5,72	4,03	5,69	4,06	5,68	4,09	5,68	4,09
8	6,63	4,47	6,61	4,53	6,59	4,56	6,55	4,59	6,55	4,59	6,53	4,62	6,51	4,65	6,49	4,67	6,49	4,67
9	7,46	5,03	7,44	5,10	7,42	5,13	7,39	5,16	7,39	5,16	7,35	5,19	7,33	5,23	7,30	5,26	7,30	5,26
10	8,29	5,59	8,27	5,66	8,24	5,70	8,19	5,74	8,19	5,74	8,17	5,77	8,14	5,81	8,12	5,84	8,12	5,84
11	9,12	6,15	9,09	6,23	9,06	6,27	9,01	6,31	9,01	6,31	8,98	6,34	8,95	6,39	8,93	6,43	8,93	6,43
12	9,95	6,71	9,92	6,80	9,89	6,86	9,84	6,88	9,84	6,88	9,80	6,93	9,77	6,97	9,74	7,01	9,74	7,01
13	10,78	7,27	10,75	7,36	10,71	7,36	10,68	7,40	10,65	7,40	10,62	7,50	10,58	7,55	10,55	7,59	10,55	7,59
14	11,61	7,83	11,57	7,88	11,54	7,93	11,50	7,98	11,47	8,03	11,43	8,08	11,40	8,13	11,36	8,18	11,36	8,18
15	12,44	8,39	12,40	8,44	12,36	8,49	12,33	8,55	12,29	8,60	12,25	8,66	12,21	8,71	12,17	8,76	12,17	8,76
16	13,26	8,95	13,23	9,01	13,19	9,06	13,15	9,12	13,11	9,18	13,07	9,23	13,02	9,29	12,99	9,35	12,99	9,35
17	14,09	9,51	14,05	9,57	14,01	9,63	13,98	9,69	13,92	9,75	13,88	9,81	13,84	9,87	13,80	9,93	13,80	9,93
18	14,92	10,06	14,88	10,13	14,83	10,19	14,79	10,26	14,74	10,33	14,70	10,39	14,65	10,45	14,61	10,51	14,61	10,51
19	15,75	10,62	15,71	10,69	15,66	10,76	15,61	10,83	15,56	10,90	15,52	10,96	15,47	11,03	15,42	11,10	15,42	11,10
20	16,58	11,18	16,53	11,26	16,48	11,33	16,43	11,40	16,38	11,47	16,33	11,54	16,28	11,61	16,23	11,68	16,23	11,68
21	17,41	11,74	17,36	11,82	17,31	11,89	17,25	11,97	17,20	12,05	17,15	12,12	17,10	12,19	17,04	12,27	17,04	12,27
22	18,24	12,30	18,19	12,38	18,13	12,46	18,08	12,54	18,02	12,62	17,98	12,70	17,91	12,78	17,85	12,85	17,85	12,85
23	19,07	12,86	19,01	12,95	18,95	13,03	18,90	13,11	18,84	13,19	18,78	13,27	18,72	13,36	18,67	13,44	18,67	13,44
24	19,90	13,42	19,84	13,51	19,78	13,59	19,72	13,68	19,66	13,77	19,60	13,85	19,54	13,94	19,48	14,02	19,48	14,02
25	20,73	13,98	20,67	14,08	20,60	14,16	20,54	14,25	20,48	14,34	20,41	14,43	20,35	14,52	20,29	14,60	20,29	14,60
26	21,56	14,54	21,49	14,64	21,43	14,72	21,36	14,82	21,30	14,92	21,23	15,00	20,17	15,10	20,10	15,19	20,10	15,19
27	22,38	15,10	22,32	15,20	22,25	15,29	22,19	15,39	22,12	15,49	22,05	15,58	21,98	15,68	21,91	15,77	21,91	15,77
28	23,21	15,66	23,15	15,77	23,07	15,86	23,01	15,96	22,93	16,06	22,87	16,17	22,79	16,26	22,73	16,38	22,73	16,38
29	24,04	16,22	23,97	16,33	23,90	16,42	23,83	16,53	23,75	16,64	23,69	16,75	23,61	16,84	23,54	16,94	23,54	16,94
30	24,87	16,78	24,80	16,88	24,72	16,99	24,65	17,10	24,57	17,21	24,50	17,31	24,42	17,42	24,35	17,53	24,35	17,53
31	25,70	17,33	25,62	17,45	25,55	17,56	25,47	17,67	25,39	17,78	25,32	17,89	25,24	18,00	25,16	18,11	25,16	18,11
32	26,53	17,89	26,45	18,01	26,37	18,12	26,29	18,24	26,21	18,35	26,13	18,47	26,05	18,58	25,97	18,70	25,97	18,70
33	27,36	18,45	27,28	18,57	27,20	18,69	27,12	18,81	27,03	18,93	26,95	19,05	26,87	19,16	26,78	19,28	26,78	19,28
34	28,19	19,01	28,11	19,14	28,02	19,25	27,94	19,38	27,85	19,50	27,77	19,62	27,68	19,74	27,59	19,86	27,59	19,86
35	29,02	19,57	28,93	19,70	28,84	19,82	28,76	19,95	28,67	20,08	28,58	20,20	28,49	20,33	28,41	20,45	28,41	20,45
36	29,84	20,13	29,76	20,26	29,67	20,39	29,58	20,52	29,49	20,65	29,40	20,78	29,31	20,91	29,22	21,05	29,22	21,05
37	30,67	20,69	30,59	20,82	30,49	20,95	30,40	21,09	30,31	21,22	30,22	21,35	30,12	21,49	30,03	21,62	30,03	21,62
38	31,50	21,25	31,41	21,39	31,32	21,52	31,23	21,66	31,13	21,80	31,03	21,93	30,94	22,07	30,84	22,20	30,84	22,20
39	32,33	21,81	32,24	21,95	32,14	22,09	32,05	22,23	31,95	22,37	31,85	22,51	31,75	22,65	31,65	22,78	31,65	22,78
40	33,16	22,37	33,06	22,51	32,96	22,66	32,87	22,80	32,77	22,94	32,67	23,09	32,56	23,23	32,46	23,37	32,46	23,37
41	33,99	22,93	33,89	23,07	33,79	23,22	33,69	23,37	33,58	23,52	33,48	23,66	33,38	23,81	33,27	23,95	33,27	23,95
42	34,82	23,49	34,72	23,64	34,61	23,79	34,51	23,94	34,40	24,09	34,30	24,24	34,19	24,39	34,09	24,54	34,09	24,54
43	35,65	24,04	35,54	24,20	35,44	24,35	35,33	24,51	35,22	24,66	35,12	24,82	35,01	24,97	34,90	25,12	34,90	25,12
44	36,48	24,60	36,37	24,76	36,26	24,92	36,15	25,08	36,04	25,24	35,93	25,39	35,82	25,55	35,71	25,71	35,71	25,71
45	37,31	25,16	37,20	25,33	37,08	25,49	36,98	25,65	36,86	25,81	36,75	25,97	36,63	26,13	36,52	26,29	36,52	26,29
46	38,14	25,72	38,03	25,89	37,91	26,05	37,80	26,22	37,68	26,39	37,57	26,55	37,45	26,71	37,33	26,87	37,33	26,87
47	38,96	26,27	38,85	26,45	38,73	26,62	38,62	26,79	38,50	26,96	38,38	27,12	38,26	27,29	38,15	27,46	38,15	27,46
48	39,79	26,84	39,68	27,02	39,56	27,18	39,44	27,36	39,32	27,53	39,20	27,70	39,07	27,88	38,96	28,04	38,96	28,04
49	40,62	27,40	40,51	27,58	40,39	27,75	40,26	27,93	40,14	28,11	40,02	28,28	39,89	28,46	39,77	28,63	39,77	28,63
50	41,45	27,96	41,33	28,14	41,21	28,32	41,08	28,50	40,96	28,68	40,83	28,86	40,71	29,03	40,58	29,21	40,58	29,21
51	42,28	28,52	42,16	28,70	42,03	28,89	41,90	29,07	41,78	29,25	41,65	29,39	41,52	29,62	41,39	29,80	41,39	29,80
52	43,11	29,08	42,98	29,27	42,85	29,45	42,73	29,64	42,60	29,83	42,47	30,01	42,33	30,20	42,20	30,38	42,20	30,38
53	43,94	29,64	43,81	29,83	43,68	30,02	43,55	30,21	43,41	30,40	43,28	30,59	43,15	30,78	43,01	30,96	43,01	30,96
54	44,77	30,20	44,64	30,39	44,50	30,58	44,37	30,78	44,23	30,97	44,10	31,17	44,06	31,36	43,83	31,55	43,83	31,55
55	45,60	30,75	45,46	30,95	45,33	31,15	45,19	31,35	45,05	31,55	44,92	31,74	45,78	31,94	44,64	32,13	44,64	32,13
56	46,43	31,31	46,29	31,52	46,15	31,72	46,01	31,92	45,87	32,12	45,73	32,32	46,59	32,52	45,45	32,72	45,45	32,72
57	47,25	31,87	47,12	32,08	46,97	32,28	46,84	32,49	46,69	32,70	46,55	32,90	46,40	33,10	46,26	33,30	46,26	33,30
58	48,08	32,43	47,95	32,64	47,80	32,85	47,66	33,06	47,51	33,27	47,37	33,43	47,22	33,68	47,07	33,88	47,07	33,88
59	48,91	32,99	48,77	33,20	48,62	33,41	48,48	33,63	48,33	33,84	48,18	34,05	48,03	34,26	47,86	34,47	47,86	34,47
60	49,74	33,55	49,59	33,77	49,45	33,98	49,39	34,26	49,15	34,41	49,00	34,63	48,85	34,84	48,66	35,05	48,66	35,05

34 Degrees												35 Degrees											
0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.		
Distance	Dep.	Lat.	Dep.	Lat.	Dep.	Distance	Dep.	Lat.	Dep.	Lat.	Dep.	Distance	Dep.	Lat.	Dep.	Lat.	Distance	Dep.	Lat.	Dep.	Lat.		
61	50.57	34.11	50.42	34.33	50.27	34.55	50.12	34.77	49.97	34.99	49.81	35.21	49.66	35.42	49.51	35.64	61						
62	51.40	34.67	51.25	34.39	51.10	35.12	50.94	35.34	50.79	35.56	50.63	35.78	50.47	36.00	50.32	36.22	62						
63	52.23	35.23	52.08	35.40	51.92	35.68	51.76	35.91	51.61	36.14	51.45	36.37	51.29	36.58	51.13	36.81	63						
64	53.06	35.79	52.90	36.02	52.74	36.48	52.58	36.71	52.42	36.94	52.26	37.17	52.10	37.37	51.94	37.39	64						
65	53.89	36.35	53.73	36.58	53.56	36.81	53.41	37.05	53.24	37.28	53.07	37.51	52.90	37.75	52.75	37.98	65						
66	54.72	36.90	54.56	37.15	54.39	37.38	54.23	37.62	54.06	37.86	53.90	38.09	53.73	38.33	53.56	38.56	66						
67	55.54	37.46	55.38	37.71	55.22	37.95	55.05	38.19	54.88	38.43	54.72	38.67	54.54	38.91	54.38	39.14	67						
68	56.37	38.02	56.21	38.27	56.04	38.51	55.87	38.76	55.70	39.01	55.53	39.24	55.36	39.49	55.20	39.73	68						
69	57.20	38.58	57.04	38.83	56.86	39.08	56.70	39.33	56.52	39.58	56.35	39.82	56.17	40.07	55.99	40.31	69						
70	58.03	39.14	57.86	39.40	57.69	39.65	57.52	39.90	57.34	40.15	57.16	40.40	56.99	40.65	56.81	40.89	70						
71	58.86	39.70	58.69	39.96	58.51	40.21	58.34	40.47	58.16	40.72	57.98	40.98	57.80	41.23	57.62	41.48	71						
72	59.69	40.26	59.51	40.52	59.34	40.78	59.16	41.04	58.98	41.30	58.80	41.55	58.62	41.81	58.43	42.07	72						
73	60.52	40.82	60.34	41.08	60.16	41.33	59.98	41.61	59.80	41.87	59.61	42.13	59.43	42.39	59.25	42.65	73						
74	61.35	41.38	61.17	41.65	60.98	41.91	60.80	42.18	60.62	42.45	60.43	42.71	60.24	42.97	60.06	43.23	74						
75	62.18	41.94	62.00	42.21	61.81	42.48	61.63	42.75	61.44	43.02	61.25	43.29	61.06	43.55	60.87	43.82	75						
76	63.01	42.50	62.83	42.77	62.65	43.04	62.45	43.32	62.25	43.59	62.06	43.86	61.87	44.13	61.68	44.40	76						
77	63.84	43.06	63.65	43.37	63.46	43.61	63.27	43.89	63.07	44.17	62.88	44.44	62.69	44.72	62.49	44.99	77						
78	64.67	43.62	64.48	43.90	64.28	44.18	64.09	44.46	63.89	44.74	63.70	45.02	63.50	45.30	63.31	45.57	78						
79	65.50	44.17	65.30	44.46	65.10	44.74	64.91	45.03	64.71	45.32	64.52	45.59	64.33	45.88	64.12	46.15	79						
80	66.33	44.73	66.13	45.02	65.93	45.31	65.73	45.60	65.53	45.89	65.35	46.17	65.16	46.46	64.93	46.74	80						
81	67.15	45.29	66.95	45.59	66.75	45.88	66.56	46.17	66.36	46.46	66.15	46.74	65.94	47.04	65.74	47.32	81						
82	67.98	45.85	67.78	46.15	67.58	46.44	67.38	46.74	67.17	47.03	66.96	47.32	66.76	47.62	66.55	47.91	82						
83	68.81	46.41	68.61	46.71	68.40	47.01	68.20	47.31	67.99	47.61	67.78	47.90	67.57	48.20	67.36	48.49	83						
84	69.64	46.97	69.43	47.28	69.23	47.58	69.02	47.88	68.81	48.18	68.60	48.47	68.39	48.78	68.17	49.08	84						
85	70.47	47.53	70.26	47.84	70.05	48.19	69.84	48.45	69.63	48.76	69.42	49.05	69.20	49.36	68.99	49.66	85						
86	71.30	48.09	71.09	48.40	70.87	48.71	70.66	49.02	70.45	49.33	70.23	49.63	70.01	49.94	69.80	50.24	86						
87	72.13	48.65	71.92	48.96	71.70	49.27	71.49	49.59	71.26	49.90	71.03	50.20	70.83	50.52	70.61	50.83	87						
88	72.96	49.21	72.74	49.53	72.52	49.84	72.31	50.16	72.08	50.48	71.87	50.78	71.64	51.10	71.42	51.41	88						
89	73.79	49.77	73.57	50.09	73.35	50.41	73.13	50.73	72.90	51.05	72.68	51.36	72.46	51.68	72.23	52.00	89						
90	74.61	50.33	74.39	50.65	74.17	50.98	73.95	51.30	73.72	51.62	73.50	51.94	73.27	52.26	73.04	52.58	90						
91	75.44	50.89	75.22	51.21	75.00	51.54	74.77	51.87	74.54	52.20	74.31	52.52	74.08	52.84	73.85	53.17	91						
92	76.27	51.44	76.05	51.78	75.82	52.11	75.59	52.44	75.36	52.77	75.13	53.10	74.90	53.42	74.67	53.75	92						
93	77.10	52.00	76.87	52.34	76.64	52.67	76.41	53.01	76.18	53.34	75.95	53.67	75.71	54.01	75.48	54.34	93						
94	77.93	52.56	77.70	52.90	77.47	53.24	77.24	53.58	77.00	53.92	76.77	54.25	76.53	54.59	76.29	54.92	94						
95	78.76	53.12	78.53	53.47	78.29	53.81	78.06	54.15	77.82	54.49	77.58	54.83	77.34	55.17	77.10	55.50	95						
96	79.59	53.68	79.35	54.03	79.12	54.37	78.88	54.72	78.64	55.07	78.40	55.41	78.15	55.75	77.91	56.09	96						
97	80.42	54.24	80.18	54.59	79.94	54.94	79.70	55.29	79.46	55.64	79.22	55.98	78.97	56.33	78.72	56.67	97						
98	81.25	54.80	81.01	55.16	80.76	55.51	80.52	55.86	80.28	56.21	80.03	56.56	79.78	56.91	79.53	57.26	98						
99	82.07	55.36	81.84	55.72	81.59	56.07	81.35	56.43	81.10	56.79	80.85	57.14	80.60	57.49	80.35	57.84	99						
100	82.90	55.92	82.66	56.28	82.41	56.64	82.16	57.00	81.91	57.36	81.66	57.71	81.41	58.07	81.16	58.43	100						
101	83.73	56.48	83.49	56.84	83.24	57.21	82.99	57.57	82.73	57.93	82.48	58.29	82.23	58.65	81.97	59.01	101						
102	84.56	57.03	84.31	57.41	84.06	57.77	83.81	58.14	83.55	58.49	83.30	58.87	83.04	59.23	82.78	59.59	102						
103	85.39	57.59	85.14	57.97	84.88	58.34	84.63	58.71	84.37	59.06	84.11	59.45	83.85	59.83	83.59	60.18	103						
104	86.22	58.14	85.97	58.53	85.71	58.91	85.45	59.28	85.19	59.63	84.93	60.02	84.67	60.39	84.41	60.76	104						
105	87.05	58.70	86.79	59.09	86.53	59.47	86.27	59.85	86.01	60.21	85.75	60.60	85.48	60.97	85.21	61.35	105						
106	87.88	59.26	87.62	59.66	87.36	60.04	87.10	60.42	86.83	60.78	86.57	61.18	86.30	61.56	86.03	61.93	106						
107	88.71	59.82	88.45	60.22	88.18	60.60	87.92	60.99	87.65	61.36	87.38	61.75	87.11	62.14	86.84	62.51	107						
108	89.54	60.38	89.27	60.78	89.00	61.17	88.74	61.56	88.47	61.93	88.20	62.33	87.92	62.72	87.65	63.10	108						
109	90.37	60.94	90.10	61.35	89.83	61.74	89.56	62.13	89.29	62.50	89.02	62.91	88.74	63.30	88.47	63.68	109						
110	91.19	61.51	90.92	61.91	90.65	62.30	90.38	62.70	90.11	63.09	89.83	63.49	89.55	63.88	89.27	64.27	110						
111	92.02	62.07	91.75	62.47	91.48	62.87	91.20	63.27	90.93	63.66	90.65	64.06	90.37	64.46	90.08	64.85	111						
112	92.85	62.63	92.58	63.03	92.30	63.44	92.03	63.84	91.74	64.24	91.46	64.64	91.18	65.04	90.90	65.44	112						
113	93.68	63.19	93.41	63.60	93.13	64.00	92.85	64.41	92.56	64.82	92.28	65.22	91.99	65.62	91.71	66.02	113						
114	94.51	63.75	94.23	64.16	93.95	64.57	93.67	64.98	93.38	65.39	93.10	65.79	92.81	66.20	92.52	66.60	114						
115	95.34	64.31	95.06	64.72	94.77	65.15	94.49	65.55	94.20	65.96	93.91	66.37	93.62	66.78	93.33	67.19	115						
116	96.17	64.86	95.89	65.29	95.60	65.71	95.31	66.12	95.02	66.54	94.73	66.95	94.44	67.36	94.15	67.77	116						
117	96.99	65.42	96.71	65.85	96.42	66.28	96.14	66.69	95.84	67.11	95.55	67.53	95.25	67.94	94.96	68.36	117						
118	97.82	65.98	97.54	66.41	97.25	66.84	96.96	67.32	96.66	67.69	96.37	68.10	96.08	68.52	95.77	68.94	118						
119	98.65	66.54	98.37	66.97	98.07	67.41	97.78	67.83	97.48	68.26	97.18	68.68	96.88	69.11	96.58	69.52	119						
120	99.48	67.10	99.19	67.54	98.90	67.97	98.60	68.40	98.30	68.83	98.00	69.26	97.69	69.68	97.39	70.11	120						
												54 Degrees											
0 Min.			15 Min.			30 Min.			45 Min.			0 Min.			15 Min.			30 Min.			45 Min.		
Distance			Dep.			Lat.			Dep.			Lat.			Distance			Dep.			Lat.		
Distance			Dep.			Lat.			Dep.			Lat.			Distance			Dep.			Lat.		
Distance			Dep.			Lat.			Dep.			Lat.			Distance			Dep.			Lat.		
Distance			Dep.			Lat.			Dep.			Lat.			Distance			Dep.			Lat.		
Distance			Dep.			Lat.																	

TRAVERSE TABLE. 67.

36 Degrees			37 Degrees			52 Degrees			53 Degrees			54 Deg.		
Distance		15 Min.		30 Min.		45 Min.		15 Min.		30 Min.		45 Min.		15 Min.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
0	0.81	0.59	0.81	0.59	0.81	0.59	0.81	0.59	0.81	0.59	0.81	0.59	0.81	0.59
1	1.62	1.18	1.61	1.17	1.60	1.16	1.59	1.21	1.58	1.20	1.57	1.21	1.56	1.20
2	2.43	1.76	2.42	1.77	2.40	1.79	2.39	1.80	2.38	1.81	2.37	1.82	2.36	1.83
3	3.24	2.35	3.23	2.36	3.20	2.39	3.19	2.40	3.18	2.41	3.17	2.42	3.16	2.43
4	4.04	2.94	4.03	2.96	4.01	2.99	4.00	3.00	3.99	3.01	3.98	3.02	3.97	3.03
5	4.85	3.53	4.84	3.55	4.82	3.57	4.81	3.58	4.80	3.59	4.79	3.60	4.78	3.61
6	5.66	4.11	5.64	4.14	5.63	4.16	5.61	4.19	5.59	4.21	5.57	4.24	5.55	4.26
7	6.47	4.70	6.45	4.73	6.43	4.76	6.41	4.79	6.39	4.81	6.37	4.84	6.35	4.87
8	7.28	5.29	7.26	5.32	7.23	5.35	7.21	5.38	7.19	5.42	7.16	5.45	7.14	5.48
9	8.09	5.88	8.06	5.91	8.04	5.95	8.01	5.98	7.99	6.02	7.96	6.05	7.93	6.09
10	8.90	6.46	8.87	6.50	8.84	6.54	8.81	6.58	8.78	6.62	8.76	6.66	8.73	6.70
11	9.71	7.05	9.68	7.10	9.65	7.14	9.61	7.18	9.58	7.22	9.55	7.26	9.52	7.30
12	10.52	7.64	10.48	7.69	10.45	7.73	10.43	7.78	10.38	7.82	10.35	7.87	10.31	7.91
13	11.33	8.23	11.29	8.28	11.25	8.33	11.21	8.37	11.18	8.43	11.14	8.47	11.11	8.52
14	12.14	8.82	12.09	8.87	12.06	8.93	12.01	8.96	11.98	9.03	11.94	9.08	11.90	9.13
15	12.94	9.40	12.90	9.46	12.86	9.52	12.82	9.56	12.77	9.63	12.74	9.68	12.69	9.74
16	13.75	9.99	13.71	10.06	13.67	10.11	13.62	10.16	13.57	10.23	13.53	10.29	13.48	10.35
17	14.56	10.58	14.51	10.64	14.47	10.71	14.42	10.76	14.37	10.83	14.33	10.89	14.28	10.96
18	15.37	11.17	15.32	11.23	15.27	11.30	15.22	11.36	15.17	11.44	15.12	11.50	15.07	11.55
19	16.18	11.76	16.13	11.83	16.08	11.90	16.02	11.97	15.97	12.04	15.92	12.11	15.87	12.17
20	16.99	12.34	16.93	12.42	16.88	12.49	16.83	12.56	16.77	12.64	16.72	12.71	16.66	12.78
21	17.80	12.93	17.74	13.01	17.68	13.09	17.63	13.16	17.57	13.24	17.51	13.31	17.45	13.39
22	18.61	13.52	18.55	13.60	18.49	13.68	18.43	13.76	18.37	13.84	18.31	13.92	18.25	14.00
23	19.42	14.11	19.35	14.19	19.29	14.28	19.23	14.36	19.17	14.44	19.10	14.52	19.04	14.52
24	20.23	14.70	20.16	14.78	20.10	14.87	20.03	14.96	19.97	15.05	19.90	15.13	19.83	15.22
25	21.03	15.28	20.96	15.37	20.89	15.47	20.83	15.55	20.77	15.65	20.70	15.74	20.62	15.83
26	21.84	15.87	21.77	15.96	21.69	16.06	21.63	16.15	21.57	16.25	21.49	16.34	21.42	16.44
27	22.65	16.46	22.58	16.55	22.50	16.66	22.43	16.75	22.36	16.85	22.29	16.95	22.21	17.05
28	23.46	17.05	23.38	17.14	23.31	17.25	23.23	17.35	23.16	17.45	23.08	17.55	23.00	17.66
29	24.27	17.63	24.19	17.74	24.12	17.84	24.04	17.95	23.96	18.05	23.88	18.16	23.79	18.26
30	25.08	18.22	25.00	18.33	24.92	18.44	24.84	18.55	24.76	18.66	24.68	18.76	24.59	18.87
31	25.89	18.81	25.80	18.92	25.72	19.03	25.64	19.15	25.56	19.26	25.47	19.37	25.39	19.48
32	26.70	19.40	26.61	19.51	26.53	19.63	26.44	19.74	26.36	19.86	26.27	19.97	26.18	20.09
33	27.51	19.99	27.42	20.10	27.33	20.22	27.24	20.34	27.15	20.46	27.06	20.58	26.97	20.70
34	28.32	20.57	28.22	20.69	28.14	20.82	28.04	20.94	27.95	21.06	27.86	21.18	27.77	21.31
35	29.12	21.16	29.03	21.28	28.94	21.41	28.84	21.54	28.75	21.67	28.66	21.79	28.56	21.92
36	29.93	21.75	29.83	21.88	29.74	22.01	29.64	22.14	29.55	22.27	29.45	22.39	29.35	22.53
37	30.74	22.34	30.64	22.47	30.55	22.60	30.45	22.73	30.35	22.87	30.25	23.00	30.14	23.14
38	31.55	22.93	31.45	23.06	31.35	23.20	31.25	23.33	31.15	23.47	31.05	23.60	30.94	23.75
39	32.36	23.51	32.26	23.65	32.15	23.79	32.05	23.93	31.95	24.07	31.84	24.21	31.73	24.35
40	33.17	24.10	33.06	24.24	32.96	24.39	32.85	24.53	32.74	24.67	32.64	24.82	32.53	24.96
41	33.98	24.69	33.87	24.83	33.76	24.98	33.65	25.13	33.54	25.28	33.43	25.42	33.32	25.57
42	34.79	25.28	34.68	25.42	34.57	25.58	34.45	25.73	34.34	25.88	34.23	26.03	34.11	26.18
43	35.60	25.86	35.48	26.02	35.37	26.17	35.25	26.32	35.14	26.48	35.02	26.63	34.91	26.79
44	36.41	26.45	36.29	26.61	36.17	26.77	36.05	26.92	35.94	27.08	35.82	27.24	35.70	27.39
45	37.22	27.04	37.09	27.20	36.98	27.36	36.86	27.52	36.74	27.68	36.62	27.84	36.49	28.00
46	38.03	27.63	37.90	27.79	37.78	27.96	37.66	28.12	37.54	28.29	37.41	28.45	37.28	28.61
47	38.83	28.22	38.71	28.38	38.59	28.55	38.46	28.72	38.34	28.89	38.21	29.05	38.08	29.22
48	39.64	28.80	39.51	28.97	39.39	29.15	39.26	29.31	39.14	29.49	39.00	29.66	38.87	29.83
49	40.45	29.39	40.32	29.57	40.19	29.74	40.06	29.91	39.93	30.09	39.80	30.26	39.67	30.44
50	41.26	30.08	41.13	30.16	41.00	30.34	40.86	30.51	40.73	30.69	40.60	30.87	40.46	31.05
51	42.07	30.67	41.93	30.75	41.80	30.93	41.66	31.14	41.52	31.30	41.39	31.47	41.25	31.66
52	42.87	31.25	42.74	31.34	42.60	31.53	42.47	31.71	42.32	31.90	42.19	32.08	42.05	32.26
53	43.68	31.84	43.55	31.93	43.41	32.12	43.27	32.31	43.12	32.50	42.98	32.68	42.84	32.87
54	44.49	32.43	44.35	32.52	44.21	32.72	44.07	32.91	43.92	33.10	43.78	33.29	43.63	33.48
55	45.30	33.01	45.16	33.11	45.02	33.31	44.87	33.51	44.72	33.70	44.58	33.89	44.43	34.09
56	46.11	33.60	45.96	33.70	45.82	33.91	45.67	34.11	45.51	34.30	45.37	34.50	45.22	34.70
57	46.92	34.19	46.77	34.29	46.62	34.50	46.47	34.70	46.31	34.91	46.17	35.10	46.01	35.31
58	47.73	34.78	47.58	34.88	47.43	35.10	47.27	35.30	47.11	35.51	46.96	35.71	46.80	35.92
59	48.54	35.37	48.39	35.48	48.23	35.69	48.07	35.90	47.92	36.11	47.76	36.32	47.60	36.53
60	49.35	36.00	49.20	36.11	49.04	36.30	48.88	36.51	48.71	36.71	48.54	36.92	48.37	37.12
Distance	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
		0 Min.		45 Min.		30 Min.		15 Min.		53 Degrees		45 Min.		30 Min.
		54 Deg.		53 Degrees		52 Degrees		45 Min.		30 Min.		15 Min.		Distance

TRAVERSE TABLE 67.

123

36 Degrees										37 Degrees										52 Degrees									
0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance		Distance	
61	49.35	35.85	49.19	36.07	49.04	36.28	48.88	36.50	48.72	36.71	48.56	36.92	48.39	37.13	48.23	37.34	48.10	37.55	48.00	37.74	47.87	38.00	47.74	38.27	47.61	38.54	47.48	38.81	
62	50.16	36.44	49.99	36.66	49.84	36.88	49.68	37.09	49.52	37.31	49.35	37.53	49.19	37.74	49.02	37.96	48.85	38.21	48.68	38.45	48.31	38.67	48.18	38.94	48.00	39.21	47.87	39.48	
63	50.97	37.03	50.80	37.25	50.64	37.47	50.48	37.69	50.31	37.91	50.15	38.13	49.98	38.35	49.81	38.57	49.64	38.82	48.69	39.09	48.46	39.36	48.23	39.63	48.00	39.90	47.77	40.17	
64	51.78	37.62	51.61	37.84	51.45	38.07	51.28	38.29	51.11	38.52	50.94	38.74	50.77	38.96	50.60	39.18	50.43	39.45	50.30	39.72	50.17	39.99	50.00	39.81	49.62	39.58	39.45	39.32	
65	52.59	38.21	52.42	38.43	52.25	38.66	52.08	38.89	51.91	39.12	51.74	39.34	51.57	39.57	51.40	39.79	51.23	40.11	51.08	40.43	50.90	40.68	50.77	40.55	50.42	40.29	50.16	40.03	
66	53.39	38.79	53.22	39.01	53.06	39.26	52.88	39.49	52.71	39.72	52.54	39.95	52.36	40.18	52.19	40.40	52.02	40.78	51.85	41.03	51.72	41.26	51.61	41.00	50.89	40.76	50.65	50.54	
67	54.20	39.38	54.03	39.62	53.86	39.85	53.68	40.08	53.51	40.32	53.33	40.55	53.15	40.79	52.98	41.06	52.81	41.42	52.69	41.69	52.57	41.88	52.45	41.74	52.62	41.51	52.40	52.29	
68	55.01	39.97	54.84	40.21	54.66	40.45	54.48	40.68	54.31	40.92	54.13	41.16	53.94	41.40	53.77	41.63	53.60	41.84	53.49	42.12	53.37	42.31	53.25	42.10	53.04	41.93	52.82	52.71	
69	55.82	40.56	55.65	40.80	55.47	41.04	55.28	41.28	55.11	41.53	54.92	41.76	54.74	42.00	54.56	42.24	54.39	42.47	54.26	42.65	54.14	42.84	54.02	42.71	53.60	42.49	53.38	53.27	
70	56.63	41.14	56.45	41.39	56.27	41.64	56.09	41.88	55.92	42.13	55.74	42.37	55.57	42.61	55.39	42.84	55.22	43.08	55.09	43.27	54.97	43.46	54.85	43.64	54.73	43.52	54.41	54.30	
71	57.44	41.73	57.26	41.98	57.07	42.23	56.89	42.48	56.70	42.73	56.52	42.97	56.35	43.22	56.17	43.47	55.99	43.72	55.80	43.91	55.62	44.10	55.44	44.29	55.27	44.46	55.10	54.99	
72	58.25	42.32	58.06	42.57	57.88	42.83	57.69	43.08	57.50	43.33	57.31	43.58	57.14	43.83	56.95	44.08	56.77	44.33	56.58	44.52	56.39	44.71	56.21	44.90	56.03	44.72	55.55	55.44	
73	59.06	42.91	58.87	43.16	58.68	43.42	58.49	43.68	58.30	43.94	58.11	44.18	57.93	44.44	57.74	44.69	57.56	44.94	57.37	45.13	57.18	45.32	57.00	45.49	56.81	45.29	56.12	56.01	
74	59.87	43.50	59.69	43.76	59.49	44.01	59.29	44.27	59.10	44.53	58.90	44.79	58.71	45.05	58.52	45.30	58.33	45.55	58.14	45.74	57.95	45.93	57.76	46.10	57.57	45.98	57.38	57.27	
75	60.68	44.08	60.48	44.35	60.29	44.61	60.09	44.87	59.90	45.14	59.70	45.39	59.51	45.66	59.32	45.91	59.13	46.16	58.94	46.35	58.75	46.54	58.56	46.73	58.37	46.58	58.19	58.08	
76	61.48	44.67	61.29	44.93	61.09	45.21	60.89	45.47	60.70	45.74	60.49	46.00	60.31	46.27	60.12	46.48	59.94	46.67	60.00	46.86	59.81	47.05	59.62	47.24	59.44	47.05	58.86	58.75	
77	62.29	45.26	62.09	45.53	61.90	45.80	61.69	46.07	61.50	46.34	61.29	46.60	61.11	46.88	60.88	47.14	60.69	47.33	60.48	47.52	60.27	47.71	60.06	47.90	59.85	47.69	59.53	59.42	
78	63.10	45.85	62.90	46.12	62.70	46.40	62.50	46.67	62.30	46.94	62.09	47.21	61.88	47.48	61.67	47.75	61.46	47.94	61.25	48.13	61.04	48.32	60.83	48.50	60.62	48.31	60.11	60.00	
79	63.91	46.44	63.70	46.71	63.51	46.99	63.30	47.26	63.10	47.54	62.88	47.81	62.67	48.09	62.46	48.36	62.25	48.54	62.03	48.73	61.82	48.92	61.61	49.10	60.90	48.89	60.69	60.58	
80	64.73	47.01	64.51	47.28	64.31	47.59	64.10	47.87	63.89	48.15	63.68	48.42	63.47	48.70	63.26	48.98	63.05	49.17	62.84	49.36	62.63	49.55	62.42	49.74	62.53	49.52	62.31	62.20	
81	65.53	47.61	65.32	47.90	65.11	48.18	64.90	48.46	64.69	48.75	64.48	49.03	64.26	49.31	64.05	49.59	63.84	49.78	63.57	49.97	63.36	50.16	63.15	50.34	63.04	50.23	63.02	62.91	
82	66.34	48.20	66.13	48.49	65.92	48.78	65.70	49.06	65.49	49.35	65.27	49.63	65.05	49.92	64.84	50.20	64.63	50.47	64.42	50.66	64.21	50.85	64.00	51.04	63.89	50.88	63.68	63.57	
83	67.15	48.79	66.94	49.08	66.72	49.37	66.50	49.66	66.29	49.95	66.07	50.24	65.85	50.53	65.63	50.81	65.42	51.09	65.18	51.38	64.97	51.67	64.66	51.96	64.55	51.84	64.34	64.23	
84	67.96	49.37	67.74	49.67	67.52	49.97	67.30	50.26	67.09	50.55	66.86	50.84	66.64	51.14	66.42	51.43	66.21	51.72	66.00	51.99	65.79	52.28	65.48	52.57	65.37	52.26	65.16	65.05	
85	68.77	49.96	68.54	50.26	68.33	50.56	68.10	50.86	67.89	51.16	67.66	51.45	67.43	51.75	67.21	52.04	66.99	52.32	66.68	52.61	66.40	52.89	66.19	53.18	66.08	52.97	66.86	66.75	
86	69.57	50.55	69.35	50.85	69.13	51.16	68.91	51.45	68.68	51.76	68.46	52.05	68.23	52.33	68.00	52.65	67.77	52.94	67.56	53.23	67.35	53.52	67.14	53.81	66.93	53.70	66.79	66.68	
87	70.38	51.14	70.16	51.44	69.94	51.73	69.71	52.05	69.48	52.36	69.25	52.66	69.02	52.96	68.79	53.26	68.56	53.54	68.33	53.82	68.11	54.11	67.90	54.40	67.69	54.29	67.58	67.47	
88	71.19	51.73	70.96	52.03	70.74	52.35	70.51	52.66	70.28	52.96	70.05	53.26	69.81	53.57	69.58	53.87	69.35	54.15	69.14	54.44	69.00	54.73	68.79	55.02	68.58	54.91	68.47	68.36	
89	72.00	52.32	71.77	52.62	71.54	52.95	71.31	53.26	71.08	53.56	70.84	53.87	70.60	54.18	70.37	54.49	70.15	54.78	69.94	55.07	69.73	55.36	69.50	55.65	69.34	55.94	69.20	69.09	
90	72.81	52.90	72.58	53.22	72.35	53.53	72.11	53.85	71.88	54.16	71.64	54.48	71.40	54.79	71.16	55.10	70.94	55.40	70.69	55.69	70.48	55.98	70.27	56.27	70.06	56.56	69.85	69.74	
91	73.61	53.49	73.39	53.81	73.15	54.13	72.91	54.45	72.68	54.77	72.44	55.08	72.19	55.40	71.95	55.71	71.70	56.00	71.49	56.29	71.28	56.58	71.07	56.87	70.86	57.16	70.65	70.54	
92	74.42	54.08	74.19	54.40	73.95	54.72	73.71	55.04	73.48	55.37	73.23	55.69	72.99	56.01	72.74	56.32	72.49	56.61	72.28	56.90	72.07	57.19	71.86	57.48	71.65	57.27	71.06	70.95	
93	75.23	54.66	74.99	54.99	74.76	55.32	74.52	55.64	74.27	55.97	74.03	56.29	73.78	56.61	73.53	56.94	73.28	57.17	73.07	57.46	72.86	57.75	72.64	58.04	72.43	58.33	72.22	72.11	
94	76.04	55.25	75.80	55.58	75.56	55.91	75.32	56.24	75.07	56.57	74.82	56.90	74.57	57.22	74.32	57.55	74.07	57.81	73.87	58.10	73.59	58.39	73.38	58.68	73.17	58.97	72.96	72.85	
95	76.85	55.84	76.61	56.17	76.37	56.51	76.12	56.84	75.87	57.17	75.62	57.50	75.37	57.83	75.12	58.16	74.87	58.11	74.66	58.34	74.45	58.54	74.20						

TRAVERSE TABLE. 67.

38 Degrees			39 Degrees			40 Degrees			45 Degrees			50 Degrees			51 Degrees			52 Deg.					
0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.		0 Min.		15 Min.		30 Min.		45 Min.	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0.78	0.62	0.79	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78	0.63	0.78
2	1.58	1.23	1.57	1.24	1.57	1.25	1.56	1.25	1.57	1.25	1.57	1.25	1.57	1.25	1.57	1.25	1.57	1.25	1.57	1.25	1.57	1.25	1.57
3	2.36	1.85	2.36	1.86	3.15	2.49	3.12	2.50	3.11	2.49	3.12	2.50	3.11	2.49	3.12	2.50	3.11	2.49	3.12	2.50	3.11	2.49	3.12
4	3.15	2.46	3.14	2.48	3.93	3.13	3.91	3.11	3.90	3.13	3.89	3.15	3.87	3.16	3.86	3.18	3.84	3.20	3.82	3.18	3.84	3.20	3.82
5	3.94	3.08	3.93	3.10	4.71	3.72	4.70	3.74	4.68	3.76	4.67	3.78	4.65	3.80	4.63	3.82	4.61	3.84	4.59	3.86	4.57	3.90	4.54
6	4.73	3.69	4.71	3.72	5.50	4.33	5.48	4.36	5.46	4.38	5.44	4.41	5.42	4.43	5.40	4.45	5.38	4.48	5.35	4.51	5.32	4.54	5.30
7	5.52	4.31	5.50	4.33	6.28	4.95	6.26	4.98	6.24	5.01	6.22	5.04	6.19	5.07	6.17	5.09	6.15	5.12	6.12	5.15	6.09	5.17	6.06
8	6.30	4.93	6.27	4.95	7.07	5.57	7.04	5.60	7.02	5.63	6.99	5.66	6.97	5.69	6.95	5.73	6.92	5.76	6.90	5.78	6.87	5.80	6.77
9	7.09	5.54	7.07	5.57	7.85	6.19	7.83	6.23	7.80	6.26	7.77	6.29	7.74	6.33	7.72	6.36	7.69	6.39	7.66	6.42	7.63	6.45	7.60
10	7.88	6.16	7.85	6.19	8.64	6.81	8.61	6.85	8.58	6.89	8.55	6.92	8.52	6.96	8.49	6.99	8.46	7.03	8.43	7.06	8.40	7.09	8.37
11	8.67	6.77	8.64	6.81	9.42	7.43	9.39	7.47	9.36	7.51	9.33	7.55	9.29	7.59	9.26	7.63	9.23	7.67	9.20	7.70	9.17	7.73	9.14
12	9.46	7.39	9.42	7.43	10.21	8.05	10.17	8.09	10.14	8.14	10.10	8.18	10.07	8.23	10.03	8.27	10.00	8.31	9.97	8.34	9.94	8.37	9.91
13	10.24	8.00	10.21	8.05	10.99	8.67	10.96	8.72	10.92	8.76	10.88	8.81	10.84	8.87	10.80	8.91	10.76	8.95	10.72	8.98	10.69	8.99	10.66
14	11.03	8.62	10.99	8.67	11.78	9.29	11.74	9.34	11.70	9.39	11.65	9.44	11.62	9.49	11.58	9.54	11.53	9.59	11.49	9.62	11.46	9.65	11.43
15	11.82	9.24	11.78	9.29	12.56	9.91	12.52	9.96	12.48	10.02	12.43	10.07	12.39	10.13	12.33	10.18	12.30	10.23	12.26	10.29	12.22	10.32	12.19
16	12.62	9.85	12.56	9.91	13.31	10.59	13.26	10.64	13.21	10.70	13.16	10.76	13.12	10.81	13.07	10.87	13.02	10.92	12.95	10.98	12.90	11.01	12.87
17	13.40	10.47	13.35	10.52	14.09	11.21	14.04	11.27	13.99	11.33	13.94	11.39	13.89	11.45	13.84	11.51	13.78	11.57	13.72	11.60	13.65	11.63	13.59
18	14.18	11.08	14.13	11.14	14.87	11.83	14.82	11.89	14.77	11.96	14.71	12.02	14.66	12.09	14.61	12.15	14.56	12.22	14.49	12.27	14.43	12.34	14.39
19	14.97	11.70	14.92	11.76	15.61	12.45	15.56	12.52	15.51	12.59	15.46	12.65	15.43	12.72	15.38	12.79	15.33	12.86	15.29	15.38	15.22	15.47	15.24
20	15.76	12.31	15.71	12.38	16.44	13.07	16.38	13.14	16.32	13.22	16.26	13.29	16.20	13.36	16.15	13.43	16.10	13.50	16.05	13.57	15.99	13.64	15.94
21	16.55	12.93	16.49	13.09	17.22	13.70	17.16	13.77	17.10	13.85	17.04	13.92	16.98	13.99	16.92	14.07	16.87	14.14	16.82	14.21	16.75	14.28	16.70
22	17.34	13.55	17.27	13.62	18.06	14.32	18.00	14.39	17.94	14.47	17.87	14.55	17.75	14.63	17.68	14.71	17.62	14.78	17.57	14.85	17.52	14.92	17.47
23	18.12	14.16	18.06	14.24	18.85	14.94	18.78	15.02	18.72	15.10	18.65	15.18	18.58	15.26	18.51	15.35	18.45	15.43	18.38	15.51	18.32	15.59	18.29
24	18.91	14.78	18.85	14.86	19.65	15.57	19.58	15.65	19.52	15.73	19.45	15.82	19.39	15.90	19.32	16.00	19.25	16.07	19.18	16.15	19.10	16.23	19.07
25	19.70	15.39	19.63	15.46	20.41	16.19	20.34	16.27	20.28	16.35	20.21	16.43	20.13	16.55	20.06	16.63	19.99	16.70	19.92	16.77	19.84	16.84	19.79
26	20.49	16.01	20.41	16.08	21.20	16.81	21.13	16.89	21.06	16.99	20.98	17.08	20.88	17.17	20.78	17.27	20.68	17.37	20.58	17.47	20.48	17.57	20.39
27	21.28	16.63	21.20	16.70	21.99	17.43	21.92	17.51	21.84	17.62	21.76	17.72	21.66	17.85	21.58	17.94	21.51	18.03	21.44	18.11	21.35	18.18	21.30
28	22.06	17.24	22.00	17.31	22.70	18.06	22.62	18.15	22.54	18.25	22.46	18.35	22.38	18.44	22.29	18.54	22.20	18.63	22.11	18.72	22.02	18.83	21.93
29	22.85	17.86	22.77	17.93	23.46	18.68	23.39	18.78	23.32	18.88	23.23	18.98	23.15	19.08	23.07	19.18	22.98	19.29	22.90	19.40	22.81	19.51	22.72
30	23.64	18.47	23.56	18.57	24.16	19.30	24.10	19.40	24.03	19.51	23.96	19.61	23.93	19.72	23.83	19.82	23.73	19.93	23.64	20.04	23.55	20.15	23.46
31	24.43	19.09	24.35	19.19	24.86	20.04	24.78	20.14	24.70	20.25	24.62	20.35	24.54	20.47	24.46	20.60	24.37	20.73	24.28	20.84	24.19	20.95	24.06
32	25.22	19.70	25.13	19.81	25.57	20.68	25.49	20.78	25.40	20.89	25.31	21.01	25.22	21.12	25.13	21.23	25.04	21.34	24.95	21.45	24.86	21.56	24.77
33	26.00	20.32	25.92	20.43	26.31	21.32	26.22	21.41	26.13	21.52	26.04	21.63	25.95	21.73	25.86	21.84	25.77	21.95	25.68	22.06	25.59	22.17	25.49
34	26.79	20.93	26.70	21.05	27.02	21.94	26.92	22.03	26.83	22.14	26.74	22.25	26.65	22.36	26.56	22.47	26.47	22.58	26.38	22.69	26.29	22.80	26.20
35	27.58	21.55	27.49	21.67	27.73	22.56	27.64	22.67	27.55	22.78	27.46	22.91	27.37	23.02	27.28	23.17	27.18	23.28	27.09	23.39	27.00	23.50	26.81
36	28.37	22.17	28.28	22.29	28.42	23.18	28.33	23.31	28.24	23.42	28.15	23.53	28.06	23.64	27.97	23.75	27.88	23.84	27.79	23.96	27.70	24.00	27.61
37	29.16	22.78	29.06	22.91	29.16	23.79	29.01	23.97	28.86	24.04	28.77	24.13	28.68	24.22	28.59	24.31	28.50	24.40	28.40	24.49	28.31	24.58	28.22
38	29.94	23.40	29.84	23.53	29.74	24.36	29.64	24.45	29.55	24.54	29.46	24.63	29.37	24.72	29.28	24.81	29.19	24.90	29.10	25.00	29.01	25.11	28.92
39	30.73	24.01	30.63	24.14	30.53	24.98	30.42	25.07	30.33	25.17	30.24	25.26	30.15	25.37	30.06	25.48	29.97	25.59	29.88	25.70	29.79	25.81	29.70
40	31.52	24.63	31.41	24.76	31.33	25.60	31.20	25.74	31.09	25.85	30.98	25.96	30.89	26.07	30.80	26.18	30.71	26.29	30.62	26.40	30.53	26.51	30.44
41	32.31	25.24	32.20	25.38	32.09	26.23	31.98	26.37	31.87	26.50	31.75	26.63	31.64	26.76	31.55	26.88	31.46	26.99	31.37	27.10	31.28	27.21	31.19
42	33.10	25.86	32.98	26.00	32.87	26.85	32.76	26.99	32.64	27.03	32.52	27.16	32.41	27.29	32.30	27.42	32.21	27.55	32.12	27.68	32.03	27.79	31.90
43	33.88	26.47	33.77	26.62	33.65	27.47	33.54	27.61	33.42	27.76	33.30	27.90	33.18	28.04	33.06	28.17	32.95	28.30	32.86	28.41	32.77	28.52	32.68
44	34.67	27.09	34.55	27.24	3																		

TRAVERSE TABLE. 67.

40 Degrees			41 Degrees			42 Degrees			43 Degrees			44 Degrees			45 Degrees			46 Degrees			47 Degrees			48 Degrees			49 Degrees			50 Degrees																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
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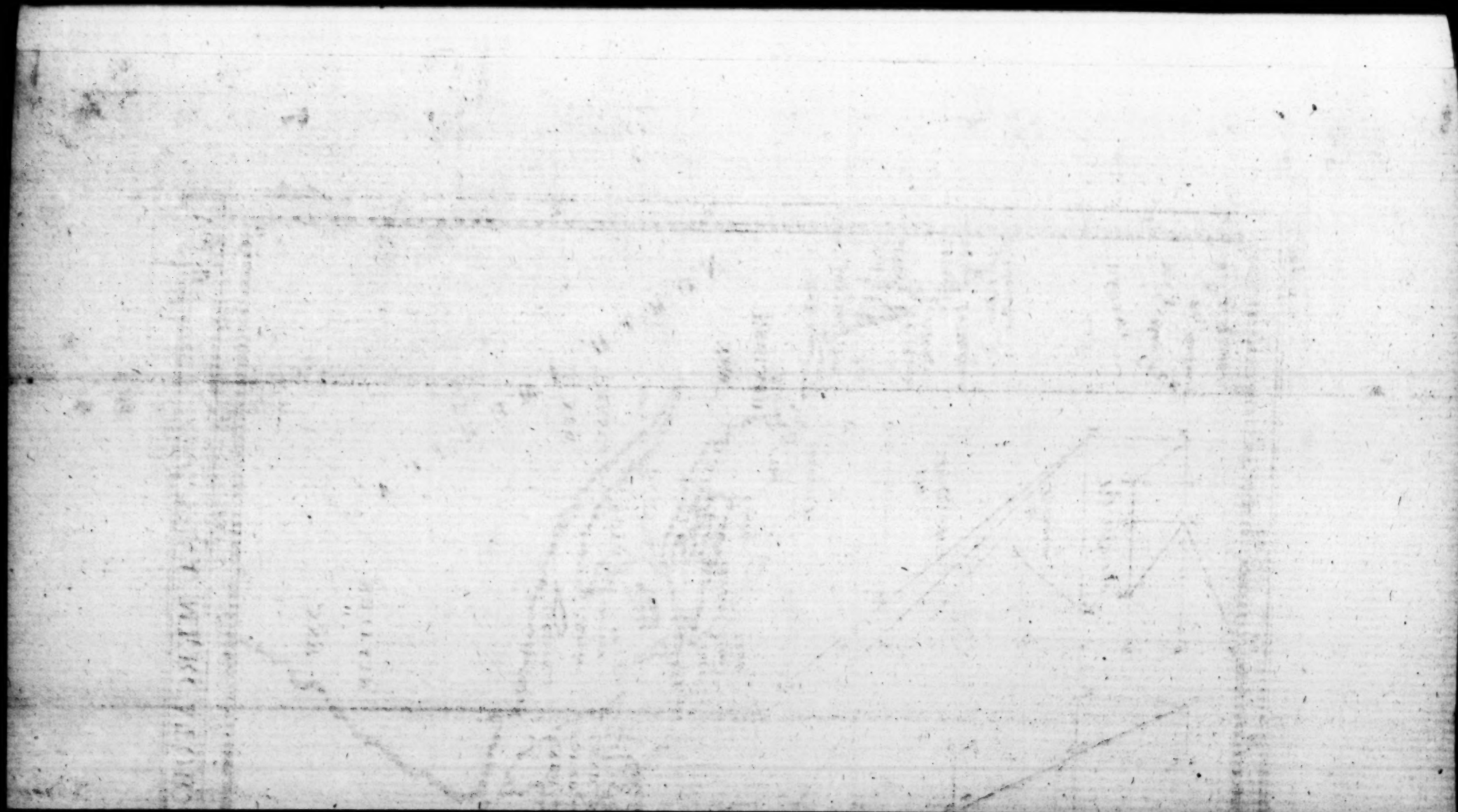
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THE
ELEMENTS
OF
NAVIGATION.

BOOK VIII.
OF GLOBULAR SAILING.

SECTION I.
Definitions and Principles.

I. BY Globular sailing, is meant the manner of estimating either the difference of latitude, difference of longitude, departure, course or distance, from any two of them being known, upon principles deduced from the spherical figure of the Earth.

In the preceding parts of sailing, the operations were performed as if the courses and distances of ships had been run on a plane, or flat surface: But that this cannot be true, appears from what was said in Book VI, Art. 2. To which, as farther proofs, there may be added; 1st. that mariners, as they sail from northern or southern latitudes towards the equator, observe the stars about the poles constantly to diminish their altitude or height above the apparent horizon: 2^d. It is also well known, that in eclipses of the moon, the darkened part is bounded by a circular curve; and consequently, the body which casts the shade, or obstructs the light, must be bounded by a like curve: But as this obscuration is caused by the Earth, consequently its surface must be limited by a circular figure, that is, it must be globular.

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2. As meridians are circles on the terraqueous globe, meeting in the poles; it is obvious, that any two of these circles must recede more from one another at greater distances from the poles; and at equal distances from both poles, or at the equator, the distance between the meridians is the greatest.

The true place of a ship at sea, depends both upon its distance from the equator, and from some noted meridian; and because the meridional distance, that is, the distance between two meridians, varies in every latitude; therefore it is convenient this distance should be reckoned in a fixed latitude, and where the degrees are of the same magnitude with those of the meridian; which can be no where but at the equator, where 60 nautical miles make a degree.

3. Now a ship sailing from any latitude upon an oblique rhumb, until she has made a known departure, will, by the rules of plane sailing, reckon herself under the same meridian, whether she sails to the northward or southward: But from what has been said, this cannot be true; for the same meridional distance, which in this case is taken as departure, will not serve two different latitudes on the same side of the equator: Therefore the common methods of plane sailing alone, are not sufficient to shew the ship's true place, a necessary correction being wanted with respect to her longitude.

The incorrect method of plane sailing was probably founded on the manner in which the sea-charts were at first constructed: For it being the most convenient way to represent the meridians, parallels of latitude, and rhumb lines in such charts by right lines, where the meridians should be parallel, and the east and west lines cut them at right angles: Consequently the distances of the meridian in every parallel were equal to the distances at the equator; and a degree on any parallel of latitude, was of the same size as a degree on the equator; whereby each degree of longitude in those parallels was increased beyond its just size, in proportion as the equator, or its radius, is greater than the parallel, or its radius.

4. Now to remove these material objections against the sea-charts, a different method of construction should be used; and it had been hinted, some hundred years since, by several persons, that the degrees of latitude in such charts should be lengthened as they approach the poles; and accordingly, Mr. *Gerrard Mercator*, a Flemming, published in the year 1556 the first chart of this kind, wherein the degrees of latitude were gradually lengthened from the equator towards the poles; but in what manner, or how the chart was constructed, he did not shew; neither were those degrees increased in the true ratio.

5. About the year 1590 Mr. *Edward Wright*, an Englishman, discovered the true principles upon which such a chart should be constructed; which was to increase the degrees of latitude as they approached the poles, in the same proportion as the degrees of longitude between any two meridians decreased on the globes: But it was not till the year 1599 that

that he published his errors in Navigation; wherein he shews the principles of the true sea-chart, with the method of constructing it: Nevertheless, the charts made on this construction are called Mercator charts.

A true method of computing a ship's place at sea does not appear to have been known till Mr. Wright published the said book; therefore although the chart may properly enough be called *Mercator's*, yet the method of sailing called *Mercator's*, should, in justice, be named from Mr. *Wright*.

6. The difficulty in constructing a true sea-chart, seems to have consisted in the finding a proper manner how to apply the surface of a globe to a plane, which Mr. *Wright* happily accomplished by a most ingenious conception; the substance whereof is as follows.

1st. Suppose a rectangular plane was rolled about the globe until the edges of the plane met, and formed a kind of concave cylinder inclosing the globe, and touching its equator.

2d. Conceive the surface of this globe to swell (like a bladder while it is blowing up), from the equator towards the poles, proportionally in latitude as it does in longitude, until every part of its surface meets that of the concave cylinder, and thereon imprints the lines that were drawn on the globular surface.

3d. Then the cylinder, or rectangular plane, being unrolled, it will represent a sea-chart, whose parts bear the same proportion to one another, as do the corresponding parts on the globe: But herein all the lines will be right lines.

For, in this formation of the nautical chart, every parallel of latitude on the globe will be increased till it is equal to the equator; and so the distance of the meridians in those parallels will become equal to their distance at the equator; consequently the meridians on the chart are expressed by parallel right lines.

Also, the meridians being lengthened as the parallels increased, every degree of latitude is lengthened in the same proportion as the degrees of longitude are increased; therefore the distances of the parallels of latitude become wider and wider as they approach the poles: But these parallels are also right lines.

Again. As the rhumb lines on the globe cut the meridians at equal angles, they will also on the chart cut the meridians at equal angles, and consequently be expressed by right lines; since none but right lines can cut several parallel right lines, at equal angles.

The construction of such a chart, Mr. Wright deduced from the following principles.

SECTION II.

Of the Reckoning of Longitude.

1. PROPOSITION I.

*The distance between any two meridians at the equator,
 is in proportion to their distance in any parallel of latitude;
 As the Radius,
 To the co-sine of that latitude.*

DEMONSTRATION. See Fig. 1. Pl. XIII.

Let P and E represent the fourth part of a sphere; E being the center, P the pole, ED the radius of the equator, AB the radius of a parallel of latitude: Then the arcs PBD , PCd , will each represent a quadrant of a meridian; Dd is an arc of the equator, and BC an arc of a parallel of latitude; also the arc BB expresses the latitude, and PE the complement of the latitude, whose right sine is BA . (III. 3)

Now (II. 197.) the circumference of a circle, whose radius is ED , is to the circumference of a circle whose radius is AB , as ED is to AB ; that is, as the radius of the equator, to the radius of the parallel; or, as radius to the co-sine of the latitude.

But like arcs, are in the same ratio of their circumferences. (II. 150, 197) And like arcs Dd , BC , are intercepted between the same two meridians; therefore

An arc of the equator, is to a like arc in any parallel of latitude;
 As the radius, to the co-sine of that latitude.

8. COROL. Hence it is easy to construct a table shewing, from the equator to the poles, the approach of two meridians in any latitude; or in what proportion the degrees of longitude diminish in every latitude.

9. PROPOSITION II.

Any part of a parallel of latitude, is to a like part of a meridian, as radius to the secant of the latitude to that parallel.

DEMONSTRATION. See Fig. 2. Pl. XIII.

Let PDE represent a quadrant of a meridian, where P is the pole, and DE the radius of the equator; AB is the radius of a parallel of latitude, or is the co-sine of the latitude whose sine is BE , and secant EC .

Then EF : EB :: ED : EC . (III. 34.)

Or $\cos. lat.$: $Rad.$:: $Rad.$: secant of the lat. in that parallel.

But $\cos. lat.$: $Rad.$:: part of a parallel : like part of the equator. (7)

Therefore, part of a parallel of latitude, is to a like part of the equator;
 As the Radius is to the secant of the lat. to that parallel.

Now like parts of the meridian and equator are equal, as great circles. Therefore any part of a parallel of lat. is to a like part of a meridian; as Radius, to the secant of the lat. to that parallel.

10. PROPOSITION III.

The distance of any parallel of latitude (A) from the equator, is expressed by the sum of the secants of all the arcs between the equator and that parallel.

DEMONSTRATION.

Now (9) As Radius, to the secant of the latitude A;
So is a diminished deg. of long. in lat. A, or a degree of that parallel, to a degree of the meridian.

But the degrees of latitude, or of the meridian, are to be lengthened in proportion as the degrees of longitude diminish. (5)

Therefore, As Radius, to the secant of the latitude A;

So is a natural deg. of the meridian, to a lengthened deg. in lat. A.

Here radius being as unity, and one natural deg. as unity also;

Therefore the length of a degree in any latitude, is as the secant of that latitude, or may be expressed by that secant.

Now the distance of any parallel from the equator, is the sum of all the successive arcs between the equator and that parallel.

Consequently the distance of that parallel, is expressed by the sum of the secants of all these arcs between the equator and that parallel of latitude.

11. Cor. Hence it appears, that by the addition of the secants of small arcs, the distances of the parallels of latitude from the equator are obtained.

And these several distances, which are called *meridional parts*, being disposed in a table corresponding to the degrees and minutes in a quadrant, form a table of meridional parts.

And this was the excellent discovery of the celebrated Mr. Wright, by which the art of Navigation received greater improvements than by whatever had been done before, the discovery of the mariner's compass excepted; and indeed, in this and other things, he so far perfected the art, that little else was wanted beside an accurate method to find the longitude at sea. And although there have been since his time many eminent men who have done excellent things towards the rendering of Navigation more perfect, yet if the real use of such things be strictly considered, it will, perhaps, be found that they amount to no more than other methods of performing the useful operations; except what has been done by Mr. Richard Norward and Dr. Halley.

12. Mr. Wright made his table for the division of the nautical meridian, or the table of meridional parts, as follows.

The meridional parts for 1 min. he made equal to the secant of 1 min.

The mer. parts to 2 m. equal to the sum of the sec. of 1 and 2 min.

The mer. parts to 3 m. equal to the sum of the sec. of 1, 2 and 3 m.

The mer. parts to 4 m. equal to the sum of the mer. parts of 3 min. and the sec. of 4 min.

And so on by a constant addition of the secants.

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Now although a table thus made is abundantly sufficient for all nautical uses, yet had the secants of smaller parts than minutes been taken, the table would have been more correct: And therefore Mr. Oughtred, Sir Jonas Moore, Dr. Wallis, Dr. Halley, and others, have been induced to find methods of constructing these tables with more accuracy than by the addition of the secants to every minute.

But a table of meridional parts constructed by the most accurate method, only shews that Mr. Wright's table does no where exceed the true meridian parts by half a minute, and this only near the pole; for in latitudes, as far as navigation is practicable, the difference is scarce sensible.

A table of meridional parts being obtained (see the table at the end of this book), the meridians and parallels of latitude for a sea chart, wherein the situation of places will bear the same proportion to one another as on the globe, may be constructed by the following.

13. PROPOSITION IV.

To construct a sea-chart which shall contain a given number of degrees of latitude and longitude.

Suppose from the latitude of 23 degrees N. to the lat. of 59 degrees N., and for to contain 80 degrees of west longitude from the meridian of London.

SOLUTION.

1st. Write the several degrees of latitude, included between the proposed limits, orderly under one another in a column, the greatest being uppermost.

2d. Then beginning at the lower one, against each degree write, in another column, the difference * between the meridional parts of the lesser latitude, and of that degree of latitude.

3d. Divide each of these meridional differences by 60, setting the quotients against their respective dividends, in another column; and these will be the meridional parts reduced to degrees of longitude, and will express the measures of the divisions for the meridional degrees.

If the meridional differences to parts of degrees, or minutes, were wanted, they are to be taken from the table of meridional parts in the same manner.

These three articles are exemplified in the following table; which, although made to serve from the equator to 72 degrees of latitude, yet it will serve for any intermediate limits, by subtracting the meridional degrees belonging to the latitude where the chart is to begin at, from those of all the latitudes the chart is to contain; of the several remainders make a table, and these will be the numbers to be transferred to the chart, as shewn in what follows.

* This is best done by writing, close to the upper edge of a bit of paper the mer. parts of the lesser lat., of the same size with the tabular figures; then this number applied to the mer. parts of the other degrees, the differences may be readily taken.

A TABLE for constructing a Mercator Chart.

Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.
72	6335	105,6	54	3865	64,4	36	2318	38,6	18	1098	18,3
71	6146	102,4	53	3764	62,7	35	2244	37,4	17	1035	17,2
70	5966	99,4	52	3665	61,1	34	2171	36,2	16	0973	16,2
69	5795	96,6	51	3569	59,5	33	2100	35,0	15	0910	15,2
68	5631	93,8	50	3475	57,9	32	2028	33,8	14	0848	14,1
67	5474	91,2	49	3382	56,4	31	1958	32,6	13	0787	13,1
66	5324	88,7	48	3292	54,9	30	1888	31,5	12	0725	12,1
65	5179	86,3	47	3203	53,4	29	1819	30,3	11	0664	11,1
64	5039	84,0	46	3116	51,9	28	1751	29,2	10	0603	10,1
63	4905	81,7	45	3030	50,5	27	1684	28,1	9	0542	9,0
62	4775	79,6	44	2946	49,1	26	1616	26,9	8	0482	8,0
61	4649	77,5	43	2863	47,7	25	1550	25,8	7	0421	7,0
60	4527	75,5	42	2782	46,4	24	1484	24,7	6	0361	6,0
59	4409	73,5	41	2702	45,0	23	1419	23,6	5	0300	5,0
58	4294	71,6	40	2623	43,7	22	1354	22,6	4	0240	4,0
57	4183	69,7	39	2545	42,4	21	1289	21,5	3	0180	3,0
56	4074	67,9	38	2468	41,1	20	1225	20,4	2	0120	2,0
55	3968	66,1	37	2393	39,9	19	1161	19,3	1	0060	1,0

4th. Draw a line AB (see Plate XII.) to represent a part of the parallel of latitude where the chart is to begin (which in this example is the parallel of 23°); and on this line, by the help of a scale of equal parts, lay a number of successive divisions to express the proposed number of degrees of longitude.

Thus 80 degrees of longitude being proposed; from some scale take one of the primary divisions, and this distance being repeated eight successive times in the line AB, will give divisions to every tenth degree of longitude, and then let the intermediate degrees be inserted, whereby AB will be divided into 80 equal parts of that scale; which, in this example, is the scale where eight subdivisions or degrees make an inch: And in like manner the parts of degrees may be inserted.

5th. From the ends A, B, draw lines AD, BC, perpendicular to AB; and on these lines lay the divisions expressed by the meridional degrees in the foregoing table, beginning at the line AB.

Thus did the chart begin at the equator, and go to 70 degrees of latitude; for 70 deg. take 99,4; for 69 deg. take 96,6; for 68 deg. take 93,8, &c.; and these divisions taken from the same scale, and laid severally on AD, BC, will give the divisions for the degrees of latitude increased in their true proportion; and the like for the intermediate parts of degrees.

But

15. PROPOSITION V.

If a ship sails on any oblique rhumb, the proper difference of latitude to the departure, the meridional difference of latitude to the difference of longitude, and the radius to the tangent of the course are all in the same ratio. See Fig. 1. Pl. XII.

DEMONSTRATION.

Suppose a ship from P in the latitude of 30 deg. N. should sail on the rhumb PS, to S in the latitude of 40 degrees north,

Now because she has made 10 deg. of diff. lat.; make PO, in the meridian, equal to 10 degrees of the equator; and draw OR parallel to QS, meeting the rhumb PS in R.

Then PO expresses the proper difference of latitude.
OR the departure.

PR the distance sailed in equatorial measure.
∠OPR is the angle of the course.

And R is the apparent place of the ship.

But she having made 10 deg. diff. lat. should be in the parallel of 40 degrees; therefore her place, when corrected by longitude, will be in S.

Then QS will be the difference of longitude.

And PQ the meridional difference of latitude.

Now the triangles POR, PQS, are similar,

Therefore PO : OR :: PQ : QS.

Also PO : OR :: Rad. : t.∠OPR.

(II. 167)
(III. 46)

Consequently. As the proper diff. lat., to the departure;

So the merid. diff. lat., to the diff. longitude.

And So is the Radius , to the tang. of the course.

16. PROPOSITION VI.

The tangents of the angles which different rhumbs make with the meridians, are directly proportional to the differences of longitude made on those rhumbs to equal meridional differences of latitude: Or, are reciprocally proportional to unequal meridional differences of latitudes on those rhumbs, when the differences of longitude are equal. See Fig. 1. Plate XII.

DEMONSTRATION.

Let QS, QV, be the differences of longitude made on the rhumbs PS, PV, to the common merid. diff. lat. PQ; also PW, PQ, be the meridional differences of latitudes, on the same rhumbs, to the equal differences of longitudes WX, QV.

Now PQ : QV :: R : t.∠QPV

And PQ : QS :: R : t.∠QPS

Consequently QV : QS :: t.∠QPV : t.∠QPS.

Again. Since R : t.∠QPV :: PQ : QV

And R : t.∠WPX :: PW : QV (= WX)

Then R x QV = PQ x t.∠QPV = PW x t.∠WPX,

Consequently PW : PQ :: t.∠QPV : t.∠WPX.

} (III. 46)

(II. 155)

} (III. 46)

(II. 162)

(II. 46, 163)

17. PRQ.

17.

PROPOSITION VII.

If a ship sails on several different rhumbs, the whole meridional diff. lat. is equal to the sum, or difference of all the successive meridional differences of lat.: And the whole diff. long. is also equal to the sum, or differences of all the successive differences of longitudes: Observing to take the sums, when of the same name; and the differences when of contrary names. See Fig. 2. Plate XII.

DEMONSTRATION.

1st. Suppose a ship sails from E in the lat. of 23° N. to G in the parallel of 26° deg. N. Then $eg = \text{mer. diff. lat.}$ and $gg = \text{diff. lon.}$
 2d. To sail from G to H in the parallel of 28° deg. Then $gh = \text{diff. lat.}$ and $hh = \text{diff. long.}$

Now $em = eg + gh$, is the whole diff. latitude.

And $mh = mb - hh$, is the whole diff. longitude.

3d. She sails west from H to I : Here she only alters her longitude by HI ; so that her present diff. long. is mi .

4th. In sailing from I to K , she increases her diff. long. by ik , from mi to gk ; and decreases her diff. lat. by ik , from em to eg .

5th. In sailing north from K to P , the diff. long. pp continues the same: But the diff. lat. is increased by kp , from eg to ep .

6th. In running east from P to M , the diff. lat. ep does not alter But the diff. longitude is diminished from pp to pm .

7th. By going from M to N ; the diff. lat. is increased by ml , from ep to eq : And the diff. long. is changed from pm , or ql to qn .

Consequently the whole diff. lat. eq is equal to the sums, or differences of all the differences of lat.; and the same of qn , the whole difference of longitude.

18.

PROPOSITION VIII.

If the course between any two places, outwards and homewards, be on the opposite rhumbs; then in each case, the differences of latitude, departures, and differences of longitude, will always be the same. See Fig. 3. Plate XII.

DEMONSTRATION.

Suppose a ship at a in lat. 25° deg. N. should sail on one rhumb to the latitude 40° deg. N. so as at the end of every third day to alter her diff. lat. five deg. or 300 miles; and to return, on the opposite rhumb, in the same manner.

Through the intersections a of this rhumb, with the parallel of every five degrees, draw meridians; make ad , af , each equal to five common degrees of the equator; and draw db , fe , parallel to the parallels of latitude.

Then are the lines ad , the proper differences of latitudes outwards to each third day's account, which must be every where equal; and also, the equal lines af , are the differences of latitudes homewards, to the like accounts.

The

The lines db outwards, and fe homewards, are the departures to each third day's account.

The lines ab outwards, and ae homewards, are the distances run to each third day's account.

The meridian lines ac outwards, and an homewards, are the meridional differences of latitude; and the west and east lines ca outwards and na homewards, taken on the parallels of latitude, are the differences of longitude.

Now it is evident from the construction, that the triangles adb , afe , in each account, are equal to one another.

Also, that the meridional differences of latitude, and the differences of longitude, to the accounts between the same parallels of latitude outwards and homewards, are respectively equal to one another.

Consequently the truth of the proposition is manifest.

PROPOSITION IX.

The divisions of the meridian line on the Mercator's chart, is a table of the differences of longitude answering to each minute of latitude on the rhumb line making an angle of 45 degrees with the meridian. See Fig. 1. Pl. XII.

DEMONSTRATION.

Let p be a rhumb of 45 degrees.

Then, since p is at right angles to qv ,

Therefore $\angle qp v = \angle qp p$, each being half a right angle.

Consequently $qv = qp$. (II. 104)

Now p is the sum of all the meridional differences of latitude between the parallels through p and q . (11)

And qv is the sum of all the differences of longitude between the meridians through q and v .

Therefore on a rhumb of 45 degrees, the sum of all the differences of longitude, is equal to the sum of all the meridional differences of latitude.

But the sum of all the meridional differences of latitude, is the same as the nautical meridian. (12)

Consequently. On a rhumb of 45 degrees the difference of longitude, or the sum of all the intermediate differences of longitude to every minute of latitude, is equal to the corresponding parts of the nautical meridian line.

20. In the Philosophical Transactions, N^o 219, Dr. Halley has given a very curious paper relating to the division of the nautical meridian, by a quite different method from Mr. Wright's; and because it contains a method of performing the problems of sailing according to the true chart, by the help of Briggs's, or the common table of logarithmic tangents, without a table of meridional parts; therefore the substance of that paper is here annexed.

The

The Doctor says: "It was first discovered by chance, and, as far as I can learn, first published by Mr. Henry Bond, as an addition to Norwood's epitomy of Navigation, about the year 1645, that the meridian line was analogous to a scale of logarithmic tangents of half the complements of the latitude. The difficulty to prove the truth of this proposition, seemed such to Mr. Mercator," (an ingenious mathematician then living, and) "the author of the Logarithmotechnia, that he proposed to wager a good sum of money, against who would fairly undertake it, that he should not demonstrate either, that it was true or false: And about that time Mr. John Collins holding a correspondence with most of the eminent mathematicians of the age, did excite them to this enquiry."

"The first that demonstrated the said analogy, was the excellent Mr. James Gregory, in his Exercitationes Geometricæ, published anno 1668, which he did, not without a long train of consequences and complication of proportions, whereby the evidence of the demonstration is in a great measure lost, and the reader wearied before he attains it:" The Doctor proceeds and says, "nor hath any one, that I know of, yet discovered the rule for computing independently the interval of the meridional parts answering to any two given latitudes."

The Doctor then, in a very elegant manner, gives the demonstration of the said analogy; and draws several useful conclusions: But, as himself confesses, he is too concise for learners; and the writers on this subject have been rather more, or at least as concise as the Doctor; therefore in order to set this affair in a clear light, and keeping to the Doctor's principles, it was thought convenient to dispose of the subject in the following manner: But the said proposition may be proved from very different principles than those used by the Doctor.

21.

PROPOSITION X.

On the globe, the rhumb lines oblique to the meridians are spirals, which continually approach the poles.

DEMONSTRATION.

In any place on the surface of the globe, the rhumb running north and south, coincides with the meridian of that place; the east and west rhumbs are perpendicular to the meridian, and the other rhumbs are oblique to it: But this obliquity is the same under every meridian; therefore all the rhumbs, except the north and south, cut the meridians at equal angles.

When right lines are parallel to each other, a right line will cut them at equal angles (II. 95): But not so when the lines are inclined to one another; therefore several inclining lines cannot be cut at equal angles, but by a curve line bending towards the place where those lines would meet.

Now the meridians on the globe, meeting in the poles, are inclined to one another; therefore the oblique rhumbs cutting them at equal angles, are curve lines continually approaching the poles.

Schol.

SCHOL. A rhumb line revolving round the pole, so as to make a very small difference of latitude at each revolution, may make a multitude of revolutions before it meets the pole: And while that multitude is undefined, it may be said that it will not meet the pole.

22. PROPOSITION XI.

The spiral rhumbs on the globe become proportional spirals in the stereographic projection on the plane of the equator. See Fig. 3. Plate XIII.

DEMONSTRATION.

Let CAEB represent part of a stereographic projection of the sphere on the plane of the equator, where AEB is part of the equator, c the pole, CA, CD, CE, &c. are meridians, and the spiral Adef one of the rhumbs.

Now in every stereographic projection, the lines therein intersecting each other form angles equal to the angles on the sphere which they represent. (452)

Therefore the projection of the rhumb line Adef cuts the radii of meridians AD, AE, &c. at equal angles.

But this is a property also of the proportional spiral. (II. 225)
Therefore the spiral rhumbs on the globe become proportional spirals in this projection.

23. COR. Hence the differences of longitude AD, AE, AF, &c. are logarithms of the ratios of the intercepted parts cd, ce, cf, &c. of the meridians to the radius CA.

24. PROPOSITION XII.

The meridian line is a scale of logarithmic tangents of half the complements of the latitudes beginning from the radius.

DEMONSTRATION.

Let the figure CADEB be part of the stereographic projection of the sphere on the plane of the equator, as in the last proposition, and on any rhumb let the points A, d, e, f, &c. represent certain places on the globe, whereof A is a point in the equator; then will Dd, Ee, Ff, &c. express their respective latitudes, and the arcs AD, AE, AF, &c. their differences of longitude from the longitude of A: also c being the pole, the lines cd, ce, cf, &c. are the projections of the complements of their latitudes.

Now in this projection the lines cd, ce, cf, &c. are equal to the tangents of half the complements of those latitudes. (IV. 48)

And the differences of longitude AD, AE, AF, &c. are logarithms of the ratio of those tangents, or of the lines cd, ce, cf, &c. to the radius CA.

Also the lengthened degrees of the nautical meridian line are the longitudes of those degrees made on the rhumb of 45 degrees, whether expressed in minutes, according to the usual tables, or in degrees, as some writers have done.

25. PROPOSITION XIII.

The logarithmic tangent of half the complement of any latitude in Napeir's original system, divided by 0,0002908882 &c. first multiplied by the radius of the tables, exhibits in minutes the change in longitude, which will be made on the rhumb of 45 degrees from the equator to that latitude; or the lengthened degrees in the meridian line from the equator to that latitude.

DEMONSTRATION.

The differences of longitude AD, AE, AF, &c. being on the rhumb of 45° , logarithms to c d, c e, c f, &c. which begin from the radius, and proceed from thence, in like manner do as Napeir's logarithmic tangents from his radius:

Therefore his logarithmic tangent of half the complement of any latitude bears to his radius the ratio of the arc of longitude accompanying the change of latitude to the radius of that arc, consequently the ratio of the number of minutes contained in that arc to the measure of the radius in minutes.

Hence the logarithm will also be to the radius of the table multiplied by 0,0002908882 &c. (the arc of one minute, when the radius is unity) as the number of minutes in this arc to the measure of the radius; in minutes multiplied by 0,0002908882 &c. that is, to one minute; for the measure of the radius in minutes is to one minute, as unity to 0,0002908882 &c.

Therefore the number of minutes contained in the longitude, or the lengthened degrees in the meridian line, is equal to the logarithm named, divided by 0,0002908882 &c. first multiplied by the radius of the table.

26. Cor. Napeir's radius being 10000000, the logarithmic tangent will be divided by 2908,882 &c.

If the difference between the logarithmic tangents of half the complements of any two latitudes, when on the same side of the equator, or their sum, if one latitude be north and the other south, be divided in the same manner, the quotient shall give the meridional parts for the interval between the two latitudes.

27. PROPOSITION XIV.

If in Briggs's logarithms the index, or characteristic be alone considered as an integer, and all the rest as an appendage of decimals, the difference between the logarithm of the radius, and the logarithmic tangent of half the complement of the latitude of any places divided by 0,0001263114 &c. gives the minutes of the change of longitude in passing on the rhumb of 45° to the latitude of that place, or the meridional parts corresponding to that latitude.

DEMONSTRATION.

Briggs's logarithms thus interpreted are to Napeir's original logarithms, as 1 to Napeir's logarithm of 10 (or rather, to his logarithm of the ratio between 10 and 1) which is nearly 23025851*, and the differences

* The number usually set down, as Napeir's logarithm of 10, is more properly his logarithm divided by the radius of his trigonometrical table, which is the radix, from which he raised his logarithms. named

named are logarithms of the respective tangents to the radius. Therefore these divided by $\frac{2908,882}{23025851}$ ($=0,00012633114$ &c.) will produce the same quotient, as Napeir's logarithms assigned to the same tangents (which are the logarithms of the ratio of those tangents to the radius in his system) divided by 2908,882: this quotient therefore will be the minutes of the change of longitude on a rhumb of 45° , that is, the length of the meridian line.

28. Cor. If the difference from the radius of the logarithmic tangent taken from any table of logarithms of Briggs's form be considered as consisting wholly of integers, and divided by 12633114 &c. wherein so many figures shall be esteemed integers, as are four short in number of the logarithmic tangent taken, the index included, the quotient shall give the meridional parts appertaining to that latitude*.

Allo here, if in any logarithmic table of the common form, the difference between the logarithmic tangents of half the complement's of any two latitudes on the same side of the equator, or their sum if one latitude be north and the other south, be considered as an integer number, and divided in like manner, the quotient shall be the meridional parts for the interval between the two latitudes.

SCHOLIUM.

29. The numbers in Napeir's table divided by 2908,882 &c. exhibiting the change of longitude on a rhumb of 45° , whose tangent is equal to the radius, the numbers themselves will express the change in longitude on a rhumb, whose tangent is 2908,882 &c. times the radius, and the angle $89^\circ 58' 49''$; and if the logarithm be divided by 10, 100, or 1000, the angle of the rhumb will respectively be $89^\circ 48' 11''$, $88^\circ 1' 52''$, or $71^\circ 1' 42''$, which is the angle Dr. Halley has chosen.

After the same manner in Briggs's system, the difference of the tangent of half the complement of any latitude from the radius divided by 0,00012633114 &c. exhibiting the corresponding length of the meridian line, should the index of the logarithm be alone considered as an integer number, those differences will be the change of longitude on a rhumb, whose tangent is equal to the radius of any trigonometrical canon of Briggs's form multiplied by 0,0001263311438 &c. and the angle $0^\circ 0' 26''$. But if one figure beside the index be taken as integer, the tangent of the rhumb will be the radius multiplied by 0,0012633 &c. the angle being $0^\circ 4' 20\frac{1}{2}''$; if two besides the index be esteemed integers, the angle will be $43' 25\frac{1}{2}''$; if three are understood as integers, the angle will be $7^\circ 12' 0''$; and if four beside the index be accounted for integers, as Halley proposes, the angle will be $51^\circ 38' 9''$.

From the preceding propositions may all the problems relating to the true method of sailing be accurately resolved; and although they are here all contained under the general title of globular sailing, yet as the

* Thus when the tables of logarithms have eight places with the index, the divisor will be 1263,2 &c. but when the tables consist of 7, or 6 places; the divisor will be 126,33 &c. or 12,633 &c. and so on.

common custom has been to distinguish different kinds, wherein the longitude is considered; so here, in compliance with such custom, it was thought proper to retain the usual forms, *viz. parallel sailing, middle latitude sailing, and Mercator's sailing*, or more properly *Wright's sailing*; which are distinctly treated of in what follows.

SECTION III.

Of Parallel Sailing.

30. PARALLEL SAILING is the art of finding what distance a ship should run due east or west, in sailing from the meridian of one place to that of another place, in any parallel of latitude.

This kind of piloting is generally used in conducting a ship to an island detached from the main land, or other islands; and also on other occasions.

The method of performing which, is by sailing to the parallel of latitude the place is in, keeping a good account, so as to be certain whether the place is then to the eastward, or westward; and also, if possible, to know the longitude arrived at; and then to run due west, or east, until the ship comes near the longitude of the given place; where she is then sure to make the port required.

The computations in parallel sailing depend on the following

RULE.

31. *As Radius,*

To the co-sine of the latitude of any parallel;

So is the miles of longitude between any two meridians

To the distance of those meridians in that parallel.

For the demonstration, see Art. 7.

From the same proposition is easily deduced the following

$$\begin{aligned} &= R. \\ &= s. \\ &= L. \\ &= D. \end{aligned}$$

RULE.

32. *As the co-sine of one latitude,*

Is to the co-sine of another latitude;

So is a given meridional distance in the first parallel,

To corresponding meridional distance in the second parallel.

33. And hence arise the following proportions.

$$\begin{aligned} R &: s :: L : D. \\ s &: R :: D : L. \\ L &: D :: R : s. \\ s &: s :: D : d. \\ D &: d :: s : s. \end{aligned}$$

Whereby all the cases that can happen in this kind of sailing are readily resolved by the logarithms, and also by the following table, calculated for a difference of longitude of 1 degree, or 60 nautical miles of the equator, by the first rule. Thus,

As Radius, to the co-sine of any latitude; suppose 30°;

So 60 miles of longitude, to the meridional distance 51,96.

And the like to the other latitudes.

37. CASE I. Given the latitude and difference of longitude: Required the distance.

EXAM. *A ship in the latitude of 32° N. sails thence due east till her difference of longitude is 384 miles: Required the distance she has run?*

By CONSTRUCTION*. Fig. 4. Pl. XIII.

From any point P, with the radius of 90°, taken from the scale of natural fines, describe the arc EQ; draw the chord EQ equal to 384 miles taken from a scale of equal parts; and draw the radii PE, PQ.

From the same point P, with a radius equal to the sine of 58° (the comp. lat.), taken from the nat. fines, describe the arc AB; and draw the chord AB, whose measure is the distance sought.

Here P represents the north pole; the arc EQ the equator; the arc AB a parallel of latitude; the chord EQ is the difference of longitude; the chord AB is the meridional distance sought; A is the place sailed from, B the place arrived at; and the radii PE, PQ, express the meridians of the places so sailed from, and come to.

By COMPUTATION.

As Radius	= 90° 00'	10,00000
To cos. lat.	= 32° 00'	9,928,42
So miles long.	= 384	2,584,33
To miles dist.	= 325,6	2,512,75

So that she has run 325,6 miles.

By the TABLE.

Seek the miles to the given latitude; multiply them by the given diff. longitude; the quotient divided by 60 will give the meridional distance sought.

Thus. To lat. 32° answers 50,88 miles.

Multiply 384 the given diff. longitude.

$$\begin{array}{r} 20352 \\ 40704 \\ \hline 15264 \end{array}$$

$$60) 19537,92 (325,632 = \text{merid. dist. sought.}$$

* This and the construction of the following cases depend on the Orthographic projection † of the circles of the sphere on the plane of the equator, that circle being the primitive, and its center or pole representing the point into which the pole of the Earth is projected; consequently the distance from the pole of any parallel of latitude, must be expressed by the complement of that latitude.

† This projection is formed by supposing that any point on the surface of a sphere is projected on a plane passing through its center, by a line drawn through that point perpendicular to the plane; which amounts to the same, as if the visual ray came from an eye at an infinite distance.

Thus in Fig. 1. Pl. XIII. perpend. lines through B, b, project those points in t, x, on the plane r d n z: Therefore small circles on that plane are described with the radii xt, xz, = the arcs of the arcs rB, r b.

78. CASE

38. CASE II. Given the latitude and meridional distance: Required the difference of longitude.

EXAM. *A ship from the latitude 53° 36' N. longitude 10° 18' E. sails due west 236 miles: Required her present longitude?*

By CONSTRUCTION. Fig. 4. Pl. XIII.

From the point P, with the sine of 90°, and the co-sine of 53° 36', describe the arcs EQ, CD. Draw the chord CD=326; through c and D draw the radii PE, PQ; then the chord EQ being drawn, will be the difference of longitude.

By COMPUTATION.

As PC, the cof. lat.	=	53° 36'	0,22664	6,0)39,8(6° 38'.
To PE, Radius	=	90 00	10,00000	Long. from 10 18 E.
So CD, merid. dist.	=	236 m.	2,37291	Diff. long. 6 38 W.
To EQ, diff. long.	=	397,7	2,59955	Long. in 3 40 E.

By the TABLE.

Find the meridional distance to a deg. of long. in lat. 53° 36' (35), and make it a divisor to the product of the given meridional distance by 60; then will the quotient be the diff. long. sought.

Thus. The diff. merid. dist. to 53° and 54°, is 0,85; which multiplied by 36' and divided by 60', gives 0,51; this taken from 36,11, the merid. dist. to 53°, leaves 35,6 for the merid. dist. to 53° 36': Then 236 multiplied by 60, and the quotient divided by 35,6 gives 397,7 for the diff. long. sought.

39. CASE III. Given the difference of longitude, and its corresponding meridional distance: Required the parallel of latitude.

EXAM. *In what latitude does 384 miles of meridional distance answer to 500 miles of diff. longitude?*

By CONSTRUCTION. Fig. 5. Pl. XIII.

From the point P, with the sine of 90°, describe the arc EQ; draw the chord EQ=500, and draw the radii PE, PQ; make EC=384, draw CB parallel to EP; then the arc AB being described with the radius PE, is the parallel of lat. sought.

By COMPUTATION.

As EQ, the diff. long.	=	500	7,30103	So in the lat. 39° 49' 384 miles of merid. dist. answers to 500 miles of longitude.
To AB, merid dist.	=	384	2,58433	
So PE, Radius	=	90° 00'	10,00000	
To PA, the cof. lat.	=	39 49	9,88536	

By the TABLE.

The given merid. dist. 384 being multiplied by 60, and the product divided by the given diff. long. 500, gives 46,08, the merid. dist. to one deg. of long. in the lat. sought: And by article 36, the latitude will be found to be 39° 49'.

40 CASE IV. Given the distance of two places in a given parallel of latitude: Required the distance of two other places, under the same meridians with the former, in another given parallel of latitude.

EXAM. *From two ports in the lat. $32^{\circ} 20' N.$ distant 256 miles, two ships sail directly north: How far are they distant from one another, when they come to the lat. $44^{\circ} 30' N.$*

By CONSTRUCTION. Fig. 6. Pl. XIII.

From the point P, with radii equal to the co-sines of $32^{\circ} 20'$, and $44^{\circ} 30'$, describe the arcs AB, DE; draw the chord $AB=256$, and draw PA, PE; then will the chord DE be the dist. sought.

By COMPUTATION.

As PA, the cof. of lat. from	$= 32^{\circ} 20'$	0.07317
To PD, the cof. of lat. in	$= 44^{\circ} 30'$	9.85324
So AB, the first dist.	$= 256$	2.40824
To DE, the present dist.	$= 216.1$	2.33465

By the TABLE.

The merid. distances to $32^{\circ} 20'$, and $44^{\circ} 30'$, (found by 35) are 50.7 and 42.8: Then the given merid. distance 256 multiplied by 42.8, and the product 1095.68 divided by 50.7, the quotient 216.1 is the meridional distance sought.

41. CASE V. Given the distances between the same two meridians in different parallels, and also the latitude of one parallel, to find the latitude of the other.

EXAM. *If two ships in the lat. $44^{\circ} 30' N.$, distant from one another 216 miles, should both sail hence directly south until their distance is 256 miles: What latitude are they come to?*

By CONSTRUCTION. Fig. 7. Pl. XIII.

From P, with a radius of the cosine of $44^{\circ} 30'$, the lat. sailed from, describe the arc DE; draw the chord $DE=216$; and through D and E, draw PA, PB; continue DE till DF $=256$; through F draw FB parallel to PA, meeting PB in B; then an arc described from P, through B and A, will be the parallel of latitude come to.

By COMPUTATION.

As DE, the first dist.	$= 216$	7.66554
To AB, the second dist.	$= 256$	2.40824
So PD, the cof. lat. from	$= 44^{\circ} 30'$	9.85324
To PA, the cof. lat. in	$= 32^{\circ} 17'$	9.91702

By the TABLE.

To lat. $44^{\circ} 30'$, find (35) the mer. dist. 42.8; this multiplied by 256, and the product divided by 216, gives 50.72; which (by 36) will be found to be the merid. dist. to $32^{\circ} 17'$, the latitude sought.

42. Other

42. Other examples to exercise the foregoing cases.

Ex. What is the distance between *St. Mary's* in lat. $37^{\circ} 00' N.$ long $22^{\circ} 56' W.$, and *Cape Henry* in lat. $37^{\circ} 00' N.$ long $76^{\circ} 23' W.$?

As Rad : cof. lat. $37^{\circ} 00' ::$ diff. long 3207 : distance 2561.

Ex. II. *A ship from Cape Clear sails west 265 miles: What long. is she come to?*

As cof. lat. $51^{\circ} 18'$: Radius :: merid. dist. 265 : diff. long. 423,8.
Answer. The present longitude is $16^{\circ} 54' W.$

Ex. III. *A ship having run due east for 3 days, at the rate of 5 knots an hour, finds she has altered her longitude $8^{\circ} 16'$: What parallel of latitude did she sail in?*

As diff. long. 496 : merid. dist. 360 :: Rad : cof. lat. $43^{\circ} 28'$.

Ex. IV. *Suppose two ships in the parallel of $47^{\circ} 54' N.$, their difference of long. being $9^{\circ} 35'$, should both sail thence directly south 836 miles: How far were they distant, at the beginning, and end of their courses?*

As Rad : cof. lat. $47^{\circ} 54' ::$ diff. long. 575 : first merid. dist. 385,5m.
Now 836 miles = $13^{\circ} 56'$; therefore the lat. come to is $33^{\circ} 58'$.

As Rad : cof. lat. $35^{\circ} 58' ::$ diff. long. 575 : last merid. dist. 476,9m.

Ex. V. *What is the diff. lat. between two parallels on different sides of the equator, when, between the same two meridians, the meridional distance in the northern parallel is $AB=238$ miles, in the southern parallel is $CD=195$ miles, and at the equator is $EQ=352$ miles?*

As diff. long. 352 : north. mer. dist. 238 :: Rad : cof. lat. $47^{\circ} 27' N.$

As diff. long. 352 : south. mer. dist. 195 :: Rad : cof. lat. $56^{\circ} 22' S.$

Then the sum of these two latitudes, viz. $103^{\circ} 49'$, is the diff. lat. sought.

In this figure 8, Pl. XIII, *P* is the north pole, and *s* is the south pole; but the construction on one side of the equator is sufficient, as is evident by drawing *ed* parallel to *PE*, and *dc* parallel to *EQ*.

The difference of longitude and meridional distances may be considered as tangents to their respective arcs, as well as chords. (Il. 185)

Among the natural scales on the Gunter, are two, which used jointly, answer the end of the foregoing table; these are a scale of chords marked *CH*, and a scale of longitude marked *ML*, standing close to one another; and are thus used.

Find the given latitude on the scale of chords, and right against it, in the scale of longitude, is the division expressing the miles of merid. distance in that lat. to one degree of longitude.

Thus against the chord 60° , stands 30 miles; and the chord 50° , stands between 38 and 39 miles: And so of the rest.

SECTION IV.

Of Middle latitude, and Mercator's sailings.

Of Middle latitude sailing.

43. MIDDLE LATITUDE SAILING, is a manner of solving the several cases of globular sailing, by the principles of plane and parallel sailing jointly.

This method, which is not quite accurate, is founded on the following supposition.

44. *The departure is reckoned as a meridional distance in that latitude, which is a middle parallel between the latitude sailed from, and the latitude come to.*

Now altho' the arithmetical mean of the cosines, of two distant latitudes is not the cosine of the arithmetical mean of those latitudes; neither is the departure between two places on an oblique rhumb equal to the distance between their meridians in a mean latitude; yet when the parallels of those places are near the equator, or not far distant from one another, in any latitude, the error becomes so small, as not materially to affect the nautical conclusions drawn from the foregoing supposition.

This artifice seems to have been invented on account of the easy manner in which the several cases may be resolved by the Traverse Table, and where a table of meridional parts is wanting. The computations depend on the following rules.

RULE I.

45. Add the latitude sailed from to the latitude arrived at, and take half the sum for the middle latitude.

RULE II.

46. As the cosine of the middle latitude,

Is to the Radius;

So is the departure,

To the difference of longitude.

= C.
= R.
= D.
= L.

RULE III.

47. As the cosine of the middle latitude,

Is to the tangent of the course;

So is the difference of latitude,

To the difference of longitude.

= C.
= T.
= P.
= L.

The second rule is the same as the first rule in parallel sailing, which is demonstrated at art. 7.

Demonstration

Demonstration of the third rule.

Now cof. mid. lat. : Rad :: departure : diff. long. (7)
 Therefore Rad $\times$ dep. = cof. mid. lat. $\times$ diff. long. (II. 162)
 Also diff. lat. : depart. :: Rad : tang. course. (VII. 26)
 Therefore Rad $\times$ dep. = diff. lat. $\times$ tang. course. (II. 162)
 Consequently cof. mid. lat. $\times$ diff. long. = diff. lat. $\times$ t. course. (II. 46)
 Therefore cof. mid. lat. : tang course :: diff. lat. : diff. long. (II. 163)

From rules II and III arise the following proportions.

C : R :: D : L. C : T :: P : L.
 L : D :: R : C. P : L :: C : T.
 R : C :: L : D.

Where the letters signify the names they stand against in the rules.

Of Mercator's sailing.

48. MERCATOR'S SAILING is the art of resolving the several cases of Globular sailing by Plane Trigonometry, with the assistance of a table of meridional parts, or of logarithmic tangents.

The computations herein are performed by the following rules. (15)

49. RULE I.

As merid. diff. lat. = M. M : L :: R* : T.
 To diff. longitude : = L. R : T :: M : L.
 So is the Radius, = R. T : R :: L : M.
 To tang. course. = T.

Therefore

50. RULE II.

As the prop. diff. lat. = P. P : D :: M : L.
 To the departure : = D. M : L :: P : D.
 So mer. diff. lat. = M. M : L :: P : D.
 To diff. longitude. = L.

Therefore

These rules are demonstrated at Prop. IV.

51. RULE III.

As diff. log. tan $\frac{1}{2}$ co-lats. = G. G : N :: L : T.
 To tang. of $51^{\circ} 38' 09''$ = N.* N : G :: T : L.
 So is a given diff. long. = L. T : L :: N : G.
 To tang. course. = T.

Therefore

For the demonstration of this analogy, see Prop. XIII. (29)

* The log. tang. of $51^{\circ} 38' 09''$ is 10,10151; its ar. co. is 9,89849. (I. 88)
 The decimal part of this log. tang. answers also to $85^{\circ} 28' 37''$; and to $89^{\circ} 32' 47''$; and to $89^{\circ} 57' 17''$; their indices being 11, 12, 13: So that at either of these angles, the diff. of longitude will be shewn by the diff. of the log. tang. of the half co-lats failed from and come unto: Observing that the number of integer places in c becomes one more, for each unit more in the index used.

PROBLEM

PROBLEM I.

52. Given the latitudes of two places:
Required the meridional difference of latitude between those places.

CASE I. When both places are on one side of the equator.

RULE I. The difference of the meridional parts answering to each latitude, will give the meridional difference of latitude required.

RULE II. The difference of the log. tangents of the half co-latitudes divided by 12,63, will give the meridional difference of latitude required.

EXAM. I. What is the meridional difference of latitude between the island of St. Thomas and Gibraltar?

By the meridional parts.

Gibraltar lat. $36^{\circ} 13' N.$

St. Thomas's lat. $00^{\circ} 00'$

Merid. parts to 36 13

Merid. parts to $00^{\circ} 00'$

2334

0000

Merid. diff. lat. =

2334

By the log. tangents.

Co-lat. $= 53^{\circ} 47'$; $\frac{1}{2}$ co-lat. $= 26^{\circ} 53'$

Co-lat. $= 90^{\circ} 00'$; $\frac{1}{2}$ co-lat. $= 45^{\circ} 00'$

Log. tang. $10,00000$ to $45^{\circ} 00'$

Log. tang. $9,70498$ to $26^{\circ} 53'$

12,63) 29502(2335

4242

4530

7410

EXAM. II. What is the meridional difference of latitude between the Lizard and Cape Verd?

By the meridional parts.

C. Verd's lat. $14^{\circ} 43' N.$

Lizard's lat. $49^{\circ} 57' N.$

Merid. parts to $49^{\circ} 43'$

Merid. parts to $14^{\circ} 43'$

3470

893

Merid. diff. lat. =

2577

By the log. tangents.

Co-lat. $= 75^{\circ} 17'$; $\frac{1}{2}$ co-lat. $= 37^{\circ} 38'$

Co-lat. $= 40^{\circ} 03'$; $\frac{1}{2}$ co-lat. $= 20^{\circ} 01'$

Log. tang. $9,88707$ to $37^{\circ} 38'$

Log. tang. $9,56146$ to $20^{\circ} 01'$

12,63) 32561(2578=m.p.

53. CASE II. When the given places are on different sides of the equator.

RULE I. The sum of the meridional parts answering to each latitude, will be the meridional difference of latitude required.

RULE II. The sum of the log. co-tangents, abating the index, of the half co-latitudes, divided by 12,63, will be the meridional difference of latitude required.

EXAM. What is the meridional difference of latitude between Cape Verd, and the island of St. Helena?

By the meridional parts.

C. Verd's lat. $14^{\circ} 43' N.$

St. Helena $16^{\circ} 00' S.$

Merid. parts to $16^{\circ} 00'$

Merid. parts to $14^{\circ} 43'$

973

893

Merid. diff. lat. to =

1866

By the log. tangents.

Co-lat. $= 75^{\circ} 17'$; $\frac{1}{2}$ co-lat. $= 37^{\circ} 38'$

Co-lat. $= 74^{\circ} 00'$; $\frac{1}{2}$ co-lat. $= 37^{\circ} 00'$

Log. cot. $0,11293$ to $37^{\circ} 38'$

Log. cot. $0,12288$ to $37^{\circ} 00'$

12,63) 23581(1867=m.p.

PROBLEM

PROBLEM II.

54. Given the lat. of one place, and the merid. diff. lat. between that and another place:

Required the latitude of the other place?

CASE I. When the given lat and mer. diff. lat. have like names.

RULE I. To the merid. parts of the given lat. add the given meridional diff. lat.; seek the sum in the table of meridional parts, and the corresponding degrees and minutes gives the latitude sought.

RULE II. Multiply the given merid. diff. lat. by 12,63; subtract the product, rejecting all to the right of the 5th decimal place*, from the log. tang. of the given half co-lat.; seek the degrees to the remainder among the log. tang. these deg. doubled, gives the co-lat. required.

EXAM. A ship from the lat. $14^{\circ} 43' N.$ sails northward till she finds her merid. diff. lat. is 2578; What lat. is she come to?

By meridional parts.

To the lat. $14^{\circ} 43' N.$

The merid. parts = 893 N.

Add merid. diff. lat. = 2578 N.

The sum is = 3471

which answers to $49^{\circ} 57' N.$

The latitude sought.

By log. tangents.

The co-lat. is $75^{\circ} 17'$; $\frac{1}{2}$ co-lat. $37^{\circ} 38'$.

And 2578 multiplied by 12,63, gives

32560, 14.

From 9,88707 = log. tang. $37^{\circ} 38'$.

Take 0,32560

Leaves 9,56147 = log. tang. $20^{\circ} 01'$.

which double is $40^{\circ} 02'$.

Whose comp. is $49^{\circ} 58' =$ lat. sought.

55. CASE II. When the given lat. and meridional diff. lat. have unlike names.

RULE I. Take the difference between the meridional parts of the given latitude and the meridional diff. lat. the remainder found in the table of meridional parts, will give the latitude sought.

RULE II. Multiply the given meridional diff. lat. by 12,63; add the product to the log. tan. of the given half co-latitude, the degrees to the tangent of the sum being doubled, gives the co-latitude required.

EXAM. A ship from the latitude $49^{\circ} 57' N.$ sails southward till her meridional diff. lat. is 2578: What latitude is she come to?

By meridional parts.

To the lat. $49^{\circ} 57' N.$

The merid. parts = 3470 N.

Subt. merid. diff. lat. = 2578 S.

Remains merid. parts 892

which answers to $14^{\circ} 43' N.$

The latitude come to.

By log. tangents.

Co-lat. $40^{\circ} 03'$; $\frac{1}{2}$ co-lat. $20^{\circ} 01'$.

And $2578 \times 12,63 = 32560, 14.$

To 9,56146 = log. tang. $20^{\circ} 01'$.

Add 0,32560

Sum 9,88706 = log. tang. $37^{\circ} 38'$.

which doubled is $75^{\circ} 16'$.

Whole comp. is $14^{\circ} 44' =$ lat.

* That is, rejecting all below the number of places in the tables used.

PROBLEM II.

56. Given the latitudes and longitudes of two places:
Required their bearing and distance.

EXAM. *What is the course and distance from Cape Clear in Ireland, to the island of St. Mary, one of the Azores?*

By MIDDLE LATITUDE SAILING.

C. Clear's lat. $51^{\circ} 18' N.$ Long. $9^{\circ} 50' W.$

St. Mary's lat. $37^{\circ} 00' N.$ Long. $22^{\circ} 56' W.$

Diff. lat. $14^{\circ} 18' = 858 m.$ Diff. long. $13^{\circ} 06' = 786 m.$

Sum latitudes $88^{\circ} 18'$; Mid. lat. $44^{\circ} 09'$; Co-mid. lat. $45^{\circ} 51'$.

By CONSTRUCTION. Fig. 9. Plate XIII.

Draw the meridian AD ; describe the quadrant $Am p$ with the sine of 90° , and the arc EF with the co-sine of the middle latitude. Make the chord MB equal to 786, the diff. longitude; draw AB cutting the arc EF in F , and draw the chord EF . Make AD equal to 858, the diff. lat. draw DC parallel to Ap and equal to EF ; and draw AC .

Then AC is the distance, and the angle DAC is the course.

For the Course.

$P : L :: C : T.$ (47)
As diff. lat. $= 858$ 7,06651
To diff. long. $= 786$ 2,89542
So cof. m. lat. $= 44^{\circ} 09'$ 9,85583

To tang. cour. $= 33^{\circ} 19'$ 9,81776

By COMPUTATION.

For the Distance.

$\sin. \angle C : AD :: R : AC.$
As cof. cour. $= 33^{\circ} 19'$ 0,97798
To diff. lat. $= 858$ 2,93349
So Rad. $= 90^{\circ} 00'$ 10,00000

To dist. $= 1027$ 3,01147

By the TRAVERSE TABLE.

1st. With the co-mid. lat. found among the degrees, and the diff. longitude, or some part thereof, (if too large) taken among the distances, find the departure, or some part thereof, in its column.

Observing to multiply this Depart. by the same number the diff. longitude was divided by.

2d. With the diff. lat. and dep. together, or their like parts, find the course among the degrees, and the distance in its column.

Let the diff. long. 786 be broke into parts; viz. into 100 taken seven times, and into 86: And as the co-mid-lat. is $45^{\circ} 51'$, or $45^{\circ} \frac{5}{6}$; proceed thus.

The departures to 100, in the columns of 45° and $46'$, are 70,7 and 71,9; the diff. is 1,2; $\frac{2}{3}$ parts thereof is 1,0; which added to 70,7 makes 71,7 for the dep. to 100; and for 700 it will be 501,9. Also, the departures to 86 in the columns of 45° and $46'$, are 60,8 and 61,9; $\frac{2}{3}$ of their diff. is 0,9, and this with 60,8 makes 61,7 the dep. to 86; and the whole dep. $= 563,6$.

Now take $\frac{2}{3}$ of the diff. lat. 858, and of the dep. 564, viz. 42,9 and 28,2; these will be found together under $33^{\circ} 15'$ for the course; and between

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between 51 and 52 diff. ; suppose $51\frac{1}{2}$; which taken 20 times gives 1030 for the distance.

By MERCATOR'S SAILING.

57. C. Clear long.	9° 50' W.	Lat. 51° 18'	Mer. parts 3597
St. Mary's long.	22 56	Lat. 37 00	Mer. parts 2393
Diff. long.	13 06	Diff. lat. 14 18	Mer. diff. lat. 1204
Or 786 miles.			

By CONSTRUCTION. Fig. 17. Plate XIII.

Draw the meridian AB, and describe the quadrant Amp ; make AD = 858, the proper diff. lat. and AB = 1204, the meridional diff. lat. ; draw DC, BE, parallel to A p; make BE equal to 786, the diff. long. and draw AE.

Then A is the place of Cape Clear, c that of St. Mary's; AC is the distance, CD the departure, and the angle DAC is the course.

COMPUTATION by Meridional Parts.

For the Course.

M : L :: R : T.	(49)	fin. $\angle C$: AD :: R : AC.
As mer. d. lat. = 1204	6,91937	As cos. cour. = $33^{\circ} 08'$ 0,07707
To diff. long. = 786	2,89542	To prop. d. lat. = 858 2,93349
So Rad. = $45^{\circ} 00'$ 10,00000		So Rad. = $90^{\circ} 00'$ 10,0000
To tan. cour. = 33 08	9,81479	To dist. = 1024 3,01056

COMPUTATION by Logarithmic Tangents.

C. Clear's lat. 51° 18'	Co-lat. 38° 42'.	$\frac{1}{2}$ co lat. 19° 21'.
St. Mary's lat. 37 00	Co-lat. 53 00.	$\frac{1}{2}$ co-lat. 26 30.
To $\frac{1}{2}$ co-lat. 26° 30', its log. tang.	=	9,69774
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.	=	9,54552
Difference	=	1522,2 = G.

For the Course.

G : N :: L : T.	(51)
As 6 1522	6,81759
To N	10,10151
So diff. long. 786	2,89542
To tang. cour. 33° 07'	9,81452

The distance will be the same as in the former computation; for the diff. of near a min. in the course would not happen in working by tables containing more places.

By the TRAVERSE TABLE.

Seek the meridional diff. lat. and diff. long. together, like diff. lat. and departure, and the course is given among the degrees.

Then with the course and proper diff. lat. find the distance.

Thus taking $\frac{1}{16}$ th of 1204 and 786, viz. 60,2 and 39,3; they will be found together under $33^{\circ} 15'$ for the course.

And taking $\frac{1}{16}$ th of the diff. lat. 858, viz. 42,9, and the course $33^{\circ} 15'$, gives about $51\frac{1}{2}$ for the dist. which taken 20 times gives 1022 diff.

PROBLEM

PROBLEM IV.

58. Given the bearing and latitudes of two places :
Required the distance and difference of longitude.

EXAM. *A ship who had taken her departure from a place in latitude $37^{\circ} 00'$ N. longitude $22^{\circ} 56'$ W. steers N. $33^{\circ} 19'$ E.; and cloudy weather coming on, she got no observation till eight days after, and then found herself in latitude $51^{\circ} 18'$ N.: Required her true distance and present longitude?*

By MIDDLE LATITUDE SAILING.

Departed lat.	$37^{\circ} 00'$ N.	Sum latitudes	$88^{\circ} 18'$
Present lat.	$51^{\circ} 18'$ N.	Mid. lat.	$44^{\circ} 09'$
Diff. lat.	$14^{\circ} 18' = 858$ m.	Co-mid. lat.	$45^{\circ} 51'$

By CONSTRUCTION. Fig. 10. Plate XIII.

Draw the meridian $AD=858$, the diff. lat. describe the quadrant $Am p$; make mc equal to $33^{\circ} 19'$, the course: Draw DC parallel to Ap meeting in C , the rhumb AC drawn through c : With the fine of the co-mid. lat. $45^{\circ} 51'$ describe the arc EF , in which apply the departure DC , from E to F : through F draw AB , and draw the chord mB . Then is AC the distance run, and mB the diff. longitude.

By COMPUTATION.

For the distance.

fin. $\angle C$:	$AD :: R : AC$.	$C : T :: P : L$.	(47)
As cos. cour.	$= 33^{\circ} 19' \quad 0,07798$	As cos. m. lat.	$= 44^{\circ} 09' \quad 0,14417$
To diff. lat.	$= 858 \quad 2,93349$	To tang. cour.	$= 33^{\circ} 19' \quad 9,81776$
So Rad.	$= 90^{\circ} 00' \quad 10,00000$	So diff. lat.	$= 858 \quad 2,93349$
To dist.	$= 1027 \quad 3,01147$	To diff. long.	$= 786 \quad 2,89542$

Now $22^{\circ} 56'$ W. = Dep. long.
 $60) 786 (13 \quad 06 \quad E. = \text{Diff. long.}$

$9 \quad 50 \quad W. = \text{Pref. long.}$

By the TRAVERSE TABLE.

1st. With the course among the degrees, and the diff. lat. in its column, find the departure and distance, in their columns.

2d. With the co-mid. lat. among the degrees, and departure in its column, find the diff. long. among the distances.

Thus taking of the $\frac{1}{20}$ diff. lat. 858 , viz. $42,9$; with it, and the course $33\frac{1}{2}^{\circ}$ the departure will be found $28,1$, and the dist. $51,3$; each being multiplied by 20 , gives 562 for the departure, and 1026 for the distance.

The co-mid. lat. $45^{\circ} 51'$, or $45\frac{1}{2}$ degrees, and $28,1 = \frac{1}{20}$ departure, will give in the column of distances $39,5$; which multiplied by 20 , gives 790 for the difference of longitude,

59. By

59. By MERCATOR'S SAILING.

Departed lat.	37° 00' N.	Mer. parts	2393
Present lat.	51 18 N.	Mer. parts	3597
Diff. lat.	14 18 = 858 m.	Mer. diff. lat.	1204

CONSTRUCTION. Fig. 18. Plate XIII.

Draw a meridian AB, in which take AB=1204, the mer. diff. lat., and AD=858, the proper diff. lat. Describe the quadrant A m p, and take mc=33° 19', the course: Through B and D, draw BE, DC, parallel to A p, meeting the rhumb drawn through A and c, in the points E, C. Then is AC the distance, and BE the diff. longitude.

COMPUTATION by Meridional Parts.

<i>For the distance.</i>		<i>For the diff. longitude.</i>	
sin. ∠DCA : AB :: R : AC.	R : T :: M : L. (49)	As Rad.	= 45° 00' 10,00000
As cos. cour. = 33° 19' 0,07798	To tang. cour. = 33 19	To tang. cour.	= 33 19 9,81776
To diff. lat. = 858 2,93349	So m. diff. lat. = 1204	So m. diff. lat.	= 1204 3,08063
So Rad. = 90° 00' 10,00000			
To dist. = 1027 3,01147	To diff. long. = 791,4		2,89839

Now 22° 56' W. = Dep. long.
60) 791 (13 11 E. = Diff. long.

Then 9 45 W. = Pres. long.

COMPUTATION by Logarithmic Tangents.

Departed lat.	37° 00' N.	Co-lat.	53° 00'.	$\frac{1}{2}$ co-lat.	26° 30'.
Present lat.	51 18 N.	Co-lat.	38 42.	$\frac{1}{2}$ co-lat.	19 21.
$10\frac{1}{2}$ co-lat. 26° 30', its log. tang.	= 9,69774				
To $10\frac{1}{2}$ co-lat. 19 21, its log. tang.	= 9,54552				
Difference	= 1522,2 = G				

For the distance.

sin. ∠DCA : AB :: R : AC.

For the diff. longitude.

N : G :: T : L. (51)	
As N	= 9,89849
To G	= 1522 3,18241
So tang. cour. = 33° 19'	9,81776

The distance is found as above.

To diff. long. = 791,8 2,89866

By the TRAVERSE TABLE.

With the course among the degrees, and the diff. lat. found in the column of lat. find the distance in its column.

Find the course among the degrees, and the mer. diff. lat. in the column of lat. and take the corresponding dep. for the diff. long.

Now $\frac{1}{16}$ th of 858, the diff. lat., and of 1204, the mer. diff. lat., are 42,9 and 60,2. Then, under 33° 15', the diff. lat. 42,9 falls between 51 and 52 in the distances, take it 51,5 ; which multiplied by 20 gives 1030 for the distance.

And under 33° 15', the mer. diff. lat. falls against 39,48 of depart., which multiplied by 20 gives 789,6 for the diff. longitude.

PROBLEM

PROBLEM V.

60. Given one latitude, course and distance :
Required the other latitude and diff. longitude.

EXAM. *A ship takes her departure from a place in lat. $51^{\circ} 18' N$. long. $9^{\circ} 50' W$.; and steers $S. 33^{\circ} 08' W$. till she has run 1024 miles : Required her present latitude and longitude ?*

By MIDDLE LATITUDE SAILING.

By CONSTRUCTION. Fig. II. Plate XIII.

Draw a meridian AD , and describe the quadrant Amp ; make the arc $mc = 33^{\circ} 08'$ the course : In the rhumb drawn through c , take $AC = 1024$, the distance : Draw cd parallel to Ap , meeting AD in D .

By COMPUTATION.

For the diff. latitude.

$R : AC :: \sin. \angle C : AD.$	Now $51^{\circ} 18' N. = \text{Dep. lat.}$
As Rad. $= 90^{\circ} 00'$ 10,00000	60) 857 (14 17 S. = Diff. lat.
To diff. $= 1024$ 3,01030	37 01 N. = Pref. lat.
So cof. cour. $= 33^{\circ} 8'$ 9,92293	88 19 = Sum lats.
To diff. lat. $= 857,5$ 2,93323	44 09 = Mid. lat.
	45 51 = Co-m. lat.

For the diff. longitude.

$As C : T :: P : L.$	Then as the arc EF , described
As cof. m. lat. $= 44^{\circ} 09'$ 0,14417	with the fin. co-mid. lat. $45^{\circ} 51'$,
To tang. cour. $= 33^{\circ} 08'$ 9,81473	apply CD from E to F : through F
So diff. lat. $= 857,5$ 2,93323	draw AB cutting the arc mc in B ;
To diff. long. $= 780,1$ 2,89213	then mB being drawn will be the

Now $9^{\circ} 50' W. = \text{Dep. long.}$	diff. long. sought.
60) 780 (13 00 W. = Diff. long.	

22 50 W. = Pref. long.	
------------------------	--

By the TRAVERSE TABLE.

With the course found among the degrees, and the distance in its column, find the dep. and diff. lat. ; then find the other lat.

Seek the co-mid. lat. among the degrees, and the dep. in its column, then the corresponding diff. is the diff. long. sought.

Thus. Under the course $33^{\circ} 15'$, and against the diff. 51, which is the 20th part of 1024, stands lat. 42,65 ; dep. 27,96.

Then $42,65 \times 20 = 853 m. = 14^{\circ} 13'$, the diff. lat.

Hence the pref. lat. is $37^{\circ} 05' N.$; and the co-mid. lat. $45^{\circ} 48'$.

Again. Over the degrees $45^{\circ} 40'$, the nearest dep. to 27,96 stands against 39 in the distances.

Then $39 \times 20 = 780 m.$ the diff. longitude.

61. By

61. By MERCATOR'S SAILING.
CONSTRUCTION. Fig. 19. Plate XIII.

Draw the meridian AB, describe the quadrant amp ; make the arc $mc = 33^\circ 08'$, the course. In the rhumb AC drawn through c , take $AC = 1024$ miles, the dist.; and draw CD parallel to Ap .

COMPUTATION by Meridional Parts.

For the diff. latitude.

As Rad : AC :: sin. $\angle C$: AD.	
As Rad = $90^\circ 00'$ 10,00000	
To diff. = 1024 3,01030	
So cos. cour. = $33^\circ 08'$ 9,92293	
To pro. diff. lat. = 857,5	2,93323

Now $51^\circ 18' N.$ = Dep. lat.	
60) 857 (14 17 S. = Diff. lat.	

37 01 N. = Pref. lat.	
-----------------------	--

To $51^\circ 18'$ merid. parts = 3597	
To 37 01 merid. parts = 2394	

Merid diff. lat. = 1203

For the diff. longitude.

As R : T :: M : L.	
As Rad. = $45^\circ 00'$ 10,00000	
To tang. cour. = $33^\circ 08'$ 9,81473	
So mer. d. lat. = 1203 3,08026	
To diff. long. = 785,2	2,89499

Make $AB = 1203$; through B draw BE parallel to Ap , meeting AC continued in E; then BE is the diff. longitude.

Now $9^\circ 50' W.$ = Dep. lat.	
60) 785 (13 05 W. = Diff. long.	
22 55 W. = Pref. long.	

COMPUTATION by Logarithmic Tangents.

The diff. lat. and present lat. being found as above.

Dep. lat. $51^\circ 18'$ Co-lat. $38^\circ 42'$ $\frac{1}{2}$ co lat. $19^\circ 21'$	
Pref. lat. 37 01. Co-lat. 52 59. $\frac{1}{2}$ co-lat. 26 29 $\frac{1}{2}$	

To $\frac{1}{2}$ co-lat. $26^\circ 21'$, its log. tang. =	9,69758
To $\frac{1}{2}$ co-lat. 19 21, its log. tang. =	9,54552

Difference

As N : G :: T : L	
As N = 9,89849	Now $9^\circ 50' W.$ = Dep. long.
To G = 1521 3,18213	60) 786 (13 06 W. = Diff. long.
So tang. cour. = $33^\circ 08'$ 9,81473	22 56 W. = Pref. long.
To diff. long. = 785,9	

By the TRAVERSE TABLE.

With the course and distance, find the diff. latitude.

And then get the present lat. and the mer. diff. latitude.

With the course, and the mer. diff. lat. taken in the column of Lat. the corresponding dep. will be the diff. longitude.

Now $\frac{2}{3}$ th of the dist. 1024 is 51,2. Then under the course $33^\circ 15'$, against 51 among the distances, stands 42,65 in col. of Lat. And 42,65 $\times$ 20 gives 853; so the diff. lat. is $14^\circ 13'$; the present lat. is $37^\circ 05' N.$ and the mer. diff. lat. is 1198; $\frac{1}{2}$ th of which is 59,9.

To the course $33^\circ 15'$, against lat. 59,9, or its nearest 60,21, stands 39,48; which multiplied by 20 gives 789,6 for the diff. long. required.

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PROBLEM VI.

62. Given the latitudes of two places, and their distance :
Required their bearing, and difference of longitude.

EXAM. *A ship in lat. $51^{\circ} 18'$, long. $22^{\circ} 06' W.$, is bound to a place in the SE. quarter, distant 1024 miles, and in lat. $37^{\circ} 00' N.$: What is her direct course, and how much must she alter her long. to arrive at the desired port?*

By MIDDLE LATITUDE SAILING.

Lat. from	$51^{\circ} 18' N.$	Sum latitudes	$88^{\circ} 18'$
Lat. to	$37^{\circ} 00' N.$	Middle latitude	$44^{\circ} 09'$
Diff. lat.	$14^{\circ} 18' = 858$ miles.	Co-middle latitude	$45^{\circ} 51'$

By CONSTRUCTION. Fig. 12. Plate XIII.

Draw the meridian $AD=858$ miles ; describe the quadrant Amp : From A with 1024 miles, the given distance, cut DC drawn parallel to Ap , in c ; draw AC . In the arc EF described with the fine of $45^{\circ} 51'$, the co-middle lat., apply DC the departure from E to F ; through F draw AB , and draw the chord Bm .

Then the course is measured by the arc mE ; and the diff. long. by the line mB .

By COMPUTATION.

For the course.

AC : Rad :	AD : fin. $\angle C$.
As dist.	$= 1024$
To Rad	$= 90^{\circ} 00'$
So diff. lat.	$= 858$

To cof. cour. $= 33^{\circ} 05'$

For the diff. longitude.

C : F :	P : L .
As cof. m. lat. $= 44^{\circ} 09'$	0.14417
To tang. cour. $= 33^{\circ} 05'$	9.81390
So diff. lat. $= 858$	2.93349

To diff. long. $= 779$

Now $22^{\circ} 06' W. =$ Long. from.
 $60) 779 (12\ 59\ E. =$ Diff. long.

Course SE. b. S. nearly.

$9\ 07\ W. =$ Pref. long.

By the TRAVERSE TABLE.

With the distance and diff. lat. in their columns, find the course among the degrees, and the dep. in its column.

With the co-middle lat. among the degrees, and the dep. in its column, find the diff. long. among the distances.

Thus. To the dist. $64 \left(= \frac{1024}{16} \right)$, and the lat. $53.62 \left(= \frac{858}{16} \right)$,

the course is $33^{\circ} 00'$; and the dep. 34.86 .

The co-mid. lat. 46° (the nearest to $45^{\circ} 51'$), the dep. 34.86 (falling between 34.53 and 35.24 , belonging to the distance 48 and 49), gives in the col. of distances 48.5 ; which mult. by 16 gives 776 for the diff. lon.

Note, $\frac{1}{16}$ th of the numbers were taken rather than $\frac{1}{10}$, because 16 divides 1024 without a fraction ; and it is more convenient to seek a diff. in whole numbers, than with a fraction annexed.

63. By

63.	By MERCATOR'S SAILING.		
Lat. from	51° 18' N.	Mer. parts	3597
Lat. to	37 00 N.	Mer. parts	2393
Diff. lat.	14 18=858 m.	Mer. diff. lat.	1204

By CONSTRUCTION. Fig. 20. Plate XIII.

In the meridian AB, take AB=1204, the meridional diff. lat. and AD=858, the proper diff. lat.: Describe the quadrant A m p; from A, with the distance 1024, cut DC, drawn parallel to A p, in c; draw AC, which continue till it meets BE, drawn parallel to A p, in E.
Then BE=diff. long. and arc m c measures the course.

COMPUTATION by Meridional Parts.

For the course.

AC : Rad :: AD : f. ∠ACD.	Rad. : T :: M : L.
As dist. = 1024	As Rad = 45° 00' 10,0000
To Rad = 90° 00' 10,0000	To tang. cour. = 33 05 9,81390
So p. diff. lat. = 858	So m. diff. lat. = 1204 3,08063
To col. cour. = 33° 05' 9,92319	To diff. long. = 784,4 2,89453

Now 22° 06' W. = Long. from.
60) 784 (13 04 E. = Diff. long.

Course SE. by S. nearly.

9 02 W. = Long. to.

COMPUTATION by Logarithmic Tangents.

Depart. lat. 51° 18'.	Co-lat. 38° 42'.	$\frac{1}{2}$ co-lat. 19° 21'.
Lat. bound to 37 00.	Co-lat. 53 00.	$\frac{1}{2}$ co-lat. 26 30.

To $\frac{1}{2}$ co lat. 26° 30', its log. tang.	=	9,69774
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.	=	9,54552

Difference = 1523,2 = G.

For the course.

AC : Rad :: AD : f. ∠ACD.

For the diff. longitude.		
N : G :: T : L.	As N =	9,89849
	To G =	3,18241
Course is S. 33° 05' E. as above.	So tang. cour. = 33° 05'	9,81390
	To diff. long. = 784,9	2,89480

By the TRAVERSE TABLE.

With the distance and diff. lat. in their columns, find the course among the degrees.

With the course among the degrees, and the meridional diff. lat. in the column of latitude, find the diff. lon. in the column of departure.

The course found as before is 33° 00'.

And $\frac{1}{2}$ th of the meridional diff. lat. viz. 60,2, found under 33° in the column of latitude, the nearest corresponding departure is 39,21; which multiplied by 20 gives 784,2 for the diff. longitude.

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PROBLEM VII.

64. Given the latitudes of two places, and the departure:
Required the course, distance, and difference of longitude.

EXAM. *A ship from the latitude 37° 00' N., longitude 48° 20' W., sails between the N. and E., and being come to the latitude 51° 18' N., finds she has made 564 miles of departure: What was her direct course, distance run, and present longitude?*

By MIDDLE LATITUDE SAILING.

Present lat.	51° 18' N.	Sum latitudes	88° 18'
Departed lat.	37 00 N.		
Diff. lat.	14 18=358 m.	Mid. lat.	44 09
		Co-mid. lat.	45 51

By CONSTRUCTION. Fig. 13. Plate XIII.

In a meridian line, take $AD=858$ m., the diff. lat. Describe the quadrant $A m p$, and parallel to $A p$ draw $DC=564$ m. the departure; also draw the distance AC : In the arc EF , described with the sine of the co-mid. lat. $45^\circ 51'$, apply the chord EF equal to DC ; through F draw AB ; then the chord mb will be the diff. long., and the arc mc measures the course.

By COMPUTATION.

To find the course.

$$AD : DC :: Rad : t. \angle DAC.$$

$$As \text{ diff. lat. } = 858$$

$$To \text{ depart. } = 564$$

$$So \text{ Rad } = 45^\circ 00'$$

$$To \text{ tang. cour. } = 33 \quad 19$$

$$9,81779$$

$$To \text{ dist. } = 1027$$

$$3,01150$$

To find the distance.

$$\sin. \angle DAC : DC :: Rad : AC.$$

$$As \text{ sin. cour. } = 33^\circ 19' \quad 0,26022$$

$$To \text{ depart. } = 564 \quad 2,75128$$

$$So \text{ Rad } = 90^\circ 00' \quad 10,00000$$

$$To \text{ dist. } = 1027 \quad 3,01150$$

To find the diff. longitude.

$$C : R :: D : L.$$

$$As \text{ cos. m. lat. } = 44^\circ 09' \quad 0,14417$$

$$To \text{ Rad } = 90^\circ 00' \quad 10,00000$$

$$So \text{ depart. } = 564 \quad 2,75128$$

$$To \text{ diff. long. } = 786$$

$$2,89545$$

Or $C : T :: P : L.$

$$\text{Now } 48^\circ 20' W. = \text{Dep. long.}$$

$$60) 786 \quad (13 \quad 06 \text{ E.} = \text{Diff. long.}$$

$$35 \quad 14 W. = \text{Pres. long.}$$

Course NE. b. N. nearly.

By the TRAVERSE TABLE.

The diff. lat. and departure being found together, will give the course among the degrees, and the distance in its column.

With the co-mid. lat. among the degrees, and the departure in its column, find the diff. longitude among the distances.

Now $\frac{1}{2}$ th of 858 , the diff. lat., and of 564 , the dep., are $42,9$ and $33^\circ 30'$, and to the distance 51 ; which multiplied by 20 , gives 1020 .

Also the departure $28,2$ being found to the co-mid. latitude 46° , gives 39 ; which multiplied by 20 gives 780 for the diff. longitude.

But the parts being proportioned, gives $51,2$ for the distance; and $39,2$ for the diff. longitude; these multiplied by 20 , gives 1024 and 784 .

By

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65. BY MERCATOR'S SAILING.

Present lat.	51° 18' N.	Mer. parts	3597
Departed lat.	37 00 N.	Mer. parts	2393
Diff. lat.	14 18 = 858 m.	Mer. diff. lat.	1204

By CONSTRUCTION. Fig. 21. Plate XIII.

In a meridian line take $AB=1204$ m., the meridional diff. lat.; and $AD=858$ m., the prop. diff. lat. Describe the quadrant $Am p$; through D draw DC parallel to Ap , and make $DC=564$ m., the dep. Through C draw AE meeting BE drawn parallel to Ap .

Then is AC the distance, BE the diff. longitude; and the arc mc measures the course.

COMPUTATION by Meridional Parts.

For the course.

AD : DC :: Rad : t. $\angle DAC$.	fin. $\angle DAC$: DC :: Rad : AC.
As diff. lat. = 858	As fin. cour. = 33° 19'
To depart. = 564	To depart. = 564
So Rad = 45° 00'	So Rad = 90° 00'
To tang. cour. = 33 19	To diff. = 1027
	3,01150

For the distance.

For the diff. long.

As prop. diff. lat. = 858	Now 48° 20' W. = Dep. long.
To depart. = 564	60) 791 (13 11 E. = Diff. long.
So mer. diff. lat. = 1204	35 09 W. = Pref. long.
To diff. long. = 791,5	Course NE. b. N. nearly.

COMPUTATION by Logarithmic Tangents

Departed lat. 37° 00' N.	Co-lat. 53° 00'.	$\frac{1}{2}$ co-lat. 26° 30'.
Present lat. 51 18 N.	Co lat. 38 42.	$\frac{1}{2}$ co-lat. 19 21.
To $\frac{1}{2}$ co-lat. 26° 30', its log. tang.	= 9,69774	
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.	= 9,54552	
Difference =	1522,2 = G	

For the diff. longitude.

The course and distance are found as above.	N : G :: T : L.
	As N = 9,89849
	To G = 1522
	So tang. cour. = 33° 19'
	To diff. long. = 791,9
	2,89866

By the TRAVERSE TABLE.

The course found as before is 30° 30'; under which against 60,2 ($=\frac{1}{5}$ of the mer. diff. lat.) in the column of latitude, stands 39,74 in the column of departure. Then $39,74 \times 20 = 794,8$, the diff. longitude.

PROBLEM VIII.

66. Given one latitude, the course and departure :
Required the other latitude, distance, and diff. longitude.

EXAM. A ship from the latitude $51^{\circ} 18' N$, longitude $9^{\circ} 50' W$, sails
S. $33^{\circ} 19' W$, until her departure is 564 miles : Required her present latitude,
longitude, and distance ?

By MIDDLE LATITUDE SAILING.

By CONSTRUCTION. Fig. 14. Plate XIII.

Draw a meridian line AD; describe the quadrant AMP; make $AP = 564$, the departure, and the arc $mc = 33^{\circ} 19'$, the course; through c draw the rhumb AC meeting AC, drawn parallel to AD, in c; and draw CD parallel to AP.

By COMPUTATION.

For the diff. latitude.

fin. $\angle DAC : CD ::$ fin. $\angle ACD : AD$.	Now $51^{\circ} 18' N. =$ Dep. lat.
As fin. cour. $= 33^{\circ} 19'$	60) 858 (14 18 S. = Diff. lat.
To depart. $= 564$	
So cos. cour. $= 33^{\circ} 19'$	37 00 N. = Present lat.
	88 18 = Sum lat.
To diff. lat. $= 858$	44 09 = Middle lat.
	45 51 = Co-m. lat.

Then in the arc EF, described with the sine of $45^{\circ} 51'$, apply CD from E to F; through F draw AB; and draw the diff. longitude BM.

To find the distance.

fin. $\angle DAC : CD ::$ Rad : AC.	As c : Rad :: D : L.
As fin. course $= 33^{\circ} 19'$	As cos. mid. lat. $= 44^{\circ} 09'$
To departure $= 564$	To Rad $= 90$
So Rad $= 90^{\circ} 00'$	So departure $= 564$
To distance $= 1027$	To diff. long. $= 786$

To find the diff. longitude.

As c : Rad :: D : L.	As cos. mid. lat. $= 44^{\circ} 09'$
As cos. mid. lat. $= 44^{\circ} 09'$	To Rad $= 90$
To Rad $= 90$	So departure $= 564$
So departure $= 564$	To diff. long. $= 786$
To diff. long. $= 786$	

Now $9^{\circ} 50' W. =$ Dep. long.
60) 786 (13 06 W. = Diff. long.

22 56 W. = Present long.

By the TRAVERSE TABLE.

With the course ($33^{\circ} 15'$ the nearest to $33^{\circ} 19'$) and the departure ($28,2 = \frac{1}{18}$ th of 564) in its column, find in the column of latitude the diff. lat. (42,53) and in the column of distance find the distance (51).

Then $42,53 \times 20 = 850,6$; and $51 \times 20 = 1020$.

Hence the present lat. is $37^{\circ} 07' N.$, and the co-mid. lat. $45^{\circ} 48'$.

The co-mid. lat. $45,40$ and departure $28,2$ gives in the column of distances 39, which multiplied by 20, gives 780, for the diff. of longitude.

But proportioning for the difference, gives 39,2; and $39,2 \times 20 = 784$, for the diff. longitude.

67. By

67. By MERCATOR'S SAILING.

By CONSTRUCTION. Fig. 22. Plate XIII.

Draw a meridian line AB ; describe the quadrant Amp ; make $Aa = 564$, and the arc $mc = 33^\circ 19'$; draw ac parallel to AB meeting the rhumb AC , drawn through c , in c ; and draw the departure CD parallel to Ap .

COMPUTATION by Meridional Parts.

<i>For the diff. latitude.</i>		<i>Now</i>	
$\sin. \angle DAC : CD :: \sin. \angle DCA : AD.$		$51^\circ 18' N.$	$=$ Dep. lat.
As $\sin. \text{cour.} = 33^\circ 19'$	$0,26022$	$60) 858$ (14 18 S.	$=$ Diff. lat.
To depart. $= 564$	$2,75128$	$37 \ 00 \ N.$	$=$ Pref. lat.
So $\text{col. cour.} = 33^\circ 19'$	$9,92202$	To $51^\circ 18'$ merid. parts $= 3598$	
		To $37 \ 00$ merid. parts $= 2394$	
To diff. lat. $= 858$	$2,93352$	Merid. diff. lat. $= 1204$	

Then make $AB = 1204$; draw BE parallel to Ap , meeting AC continued, in E ; and BE is the diff. of longitude.

To find the distance.

As $\sin. \angle DAC : DC :: \text{Rad} : AC.$	<i>P : D :: M : L.</i>		
As $\sin. \text{cour.} = 33^\circ 19'$	$0,26022$	As p. diff. lat. $= 858$	$7,06651$
To depart. $= 564$	$2,75128$	To depart. $= 564$	$2,75128$
So $\text{Rad.} = 90^\circ 00'$	$10,00000$	So mer. diff. lat. $= 1204$	$3,08063$
To distance $= 1027$	$3,01150$	To diff. long. $= 791,4$	$2,89842$

To find the diff. longitude.

$P : D :: M : L.$	
As p. diff. lat. $= 858$	$7,06651$
To depart. $= 564$	$2,75128$
So mer. diff. lat. $= 1204$	$3,08063$

Now $9^\circ 50' W. =$ Dep. long.
 $60) 791$ (13 11 W. $=$ Diff. long.

$23 \ 01 \ W. =$ Pref. long.

COMPUTATION by Logarithmic Tangents.

Find the diff. latitude and present latitude as above.

Then, Dep. lat. $51^\circ 18'.$ Co-lat. $38^\circ 42'.$ $\frac{1}{2}$ co lat. $19^\circ 21'.$
 Pref. lat. $37 \ 00.$ Co-lat. $53 \ 00.$ $\frac{1}{2}$ co-lat. $26 \ 30.$

To $\frac{1}{2}$ co-lat. $26^\circ 30'$, its log. tang. is $= 9,69774$
 To $\frac{1}{2}$ co-lat. $19 \ 21$, its log. tang. is $= 9,54552$

Difference $= 1522,2 = G.$

To find the diff. longitude. $N : G :: T : L.$

As N	$= 9,89849$	The distance and present longitude may be found as above.
To G	$= 1522$	
So tang. cour. $= 33^\circ 19'$	$9,81776$	
To diff. long. $= 791,8$	$2,89866$	

By the TRAVERSE TABLE.

Find as before the diff. lat. $= 851$; and the distance $= 1020.$

Then the present lat. is $37^\circ 07' N.$; and the mer. diff. lat. is 1197 ; $\frac{1}{2}$ th of which is $59,85.$

Now under the course $33^\circ 15'$, and against $60,21$, the nearest to $59,85$ in the column of latitude, stands in the column of dep. $39,48$; which multiplied by 20 , gives $789,6$ for the diff. longitude.

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PROBLEM IX.

68. Given one latitude, the distance and departure:
Required the other latitude, course, and diff. longitude.

EXAM. A ship from the latitude $37^{\circ} 00' N$, longitude $9^{\circ} 02' W$, having sailed between the N. and W. 1027 miles, reckons she has made 564 miles of departure: What was her direct course, and the present latitude and longitude?

By MIDDLE LATITUDE SAILING.

By CONSTRUCTION. Fig. 15. Plate XIII.

Draw the meridian line AD; describe the quadrant A m p; make A a = 564 m.; from A with 1027, the distance, cut AC drawn parallel to AD in c; draw AC, and draw the departure CD parallel to Ap.

By COMPUTATION.

To find the Course.

AC : Rad :: CD : sin. $\angle DAC$.
As dist. = 1027 6,98843
To Rad = $90^{\circ} 00'$ 10,00000
So depart. = 564 2,75128

To sin. cour. = $33^{\circ} 19'$ 9,73971

Now $37^{\circ} 00' N$. = Dep. lat.
60) 858 (14 18 N. = Diff. lat.

51 18 N. = Pref. lat.

To find the diff. latitude.

Rad : AC :: sin. $\angle DCA$: AD.
As Rad = $90^{\circ} 00'$ 10,00000
To dist. = 1027 3,01157
So cof. cour. = $33^{\circ} 19'$ 9,52202

To diff. lat. = $85^{\circ} 8,2$ 2,93359

Sum latitudes $88^{\circ} 18'$

Mid. lat. 44 09

Co-mid. lat. 45 51

Then describe the arc EF with the sine of $45^{\circ} 51'$, and therein apply CD from E to F; through F draw AB; and draw the diff. long. B m.

To find the diff. longitude.

As cof. m. lat. = $44^{\circ} 09'$ 0,14417
To tang. cour. = $33^{\circ} 19'$ 9,81776
So diff. lat. = $85^{\circ} 8,2$ 2,93359

To diff. long. = 786,2 2,89553

C : T :: P : L.

Now $9^{\circ} 02' W$. = Dep. long.
60) 786 (13 06 W. = Diff. long.

22 08 W. = Pref. long.

Course is NW. b. N. nearly.

By the TRAVERSE TABLE.

Now $\frac{1}{2}$ th of the distance, and of the departure, are 51,35, and 28,2. Find in some column of departure, the nearest number to the given departure, (28,2) standing against the given distance (51); which then gives the course ($33^{\circ} 30'$) among the degrees; and the diff. lat. (42,53) in its column.

Then $42,53 \times 20 = 850,6$: Hence the present latitude is $51^{\circ} 11' N$.; and the co-middle latitude is $45^{\circ} 55'$.

With the nearest co-middle latitude (46°) and the departure (28,2) find among the distances (39) the diff. longitude.

Then $39 \times 20 = 780$, for the diff. longitude.

But by proportioning the parts, the diff. lat. will be found to be 856; the diff. long. 784.

69. By

69. By MERCATOR'S SAILING,

By CONSTRUCTION. Fig. 23. Plate XIII.

Draw a meridian AB; describe a quadrant Δmp ; make $\Delta a = 564$; draw ac parallel to AB; from A with 1027, the distance, cut ac in c ; draw the departure CD parallel to ap , and draw the distance AC.

COMPUTATION by Meridional Parts.

To find the course.

AC : Rad :: CD : fin. $\angle DAC$.	Rad : AC :: fin. $\angle DCA$: AD.
As dist. = 1027	As Rad. = 90° 00'
To Rad = 90° 00'	To dist. = 1027
So depart. = 564	So col. cour. = 33° 19'
To fin. cour. = 33° 19'	To diff. lat. = 858,2
	2,93359

Now 37° 00' N. = Dep. lat. To 51° 18', the mer. parts = 3598
60) 858 (14 18 N. = Diff. lat. To 37° 00', the mer. parts = 2394

51 18 N. = Pref. lat.

Mer: diff. lat. = 1204

Then make AB=1204; continue AC till it meets BE drawn parallel to ap ; and BE is the diff. longitude.

To find the diff. longitude.

Rad : T :: M : L.

As Rad. = 45° 00' 10,00000 Now 9° 02' W. = Long. from.
To tang. cour. = 33 19 9,81776 60) 791 (13 11 W. = Diff. long
So m. diff. lat. = 1204 3,08063

To diff. long. = 791,4

22 13 W. = Pref. long.

Courfe NW. b. N. nearly.

COMPUTATION by Logarithmic Tangents.

Find the course and present latitude as above.

Then, Dep. lat. 37° 00' N. Co-lat. 53° 00'. $\frac{1}{2}$ co-lat. 26° 30'.
Pref. lat. 51 18 N. Co-lat. 38 42. $\frac{1}{2}$ co-lat. 19 21.

To $\frac{1}{2}$ co-lat. 26° 30', its log. tang. = 9,69774
To $\frac{1}{2}$ co-lat. 19 21, its log. tang. = 9,54552

Difference = 1522,2 = G.

To find the diff. longitude.

N : G :: T : L.

As N = 9,89849

To G = 1552

So tang. cour. = 33° 19'

To diff. long. = 791,8

The present longitude may be found as above.

By the TRAVERSE TABLE.

Find as before, by proportioning the parts, the diff. lat. = 856, and the course N. 33° 30' W.

Hence the present lat. is 51° 16' N.; and the mer. diff. lat. = 1200, $\frac{1}{2}$ th whereof is 60.

Then in the column of latitudes, under 33° 30', find 60,04, the nearest to 60; against which, in the departure, stands 39,74; which multiplied by 20, gives 794,8 for the diff. of longitude.

PROBLEM

70. PROBLEM X.

Given one latitude, departure and diff. longitude:

Required the other latitude, course and distance.

EXAM. *A ship from the latitude $51^{\circ} 18' N.$, longitude $22^{\circ} 06' W.$, having sailed between the S. and E. for several days, reckons she has made 564 miles of departure, and 786 miles of longitude: What is the lat. and long. of the place arrived at, and also her direct course and distance?*

By MIDDLE LATITUDE SAILING.

By CONSTRUCTION. Fig. 16. Plate XIII.

Draw a meridian AD ; describe the quadrant AMP ; in the arc mp apply the chord $mp = 786$, the diff. long.; draw AB ; make $ma = 564$; draw af parallel to $A m$, and with the radius AF describe the arc FE , and draw the chord EF .

Then is EF = departure, and AE = sine of the co-mid. lat.

By COMPUTATION.

To find the co-mid. latitude.

Now Co-mid. lat. = $45^{\circ} 51'$

$l : d :: Rad : c.$

Middle lat. $44^{\circ} 09'$

As diff. long. = 786
To depart. = 564
So Rad. = $90^{\circ} 00'$ 10,00000

Sum lats. 88 18
Depart. lat. $51^{\circ} 18'$

To col. m. lat. = $44^{\circ} 09'$ 9,85586

Present lat. 37 00

To col. m. lat. = $44^{\circ} 09'$ 9,85586

Diff. lat. $14^{\circ} 18' = 858m.$

Make $AD = 858$; draw DC parallel to $A p$, and equal to EF ; then AC being drawn, will be the distance; and $\angle DAC$ the course.

To find the course.

To find the distance.

$P : L :: C : T.$

fin. $\angle DAC : DC :: Rad : AC.$

As diff. lat. = 858 7,06651
To diff. long. = 786 2,89542
So col. m. lat. = $45^{\circ} 51'$ 9,85583

As fin. cour. = $33^{\circ} 19'$ 0,26022
To depart. = 564 2,75128
So Rad = $90^{\circ} 00'$ 10,00000

To tang. cour. = 3319 9,81776

To dist. = 1027 3,01150

Now $22^{\circ} 06' W.$ = Dep. long.
60) 786 (13 $06^{\circ} E.$ = Diff. long.

The course is SE. b. S. nearly.

Distance 1027 miles.

Present lat. $37^{\circ} 00' N.$

By the TRAVERSE TABLE.

Now $\frac{1}{15}$ th of the diff. long. and dep. are 39,3 and 28,2.

Seek in the table, till the given depart. 28,2 is found in its column against the given diff. long. 39 in the column of distance; and it gives the co-mid. lat. among the degrees.

Here 28,2 falls between 28,05 over 46° , and 27,89 over $45^{\circ} 40'$; therefore taking the co-mid. lat. = $45^{\circ} 50'$, the present lat. is $37^{\circ} 02' N.$; the diff. lat. is 856; and its $\frac{1}{10}$ th is 42,8.

Then 42,8 and 28,2 will be found together under $33^{\circ} 30'$, the course, and between 51 and 52 in the distances; suppose 51,2, which multiplied by 20, gives 1024 for the distance.

PROBLEM

PROBLEM XI.

- 71. Given one latitude, course, and difference of longitude :
Required the other latitude and distance.**

EXAM. A ship from the lat. $37^{\circ} 00' N$, longitude $22^{\circ} 56' W$, runs on a course $N. 33^{\circ} 19' E$. until she finds her difference of longitude is 786 miles: What is her present latitude and distance sailed?

This problem cannot be resolved by middle latitude sailing.

By MERCATOR'S SAILING.

BY CONSTRUCTION. Fig. 24. Plate XIII.

Draw the meridian AB ; describe the quadrant Amp ; continue Ap till $Aa=786$ miles, the diff. longitude, and draw Aa parallel to AB . Make the arc $mc=33^{\circ} 19'$; draw the rhumb Aa till its meets Aa in E , and draw EB parallel to Ap .

COMPUTATION by Meridional Parts.

To find the mer. diff. latitude

T	:	R	:	L	:	M.	
As tang. cour.	=	33° 19'		0,18224			
To Rad	=	45 00		10,00000			
So diff. long.	=	786		2,89542			
Tom. diff. lat.	=	1196					
					Dep. lat.	37° 00' N.	Mer. pts. 2394
						Mer. d. lat.	1196
					Pref. lat.	51 14 N.	M. pts. 3590
					Diff. lat.	14 14	= 854 miles.

Make $AD=854$; draw DC parallel to AP ; then AC is the distance.

To find the distance. $\sin. \angle DCA : AD :: \text{Rad} : AC.$

As col. cour.	= 33 ^s	19'	0,07798
To diff. lat.	= 85 ⁴		2,93146
So Rad.	= 90 ⁰	00'	10,00000
To distance	= 1022		3,00944

COMPUTATION by Logarithmic Tangents.

To find the present latitude.

T : L :: N :	G.
Astang. cour.	= 33° 19' O, 18224
To diff. long.	= 786 2, 89542
So N =	= 10, 10151
To c	= 1511
Co-lat.	38 48.

Therefore the pref. lat. is $51^{\circ}12'N$.

By the TRAVERSE TABLE.

Seek the diff. long. in the column of departure to the given course, and it gives in the column of latitude the meridional diff. lat.

Thus under $33^{\circ} 15'$, against 39,3 ($=\frac{1}{2}$ th of 786) in the departure stands 59,38; or by proportioning 59,86, which multiplied by 20, gives 1197 for the mer. diff. lat. Hence the present lat. is $51^{\circ} 14' N.$; and the diff. lat. 854; its $\frac{1}{2}$ th is 427.

Then under $33^{\circ} 15'$, 42,7 in the col. of lat. stands against 51 in the col. of distances; which multiplied by 20, gives 1020 for the distance.

PROBLEM

P R O B L E M XII.

72. Given the course, departure, and difference of longitude :
Required the latitudes departed from and arrived at ?

EXAM. From a place in north latitude, a ship sails S. $33^{\circ} 15'$ E. until she has made 564 miles of departure, and 786 miles of difference of longitude : What were the latitudes sailed from and arrived in ?

S O L U T I O N.

As the sine of the course, is to the departure

So is the cosine of the course, to the difference of the latitude $= 860$

And as the diff. longitude, is to the departure

So is the Radius, to the cosine of the middle latitude $= 44^{\circ} 09'$

Now $\frac{860}{60} = 14^{\circ} 20'$ the diff. lat. the half is $7^{\circ} 10'$

Then $44^{\circ} 09' + 7^{\circ} 10' = 51^{\circ} 19'$, the latitude sailed from ;

And $44^{\circ} 09' - 7^{\circ} 10' = 36^{\circ} 59'$, the latitude arrived in.

Hence any two of these four particulars being given, viz.

Course, distance, departure, difference of latitude,

together with the difference of longitude :

The two latitudes may thence be found.

P R O B L E M XIII.

73. Given one latitude, distance, and difference of longitude.
Required the course, and the present latitude ?

S O L U T I O N.

1. Find the meridional distance (37) to use as a first departure.

When the course is towards the equator, augment this mer. dist. for a second departure ; if towards the pole diminish it for a 2d. departure. Let the 1st. dep. be altered by about a 5th or 6th of itself for a 2d. departure.

2. With the given distance and these departures taken separately in order ; Find the respective courses, and differences of latitudes. (VII. 32)

And hence, the present latitudes and mer. differences of lat. (52)

3. To each course and respective mer. diff. lat. find a diff. long. (48)

4. Between each of these differences of longitude, taken in order, and that given, take the two differences ; which call the 1st. and 2d. errors.

Take the product of the 1st. departure by the 2d. error, and the product of the 2d. departure by the first error.

5. If the two differences of longitude found by 3d art. were both too great, or both too small; let the difference of the said two products, be divided by the difference of the errors, and the quotient may be taken for the true departure.

But if the two differences found, were one too great, and one too small; Let the sum of the said products be divided by the sum of the said errors, and the quotient may be taken for the true departure.

6. With the true dep. and dist. find the true course and diff. latitude. (VII. 32)

EXAM. *A ship from the latitude of 37° N. having sailed for some days on a direct course in the NW. quarter, finds that she has run 1027 miles, and has made 790 miles of difference of longitude: on what course did she steer, and what latitude is she come to?*

With radius and the diff. long. 790, gives a mer. dist. 631, which diminished by about its $\frac{1}{8}$ th, or by 100, gives 531.

Then 631 and 531 may be taken for the 1st. and 2d. departures.

The distance 1027, and these departures, give the courses 37° 45' and 31°, and also the differences of latitude 814 and 883; or 13° 34' and 14° 43'.

Hence the supposed latitudes are 50° 34' and 51° 43'; and the meridional differences of latitude will be 1135 and 1245.

With 1135 and 1245, taken with the courses 37° 45' and 31°, the diff. long. are 879 and 748.

And $879 - 790 = 89$, the 1st. error, too great:

Also $790 - 748 = 42$, the 2d. error, too small.

The sum of these errors is 131, the divisor.

Now $631 \times 42 = 26502$; and $531 \times 89 = 47259$; the sum of these products is 73761.

Then $73761 \div 131$ gives 563 for the true departure.

With the dist. 1027 and dep. 563, the course is N. 33° 15' W.

The diff. lat. is 860 m. $= 14^{\circ} 20' N$.

And the present latitude is $51^{\circ} 20' N$.

SECTION V.

Of compound Courses corrected by Longitude.

How to solve compound courses or traverses, on the principles of plane sailing, has been already shewn : But it is necessary also, to shew how the proper allowances for longitude are to be introduced into such accounts ; which is easily done by either of the following methods.

74.

RULE I.

1st. Complete the traverse table to each course and distance, and find the whole diff. latitude, departure, and present latitude. (VII. 36)

2d. With the diff. latitude and departure, find the direct course and distance. (VII. 33)

3d. Working by middle latitude, with the departed latitude and present latitude find the co-mid. lat. ; with which, and the departure, find the difference of longitude. (64)

Or 4th. Working by Mercator, with the departed latitude and present latitude find the meridional difference of latitude ; with which, and the direct course, find the difference of longitude. (59)

75.

RULE II. *By Middle Latitude.*

1st. To the traverse table (completed as above) annex seven other cols. ; title them dep. lats., pref. lats., sum lats., mid-lats, co-mid. lats., diff. longit.

E.	W.
----	----

2d. By the several differences of latitude found in the traverse table, fill up the first and second columns ; and hence the corresponding sum lats., mid-lats., and co-mid. lats. will be obtained.

3d. With each. co-mid. lat. and corresponding departure, find the diff. of longitude to each course (64), and set them in the columns marked E. W., according to the name of the departure used.

4th Add up both the east and west columns of longitude, and the difference of their sums will be the whole difference of longitude.

76.

RULE III. *By Mercator.*

1st. To the traverse table (completed as above) annex five other columns ; title them successive latitudes, meridional parts, mer. diff. lat., diff. long.

E.	W.
----	----

2d. By the several differences of latitude, found in the traverse table, fill up the column of successive latitudes ; set their meridional parts in its column ; and the meridional differences in its column.

3d. To each course, and its corresponding meridional diff. lat., find (59) the diff. longitude ; which set in its proper column.

4th. The difference of the sums of the east and west columns of longitude will be the whole difference of longitude.

The first of these rules is in common practice at sea ; but either of the latter gives the result more accurate.

The operations, by these rules, may be all performed by the traverse table at the end of Book VII.

77. EXAMPLE I. Yesterday noon we were in latitude $38^{\circ} 14' N.$, longitude $25^{\circ} 56' W.$, and have since run the following courses and distances.
 1st NE. b. N. $\frac{1}{2}$ E. 56 m.; 2d. NNW. 38 m.; 3d. NW. b. W. 46 m.; 4th. SSE. 30 m. 5th. S. b. W. 20 m.; 6th. NE. b. N. 60 m.
 Required the ship's place, with her direct course and distance?

By the FIRST RULE.

The TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
NE. b. N. $\frac{1}{2}$ E. 56	45,0			33,4	
NNW. 38	35,1				14,5
NW. b. W. 46	25,6				38,2
SSE. 30		27,7	11,5		
S. b. W. 20			19,6		
NE. b. N. 60	50,0		33,3		
	155,7	47,3	78,2	56,6	
Diff. lat.	108,4	Dep.	21,6		

Hence (VII. 33) the course is N. b. E., distance 111 miles.

By MIDDLE LATITUDE.

Dep. lat. = $38^{\circ} 14' N.$	Mid. lat. = $39^{\circ} 8'$
Diff. lat. = $1^{\circ} 48' N.$	Co. mid. lat. = $50^{\circ} 52'$
Present lat. = $40^{\circ} 2' N.$	Dep. long. = $25^{\circ} 56' W.$
Sum lats. = $78^{\circ} 16'$	Diff. long. = $0^{\circ} 28' E.$ (64)

By MERCATOR.

Dep. lat. $38^{\circ} 14' N.$ M. pts. = 2486,1	Depart. long. = $25^{\circ} 56' W.$
Pres. lat. $40^{\circ} 2' N.$ M. pts. = 2625,3	Diff. long. = $0^{\circ} 28' E.$ (59)
Mer. diff. lat. = 139,2	Present long. = $25^{\circ} 28' W.$

EXPLANATION.

The diff. lat. 108,4 and the dep. 21,6 being sought for, standing together, in the traverse table (VII. 67) will be found under one point, or $11^{\circ} 15'$ for the course, and against 111 miles for the distance.

The co-mid. lat. $50^{\circ} 52'$, or its nearest $50^{\circ} 45'$, being found in the traverse table (VII. 67), and the departure 21,6 in the dep. column; then right against it, in the col. of dist., stands 28 for the diff. long.

The course $11^{\circ} 15'$ being found among the deg. at top in the traverse table (VII. 67), and the mer. diff. lat. 139,2 (or its $\frac{1}{2}$, viz. 69,6) found in the col. of lat., gives a corresponding dep. 28, ($= 13,85 \times 2$) for the difference of longitude.

By

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78. EXAMPLE II. Suppose a ship from the latitude $52^{\circ} 20' N.$, and longitude $14^{\circ} 38' W.$, has sailed the following courses, viz.

1st. ESE. 43 m. 2d. SW. 32 m. 3d. SE. b. S. 58 m.

4th. SSW. 60 m.

Required her present latitude and longitude, also her direct course and distance?

By the FIRST RULE.

The TRAVERSE TABLE.					
Courses.	D.	N.	S.	E.	W.
ESE.	—	—	16,5	39,7	—
SW.	43	—	22,6	—	22,6
SE. b. S.	32	—	48,2	32,2	—
SSW.	58	—	55,4	—	23,0
	60	—	—	—	—
Diff. lat.		= 142,7		71,9	45,6
Departure =					26,3

Hence (VII. 33) the course is $S. 10^{\circ} 30' E.$ distance 146 miles.

To find the difference of Longitude.

By MIDDLE LATITUDE.

Dep. lat.	= $52^{\circ} 20' N.$	Mid. lat.	= $51^{\circ} 8'$
Diff. lat.	= $23 S.$	Co-mid. lat.	= $38 52$
Present lat.	= $49 57 N.$	Dep. long.	= $14 38 W.$
Sum lats.	= $102 17$	Diff. long.	= $0 42 E. (64)$

By MERCATOR.

Dep. lat. $52^{\circ} 20' N.$	Mer. parts 3698	Depart. long.	= $14 38 W.$
Pres. lat. $49 57 N.$	Mer. parts 3470	Diff. long.	= $0 42 E. (59)$
Mer. diff. lat. 228	Present long.	= $13 56 W. (VI. 53)$	

EXPLANATION.

The halves of the diff. lat. and dep., viz. $71,35$ and $13,15$, being sought for, standing together, in the traverse table (VII. 67), their nearest numbers, viz. $71,73$ and $13,30$, stand under $10^{\circ} 30'$ for the course; and against 73 in the column of dist., which doubled, as the table was entered with halves, give 146 for the direct distance made good.

The co-mid. lat. $38^{\circ} 52'$, or its nearest $38^{\circ} 45'$, being sought in the traverse table (VII. 67); then in the column signed Dep. the number nearest to $26,3$ (the dep. by this question), stands against 42 in the col. of dist.; which is to be taken for the difference of longitude.

Or thus, Seek in the traverse table (VII. 67) under the direct course $10^{\circ} 30'$, and in the column signed Lat., for the nearest number to the mer. diff. lat. ($= 228$, or to some part thereof, viz. its $\frac{1}{4} = 57$; then the corresponding departure is $10,57$; which taken four times, gives $42,28$ for the difference of longitude.

Vol. II.

By

By the SECOND RULE.

TRAVERSE TABLE.					LONGITUDE TABLE.						
Courses.	D.	N.	S.	E.	W.	Dep. lats.	Pref. lats.	Sum lats.	Mid. lats.	Co-m. lats.	D. lon. E. W.
ESE.	43		16,5	39,7		52° 20'	52° 04'	104° 24'	52° 12'	37° 48'	65
SW.	32		22,6		22,6	52 04	51 41	103 45	51 52	38 08	37
SE. b. S.	58		48,2	32,2		51 41	50 53	102 34	51 17	38 43	51
SSW.	60		55,4		23,0	50 53	49 58	100 51	50 26	39 34	36
Diff. lat. =					142,7						116 73
Depart. =					26,3						43
Diff lon. =											

EXPLANATION.

Dep. lat. $52^{\circ}20'N$., and diff. lat. $16 S$., give pref. lat. $52^{\circ}04'N$.
 Dep. lat. $52^{\circ}04'N$., and diff. lat. $23 S$., give pref. lat. $51^{\circ}41'N$.
 Dep. lat. $51^{\circ}41'N$., and diff. lat. $48 S$., give pref. lat. $50^{\circ}53'N$.
 Dep. lat. $50^{\circ}53'N$., and diff. lat. $55 S$., give pref. lat. $49^{\circ}58'N$.
 Which completes the first two columns of the longitude table.

The sums of the opposite dep. lat. and pref. lat. make the 3d column.

The halves of these sums make the mid. lats. in the 4th column.

And their complements make the 5th column.

Then with these co-mid. lats. and the before found departures, the differences of longitude being found (64), and set in their proper columns; the difference of the sums of the columns of longitude gives the whole differences of longitude.

By the THIRD RULE.

TRAVERSE TABLE.						LONGITUDE TABLE.			
Courses.	D.	N.	S.	E.	W.	Successf. lats.	Merid parts.	M d. lat.	Diff. long. E. W.
ESE.						52° 20'	3697,8		
SW.	43		16,5	39,7		52 04	3671,7	26,1	62,8
SE. b. S.	58		22,6		22,6	51 41	3634,4	37,3	37,3
SSW.	60		48,2	32,2		50 53	3557,7	76,7	51,2
			55,4		23,0	49 58	3471,4	86,3	
Diff. lat. =			142,7	71,9	45,6				114,0
Depart. =				26,3					41,0

EXPLANATION.

By the differences of latitude, first found, the column of successive latitudes is filled up; to these the meridional parts and their differences are next taken; the courses and corresponding meridional diff. lat. give (59) the differences of longitude.

In this example, the difference in the result, by the three methods, is very inconsiderable. 77. Ex-

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79. EXAMPLE III. Suppose a ship from the latitude $67^{\circ} 30' N.$, longitude $8^{\circ} 46' W.$, has sailed the following courses, viz.

NE. 64 m. NNE. 50 m. NW. b. N. 58 m. WNW. 72 m.
West 48 m. SSW. 38 m. S. b. E. 45 m. ESE. 40 m.
Required her present latitude and longitude?

By MIDDLE LATITUDE, according to the second Rule.

Courses.	D.	Diff. lats.		Depart.		Success. lats.	Sum lats.	Mid. lats.	Co-m. lats.	D. lon.	
		N.	S.	E.	W.					E.	W.
NE.	64	45,3		45,3		$67^{\circ} 30'$	0	0	0		
NNE.	50	46,2		19,1		68 15	135 45	67 52	22 08	120	
NW. b. N.	58	48,2				69 01	137 16	68 38	21 22	53	
WNW.	72	27,6			32,2	69 49	138 50	69 25	20 35		92
West	48				66,5	70 17	140 06	70 03	19 57		195
SSW.	38		35,1	8,8	48,0	70 17	140 34	70 17	19 43		142
S. b. E.	45		44,1		14,5	69 42	139 59	69 59	20 01	25	42
ESE.	40		15,3	36,9		68 58	138 40	69 20	20 40		
						68 43	137 41	68 50	21 10	102	
		167,3	94,5	110,1	161,2					300	471
Diff. lat. =	72,8	Depart =	51,1							Diff. long. =	171

By MERCATOR, according to the third Rule.

Courses.	D.	Diff. lat.		Depart.		Success. latitudes.	Mer. parts.	Mer. d. lat.	Diff. long.	
		N.	S.	E.	W.				E.	W.
NE.	64	45,3		45,3		$67^{\circ} 30'$	5551,6			
NNE.	50	46,2		19,1		68 15	5671,1	119,5	119,5	
NW. b. N.	58	48,2				69 01	5797,3	126,2	52,4	
WNW.	72	27,6			32,2	69 49	5933,9	136,6		91,1
West	48				66,5	70 17	6016,0	82,1		198,6
SSW.	38		35,1	8,8	48,0	70 17	6016,0	80,0		142,0
S. b. E.	45		44,1		14,5	69 42	5913,7	102,3	24,8	42,4
ESE.	40		15,3	36,9		68 58	5789,0	124,7		
						68 43	5747,4	41,6	100,7	
Diff. lat. =	72,8	Depart =	51,1						297,4	474,1
		167,3	94,5	110,1	161,2					
										Diff. longit. = 176,7

Hence the present latitude is $68^{\circ} 43' N.$; and present long. $11^{\circ} 42' W.$

According to the first Rule.

The co-mid. lat. between the first and last lats. is $21^{\circ} 54'$, with which, and the departure 51,1, the diff. long. will be 137 miles.

Also, the mer. diff. lat. between the first and last lats. is 195,8; with which, and the diff. lat. 72,8, and the dep. 51,1, gives a diff. long. 137,1.

* This diff. long. is found by Art. 38 of this Book.

Q²

78. Ex-

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80. EXAMPLE IV. Suppose a ship from the latitude of $68^{\circ} 38' N.$, longitude $8^{\circ} 40' E.$, has sailed the following courses, viz.

NE. b. N.	63 m.	NE.	38 m.	NNE.	56 m.
North.	30 m.	NW. b. N.	25 m.	NNW. $\frac{1}{2}$ W.	36 m.
N. b. E.	40 m.	NE. b. E. $\frac{1}{2}$ E.	72 m.	SE.	50 m.
ENE.	65 m.				

Required her present latitude and longitude, with her direct course and distance to the North Cape, in lat. $71^{\circ} 27' N.$, longitude $26^{\circ} 30' E.$?

By MIDDLE LATITUDE, according to the second Rule.

Courses.	D.	Diff. lats.		Depart.		Successf. lats.	Sum lats.	Mid. lats.		Co-m. lats.	D lon.	
		N.	S.	E.	W.						E.	W.
NE. b. N.	63	52,4		35,0		$68^{\circ} 38'$	0 138,8	0 69	0 420,5	0 98		
NE.	38	26,9		26,9		69 30	139 27	69 43	20 17	78		
NNE.	56	51,7		21,4		69 57	140 46	70 23	19 37	64		
North	30	30,0				70 49	142 87	71 4	18 56	00		
NW. b. N.	25	20,7			13,9	71 40	142 59	71 30	18 30	44		
NNW. $\frac{1}{2}$ W.	36	31,7			17,0	72 12	143 52	71 56	18 4	55		
N. b. E.	40	39,2		7,8		72 51	145 37	72 31	17 29	26		
NE. b. E. $\frac{1}{2}$ E.	72	33,9		63,5		73 25	146 16	73 8	16 52	220		
SE.	50		35,3	35,3		72 50	146 15	73 7	16 53	121		
ENE.	65	24,9		60,0		73 15	146 57	73 31	16 57	205		
		311,4	35,3	249,9	30,9					812	99	
Diff. lat. =	276,1			219,0	Dep.					Diff. long. =	713	

By MERCATOR, according to the third Rule.

Courses.	D.	Diff. lats.		Depart.		Successf. lats.	Mer. parts.	Mer. d. lat.	Diff. long.	
		N.	S.	E.	W.				E.	W.
NE. b. N.	63	52,4		35,0		$68^{\circ} 38'$	5733,7			
NE.	38	26,9		26,9		69 30	5879,2	145,5	97,2	
NNE.	56	51,7		21,4		69 57	5957,2	78,0	78,0	
North	30	30,0				70 49	5112,1	154,5	64,0	
NW. b. N.	25	20,7			13,9	71 19	6204,5	92,4		
NNW. $\frac{1}{2}$ W.	36	31,7			17,0	71 40	6270,7	66,2		44,0
N. b. E.	40	39,2		7,8		72 12	6373,9	103,2	26,0	55,0
NE. b. E. $\frac{1}{2}$ E.	72	33,9		63,5		72 51	6503,8	129,9		
SE.	50		35,3	35,3		73 25	6621,0	117,2	219,4	
ENE.	65	24,9		60,0		72 50	6500,4	120,6	120,6	
						73 15	6586,1	85,7	207,0	
		311,4	35,3	249,9	30,9				812,2	99,0
Dep. lat. =	276,1			219,0	Dep.				Diff. long. =	713,2

By

By the FIRST RULE.

With $\frac{1}{10}$ th of the diff. lat. and departure, viz, 27,61 and 21,9, the direct course is N. $38^{\circ} 30' E.$, and distance 353.

The co-mid. lat. is $19^{\circ} 04'$; with which, and the departure 219, the difference of longitude is 673.

The meridional diff. lat. 852,4; with which, and the direct course $38^{\circ} 30'$, the difference of longitude is 676.

To find the direct course and distance from the ship's place to the north cape.

Depart. lat.	= $68^{\circ} 38' N.$	Dep. long.	= $8^{\circ} 40' E.$
Diff. lat.	= $4 36 N.$	Diff. long.	= $11 53 E.$
Present lat.	= $73 14 N.$	Mer. parts = 6582,6	Present. long. = $20 33 E.$
N. Cape's lat.	= $71 27 N.$	Mer. parts = 6229,6	N. C.'s long. = $26 30 E.$
Diff. lat.	= $1 47$	M. d. lat. = 353,0	Diff. long. = $5 57 E.$
Or	= 107 miles.	Or	= 357 miles.

By MIDDLE LATITUDE.

Sum lats. $144^{\circ} 41'$; mid. lat. $72^{\circ} 21'$; co-mid. lat. $17^{\circ} 39'$.
 To the co-mid. lat. $17^{\circ} 39'$, and the diff. longitude 357 ($= 119 \times 3$) taken as a distance, the corresponding depart. is $(36,28 \times 3 =) 108,84$.
 As the diff. lat. 107, and depart. 108,8, are so nearly equal, therefore the course to the North Cape is SE., and the distance 152 miles.

By MERCATOR.

The meridional diff. lat. 353, and diff. long. 357, taken as a diff. lat. and depart., gives the course S. $45^{\circ} 20' E.$; and the distance 152 miles.

81.

REMARK.

The three methods of operation are sufficiently illustrated in the preceding examples. From whence it may be inferred, that in latitudes not exceeding 45° , the *first method*, or that usually practised at sea, is correct enough for any runs that can be made in one or two days; and one day's run may be safely worked by the same method in latitudes as high as 50° or 52° : But in latitudes nearer to the pole, it will be proper to find the difference of longitude to compound courses by either of the second or third methods; as may be seen by comparing the results to the third and fourth examples; the first method giving the difference of longitude too small, by a quantity considerable enough to produce fatal consequences.

SECTION VI.

The Construction and Use of a Mercator's Chart.

The linear and numeral solutions of such questions as may be proposed concerning sailing by the principles on which Mercator's charts are founded, having been sufficiently exemplified; it follows next to shew how such questions may be solved by the chart itself.

82.

PROBLEM I.

To make a true sea-chart, which shall shew the known sea-coasts, islands, rocks, shoals, &c. that are contained within any proposed limits.

SOLUTION.

1st. Construct the boundaries or limits of the chart, within the proposed latitudes and longitudes (13), and let the degrees of latitude and longitude be marked on these limits.

2d. On a piece of stiff paper, parchment, &c. make a scale of equal parts, like the degrees of longitude that are in the graduated parallel of latitude, and let the divisions on this scale be numbered in the same manner.

3d. From a table of the latitudes and longitudes of places, (VI. 137) extract those places, with their latitudes and longitudes, that lie within the proposed limits.

4th. Then by sliding the outside divisions of the scale of the graduated parallel, along the graduated meridians, so that both ends of the scale, at the same time, always cut like divisions of latitude; the points corresponding to the latitudes and longitudes of the places contained within the proposed limits, may be readily marked in the chart.

5th. Lines being properly drawn from point to point, will form the outlines of the sea-coasts, islands, &c.; to which may be annexed the *depths of water, setting of currents*, and whatever else may be judged convenient for the chart to contain.

In this manner was constructed the sea-coasts, &c. of the chart in Plate XII.; containing part of the sea-coasts of Europe, Africa, and America, with the intermediate islands; extending from the latitude of 23 degrees to 59 degrees north; and from the longitude of London, or 0 degrees, to 80 degrees of longitude west.

83. The Gunter's scales have drawn on them two lines, one marked *M. R.*, signifying the *nautical meridian*; and the other, which is placed directly under it, is marked *E. P.*, signifying *equal parts*.

These equal parts are degrees of longitude, to which the divisions of the nautical meridian are fitted, by increasing them in their true proportion; which is readily done by the table in Art. 13, taking the meridional degrees from the scale of equal parts, or degrees of longitude. These two lines, or scales, are used either to construct a Mercator's chart, or to shew nearly the meridional diff. lat. between any two parallels of latitude.

The construction of a Mercator's chart (or rather the limits or borders thereof) by these lines is easily done, by transferring the divisions, corresponding to the degrees to be used, from these scales to the paper the chart is to be drawn on. See Plate XII. Thus:

On an east and west line (AB) apply the extent, taken from the scale of longitude (EP), between the degrees of longitude where the chart is to begin and end at (suppose between the longitudes 0° and 80°).

Through these two extremities (A, B,) draw two meridians (AD, BC,) each of a length equal to the extent, taken from the line of latitudes (Mer.), between the two latitudes where the chart is to begin and end at (suppose between the latitudes 23° and 59°).

Then the several degrees both of longitude and latitude, taken from those scales, and separately applied in their respective places, will form the borders of the chart as required.

The meridional difference of latitude, is found by taking with the compasses the extent, on the meridian scale, between the two proposed latitudes, and applying it to the line of longitude; then the number of equatorial degrees, and parts of degrees, being reduced to miles, will shew the meridional diff. latitude between those parallels.

The scale of longitude put on (perhaps all) the two-feet Gunter's scales made before the year 1746, for common sale, had each degree of longitude rather less than $\frac{1}{4}$ th of an inch, and the meridian scale fitted to that size; so that about 85 meridian degrees came within the length of the scale; and consequently the charts constructed from them had their degrees rather too small to be of much use to seamen, whose instrument and hands are not, usually, so nice as to work with tolerable accuracy on such small charts; and such scales continue still to be made by some workmen: But about that time, and since, certain alterations, conceived to be useful, were made in Gunter's scales; by some of the instrument-makers, to whom those alterations were communicated †: and herein a degree of longitude is nearly half an inch, and is divided into five parts of 12 minutes each; and the meridional degrees, fitted thereto, are in two lengths, or scales; the first containing 40 degrees; and the other, beginning at 40° , extends to 68° ; most navigations being performed within those limits.

The scale of logarithmic tangents, put on the Gunter's scale, is also a scale of meridional parts, or a scale of the differences of longitude to a course on the fourth rhumb, or to the rhumb, making angles of 45 degrees, with the meridians, on a Mercator's chart.

For to the latitudes 0° , 5° , 10° , 15° , 20° , 25° , 30° , &c.; the half complements are 45° , $42\frac{1}{2}^{\circ}$, 40° , $37\frac{1}{2}^{\circ}$, 35° , $32\frac{1}{2}^{\circ}$, 30° , &c.: Therefore, if on the scale of log. tang. be set the numbers, 5, 10, 15, &c. against the divisions represented by 45° , $42\frac{1}{2}^{\circ}$, 40° , &c., and the intermediate divisions be estimated accordingly; then will those divisions, taken as latitudes, be the log. tang. of their half polar distances, or half co-latitudes; which (23) are analogous to the lengthened degrees on the meridian; and consequently (19) represent the differences of longitude on the rhumb of 45° .

† By the author of these Elements,

A scale of equal parts, or a scale of longitude proper to this scale of meridional parts, may be found by the table, art. 13. p. 137. Thus, take from the scale of log. tang marked as the scale of mer. parts, as above said, the mer. parts to any proposed latitude; then that distance being divided into as many equal parts as there are mer. deg. standing against that latitude in the table, will be the scale of equal parts required.

Thus. The first 20° of lat., which is the distance from the tangent of 45° to the tangent of 35° , being divided into 20, 4 equal parts, will be the scale of longitude required.

Also, the first 30° of lat., which is the space between 45° and 30° on the log. tangents, gives 31, 5 equatorial parts.

And (which is the most accurate among them) the distance between the tangents of 45° and $22\frac{1}{2}^{\circ}$, answering to the first 45° of latitude, gives 50, 5 equal parts, or degrees of the equator.

84. PROBLEM II.

A latitude and longitude being given, to find the corresponding place on the chart. Example. Lat. $32^{\circ} 25'$ N. long. $17^{\circ} 21'$ W. Plate XII.

SOLUTION.

1st. In the graduated meridian * take, with a pair of compasses, the distance between the proposed latitude and the nearest parallel of latitude drawn across the chart, as between the lat $32^{\circ} 25'$, and the parallel of 30° .

2d. With other compasses, take the distance in the graduated parallel + between the proposed longitude and the nearest meridian drawn across the chart, as between the long. $17^{\circ} 21'$ W. and the meridian passing through 20° of longitude.

3d. Slide the point of one compass along the meridian of 20° , and a point of the other compass along the parallel of 30° , keeping the points of each compasses perpendicular to the line they slide along; then will the meeting of the other points of the compasses shew the place corresponding to the given latitude and longitude; which in this example is the west end of the Island of Madeira.

This problem is most readily solved by the means of such a moveable scale of longitude, as is described and used in the foregoing Problem.

Or thus. A meridian drawn through the degrees of longitude given, and a parallel of latitude drawn through the given degrees of latitude, their intersection will be the place required.

* The graduated meridian, is the scale of latitudes on the right or left hand borders of the chart.

+ The graduated parallel, is the scale of longitudes on the top or bottom borders of the chart.

85. PROBLEM III.

A place being given on the chart, to find its latitude and longitude.

Suppose Palma, one of the Canary Isles. Plate XII.

SOLUTION.

1st. With the compasses, take the shortest distance between the given place and the nearest parallel of latitude; this distance applied to the graduated meridian, from the said parallel the same way the place lies, will give the latitude sought. Thus the distance from Palma to the parallel of 30° , applied from 30° southward, gives $28^{\circ} 35' N.$ for the latitude sought.

2d. In like manner, the shortest distance taken between the given place and the nearest meridian, being applied to the graduated parallel, from that meridian, will give the longitude: Thus the distance from Palma to the meridian through 20° , applied from 20° eastward, will give $17^{\circ} 41' W.$ longitude.

86. PROBLEM IV.

Two places being given on a Mercator's chart, to find their difference of latitude and difference of longitude.

Suppose Cape Finisferre and the Island of Porto Sancto. Plate XII.

SOLUTION.

Find the latitude and longitude of each place; then the required differences are readily known.

By Prob. III. $\left\{ \begin{array}{l} \text{C. Finisferre's lat.} = 43^{\circ} 15' N. \\ \text{Is. Porto Sancto lat.} = 33^{\circ} 07' N. \end{array} \right.$ Long. = $9^{\circ} 20' W.$ Long. = $15^{\circ} 53' W.$

Diff. lat. = $10^{\circ} 08'$ D. long. = $6^{\circ} 33' W.$

87. PROBLEM V.

Between two given places on a Mercator's chart, to find the bearing or course.

Suppose between Cape Clear and the Island St. Michael. Plate XII.

SOLUTION.

Lay the edge of a ruler upon Cape Clear and the middle of the Island of St. Michael; and holding the ruler in this position, take with a pair of compasses the shortest distance between the edge of the ruler and the center of the nearest compass, or flie; slide one point along the edge of the ruler, till the other point cuts the border of the flie; and the course will be found to be SW. b. S. $\frac{1}{4}$ W. nearly.

And the like for other places.

PROBLEM

PROBLEM VI.

Two places being given in a Mercator's chart, to find their distance.

88. CASE I. *When the given places are both under the same meridian or under the equator.*

SOLUTION.

Find the latitude of each place when under the same meridian, or their longitudes if under the equator.

Then the difference of the latitudes is the distance sought; or, the difference of longitudes is the distance sought.

89. CASE II. *When the given places are both under the same parallel of latitude.*

Suppose between Cape Roxant and the Island of St. George. Pl. XII.

SOLUTION.

Lay the edge of a ruler over the given places; take half the distance of those places with the compasses; set one point on the division cut by the ruler in the graduated meridian, and apply the other point upwards and downwards on the same meridian, noting the degrees of latitude the said point falls on: Then the difference of those latitudes will be the distance sought, very nearly.

Thus, half the distance between Cape Roxant and the Island of St. George, will reach from $38^{\circ} 45'$; the division cut by the edge of a ruler laid over those places, to $45^{\circ} 00'$ northward, and to $32^{\circ} 20'$ southward; the difference between these two latitudes is $12^{\circ} 40'$, or 760 miles nearly, for the distance required.

But if the proposed places lie in a parallel of latitude greater than 30 degrees, it will be nearer the truth to work as follows.

Take with the compasses that meridional degree wherein the parallel falls; and as many times as this distance is contained in a straight line between the two places, of so many degrees is their distance to be estimated; which is reduced to miles as before.

Thus the distance between the latitudes of $37^{\circ} 30' N.$ and $38^{\circ} 30' N.$ will be contained between Cape Roxant and the Island of St. George about $12\frac{2}{3}$ times, or $12^{\circ} 40'$, as before*.

90. CASE III. *When the given places differ both in latitude and longitude.*

Suppose the Lizard and the Island of Corvo. Plate XII. Fig. 4.

SOLUTION.

Find the latitudes of the two places, and their diff. latitude. Take the diff. lat. in degrees and parts from the equator.

* These operations depend on the following proportion.

As mer. diff. lat. : proper diff. lat. : : diff. long. : departure; which will differ but very little from the meridional distance, unless the places are very far asunder.

Lay the edge of a ruler over the two places; slide one point of the compasses along the edge, till the other point, held perpendicular to the ruler, falls on a parallel of latitude, and take the length of the parallel intercepted between this point and the edge of the ruler.

Then this extent, applied to the graduated parallel, gives, in degrees, the distance sought.

Thus, the difference of latitude between Corvo and the Lizard is $10^{\circ} 09'$; then $10^{\circ} 09'$ taken from the graduated parallel, and one point of the compasses applied to the edge of the ruler laid in the direction CL, slide that point along CL till the other point, kept in a perpendicular position, cuts the parallel of 50 degrees in B; then the distance BL, measured on the graduated parallel, gives about $20^{\circ} 25' = 1225$ miles, the distance required.

OR THUS. Take the difference of latitude $10^{\circ} 09'$ from the graduated parallel, apply one point of the compasses to the edge of the ruler touching c and L, slide this point along, till the other point touches a parallel (and not cutting it) in E; then the other point resting at D, take the distance DL, which measured on the graduated parallel, will give, in degrees, the distance sought.

These methods are in effect the same; for the triangles LBA, LDE, are congruous; as the angle at L is common, the angles A and E are right angles, and the sides AB, ED, are equal; therefore BL and DL are equal.

If the difference of latitude DE be taken on a meridian through c, from c to F; and the parallel of latitude FH be drawn, cutting the rhumb line CL in H; then are the triangles LDE, HCF, congruous; and CH = DL. (II. 102)

Now it is evident, that the parts of the triangles CFH, CGL, bear the same proportion to one another, as have been shewn in Art. 25. of this; where CF is the proper diff. lat., CG the mer. diff. lat., GL the diff. long., FH the departure, and CH the distance: Consequently the distance DL, or BL, found as above, is the true distance sought.

91.

PROBLEM VII.

Given the latitude and longitude of the departed place, the course, and the latitude of the present place: Required that place on the chart.

EXAMPLE.

A ship from the Island of Flores runs SE. b. S. $\frac{1}{2}$ E. till she finds herself in the latitude $27^{\circ} 29' N.$: Required her present place? Plate XII. Fig. 5.

SOLUTION.

Through the departed place, Flores, draw the given rhumb, viz. SE. b. S. $\frac{1}{2}$ E.; and its intersection with the parallel of latitude drawn through the latitude of the present place, viz. $27^{\circ} 29'$, will give the ship's place on the chart; which will be near the Island Ferro, or in lat. $27^{\circ} 48' N.$ long. $17^{\circ} 26' W.$

92. PROBLEM VIII.

Given the departed place, and also the course and distance, to find the present place on the chart.

EXAMPLE.

A ship from the Island Ferro runs NW. b. N. $\frac{1}{2}$ W. 951 miles: What is the latitude and longitude of the present place? Plate XII. Fig. 5.

SOLUTION.

Lay the edge of a ruler over Ferro in the position of the given course NW. b. N. $\frac{1}{2}$ W.; take the degrees $15^{\circ} 51'$ in the given distance, 951 miles, from the graduated parallel, and apply this extent from Ferro along the ruler's edge to R; take the nearest distance between R, and a parallel PZ drawn through Ferro, and see how many degrees this extent makes on the graduated parallel, which will be $11^{\circ} 35'$; those degrees counted northwards from Ferro will give $39^{\circ} 23' N.$ for the ship's latitude; then the point Q, where a parallel through this latitude intersects the edge of the ruler, still lying in the first position, will be the ship's present place.

93. *If the course be due east or west.*

EXAMPLE.

A ship from Cape Roxant fails due west 760 miles: Required her present place? Plate XII.

SOLUTION.

Take half the degrees (*viz.* $6^{\circ} 20'$) in the given distance, 760 miles; add them to, and subtract them from, the given lat. $38^{\circ} 42'$, noting the sum $45^{\circ} 02'$, and the remainder $32^{\circ} 22'$: Then the extent between those degrees taken in the graduated meridian, and applied from the given place westward in the parallel, will give the ship's present place; which will be near the Island of St. George.

Or thus, if the latitude be above 30° degrees.

Take the length of a degree in the latitude of the given parallel, as much above the parallel as below it; and turn this distance over in that parallel, the same way the course is, as many times as there are degrees in the given distance, and this will give the ship's place.

Thus the length of a degree taken, in the latitude $38^{\circ} 42'$, and turned over from Cape Roxant to the west $12\frac{2}{3}$ times, the compasses will fall on the Island of St. George.

94. PROBLEM IX.

Given the departed place, with the difference of latitude and distance: Required the ship's place on the chart.

EXAMPLE.

A ship from Q fails in the SE. quarter 951 miles and then finds herself in lat. $27^{\circ} 48' N.$, having altered her latitude $11^{\circ} 35'$: Required her present place? Plate XII. Fig. 5.

SOLUTION.

Take from the graduated parallel the difference of latitude $11^{\circ} 35'$; lay

lay this extent from Q along the edge of a ruler, lying north and south, to x ; turn the edge of the ruler about x till it lies in an east and west position: From the graduated parallel take $15^{\circ} 51'$, the degrees in 951 miles, the given distance, apply this extent from Q , to cut the edge of the ruler in v : Let the points of the compasses rest in Q and v , and apply the edge of the ruler close to those points; then the point P , where the parallel of $27^{\circ} 48'$ is cut by the ruler's edge, is the ship's place required.

Or, by drawing lines on the chart, let the diff. lat., taken from the graduated parallel, be applied in a meridian drawn through Q from Q to x : From Q , with the distance in degrees taken from the graduated parallel, cut, in v , a parallel drawn through x : Then the point P , where a rhumb through Q and v cuts the parallel of latitude come to, viz. $27^{\circ} 48'$, is the place sought.

PROBLEM X.

95. *Given the departed place, with the course and difference of longitude: Required the ship's place on the chart.*

EXAMPLE.

A ship from P sails NW. b. N. $\frac{1}{4}$ W.: Required her place when she has altered her longitude $12^{\circ} 53'$? Plate XII. Fig. 5.

SOLUTION.

Take the given difference of longitude $12^{\circ} 53'$ from the graduated parallel; slide one foot of the compasses along the edge of a ruler laid on P , in the position PQ of the given course, until the other foot just touches in T a meridian PT drawn through P ; then will the point Q , where the foot rests against the ruler's edge, be the place required.

96. *If the course had been due east or west.*

EXAMPLE.

A ship from Cape Roxant sails due west until she has altered her longitude $16^{\circ} 40'$: Required her place on the chart?

SOLUTION.

Take the difference of longitude $16^{\circ} 40'$ from the graduated parallel; then this extent applied from the given place, Cape Roxant due west, will give the place required, which is near the Island of St. George.

97. PROBLEM XI.

Given the departed place, with the difference of latitude and departure: Required the ship's place on the chart.

EXAMPLE.

A ship from Q , in lat. $39^{\circ} 23'N.$, sails between the south and east till she comes into the lat $27^{\circ} 48'N.$, having made by account 649 miles of departure: Required the ship's present place) Pl. XII. Fig. 5.

SOLUTION.

Take the given difference of latitude $11^{\circ} 35'$ from the graduated parallel, and apply this extent from Q , along the edge of a ruler laid in the meridian of Q , to x ; take $10^{\circ} 49' = 649$ miles, the given departure, from

from the graduated parallel; set one foot on x , and lay this extent eastward to v , so that xv be in a perpendicular position to the ruler's edge: Lay the edge of the ruler on the points Q and v , and where it cuts zP , the parallel of the ship's present latitude in P , is the ship's place.

Then the longitude, course, and distance may be found as before. See Articles 85, 87, 90.

98. PROBLEM XII.

Given the departed place, with the course and departure: Required the ship's place on the chart.

EXAMPLE.

A ship from P , in latitude $27^{\circ} 48' N.$, sails $NW.$ $b. N. \frac{3}{4} W.$, until she has made 649 miles of departure: Required her present place? Plate XII. Fig. 5.

SOLUTION.

Lay the edge of a ruler over the given place P , in the position of the given course $NW. b. N. \frac{3}{4} W.$; take from the graduated parallel $10^{\circ} 49' = 649$, the given departure, and slide one foot along the ruler's edge, until the other foot just touches in s , a meridian PT drawn through P ; the distance SP applied to the graduated parallel, gives the diff. latitude; and consequently the present latitude is known: Then where the ruler's edge, lying in the same position, cuts the present parallel of lat. TQ , as in Q , is the place sought.

99. PROBLEM XIII.

Given the departed place, with the distance and departure, to find the ship's present place on the chart.

EXAMPLE.

A ship from Q , in lat. $39^{\circ} 23' N.$, in sailing between the south and east 951 miles, found she has made 649 miles of departure: Where is the present place of the ship? Plate XII. Fig. 5.

SOLUTION.

From the graduated parallel take $10^{\circ} 49' = 649$ miles, the given departure, and lay this extent from Q to o directly east; with the extent $15^{\circ} 51' = 951$ miles, the given distance, taken from the graduated parallel, one foot being set in Q , with the other foot cut the edge of a ruler laid on o in a meridional position, as in v ; transfer the distance ov from Q to x in a meridian passing through Q ; then Qx , measured on the graduated parallel, gives the diff. latitude, and consequently the present latitude $27^{\circ} 48'$ will be found: Lay a ruler's edge over Q and v , and where it cuts the parallel of $27^{\circ} 48'$, as in P , is the ship's place required.

SECTION VII.

Of sailing on a Great Circle of the Sphere.

100. GREAT CIRCLE SAILING is the art of finding what places a ship must go through, and what courses to steer, so that her tract shall be in the arc of a great circle (or nearly so) passing through the place sailed from and that bound to.

Although Mercator's sailing resolves truly all the cases incident to a ship's course along the rhumb passing through two places; yet there are few cases, wherein the course performed on the direct rhumb line is the shortest distance between those two places.

For on the sphere, the shortest distance between two places is the arc of a great circle intercepted between them (IV. 24); consequently the spiral, or rhumb, passing through those places, is not the shortest distance, but when that rhumb coincides with a great circle, which can only be on a meridian, or on the equator.

Now it is chiefly on account of the shortest distance that this method of sailing has been proposed; and it is for the most part advantageous for a ship to reach her port by the shortest way she can, when and where it is practicable.

As the solutions of the cases in Mercator's sailing were performed by plane triangles; so the cases in great circle sailing are resolved by the solution of spheric triangles.

A great variety of cases may be proposed in this kind of sailing; but as they serve rather for exercises in the solution of spheric triangles, than for any real use towards the navigating a ship, therefore those appertaining to only one Problem will be here more particularly considered, as they are the most useful cases that usually occur.

101. The angle of position, is an angle which a great circle, passing over two places, makes with the meridian of one of them; and shews the angular position of one place from the other.

PROBLEM.

Given the latitudes and longitudes of two places on the Earth: Required their nearest distance on the surface, together with the angles of position from either place to the other.

This Problem may be branched into six Cases.

102. CASE. I. *When the two places lie under the same meridian.*

SOLUTION. Their difference of latitude will give their distance; and the position of one from the other will be directly north or south.

101. CASE

103. CASE. II. *When the two places lie under the equator.*

SOLUTION. Their distance is equal to their difference of longitude, and the angle of position is a right angle, or the course from one to the other is due east or west.

104. CASE III. *When both places are in the same parallel of latitude.*

EXAMPLE. *What is the least distance between St. Maries, in latitude $37^{\circ} 00' N.$, longitude $22^{\circ} 56' W.$, and Cape Henry, in latitude $37^{\circ} 00'$, longitude $76^{\circ} 23' W.$?*

GENERAL CONSTRUCTION. Plate XIII. Fig. 25.

1st. Describe a circle $PESQ$, representing the meridian of one of the places, suppose of the eastern one, as St. Maries; draw the line EQ representing the equator, and at right angles to it draw the line PS , for the axis of the Earth, whose extremity P is the north pole, and s is the south pole; and on this circle lay off from P to A the complement of the latitude of (St. Maries) the eastern place.

2d. On the equator, lay off from Q to c (IV. 70) the difference of longitude between the two places ($53^{\circ} 27'$); and through P , c , s , (II. 72) describe the circle PCS , which will be the meridian of the other place (Cape Henry); on which lay from P to B the co-latitude of this place; which is done, by describing the arc Aa about the pole P (IV. 68) at the distance of the co-latitude.

3d. Through the points A , B , D , describe a great circle ABD . (II. 72)

Then A represents one place (St. Maries), B the other (Cape Henry); PA and PB are their co-latitudes; the angle APB , which is measured by the arc QC (IV. 9), is the diff. long.; the arc AB is the nearest distance of those places; the angle PAB is the angle of position from A to B ; and the angle PBA is the angle of position from B to A .

The arc AB may be measured as directed in Article 70, Book IV.

And the angle PAB , or PBA , as shewn in Art. 72, Book IV.

Now the places having the same latitude; therefore $PA = PB$, and $\angle PAB = \angle PBA$ (IV. 90). Therefore, if the arc PI be described (IV. 75) making the angle $API = 26^{\circ} 43\frac{1}{2}'$, the half of the difference of longitude; then will PI be perpendicular to AB , bisecting it.

And in the triangle API , right-angled at I , there will be given the hypotenuse $AP = 53^{\circ} 00'$; the $\angle API = 26^{\circ} 43\frac{1}{2}'$; to find the leg $AI =$ half the distance sought, and the $\angle PAI =$ the angle of position.

And the solution hereof falls under Art. 130, 132, Book IV.

To find the distance.

As Radius,	Or, As Rad	= 90° 00'	10,00000
To fin. hyp. PA;	To cos. lat.	= 37 00	9,90235
So fin. giv. $\angle A P I$,	So sin. $\frac{1}{2}$ diff. long. = 26 43 $\frac{1}{2}$		9,65292
To fin. leg. AI.	To fin. $\frac{1}{2}$ distance = 21 03		9,55527

Which doubled, gives 42 66 for the dist. AB. And this 42° 06' reduced to nautical miles, is 2526, which is 35 miles less than 2561, the distance given by parallel sailing. See Art. 42 of this.

To find the angle of position.

As Radius,	Or, As Rad	= 90° 00'	10,00000
To cos. hypoth. PA;	To sin. lat.*	= 37 00	9,77946
So tang. giv. $\angle A P I$,	So tang. $\frac{1}{2}$ diff. long. = 26 43 $\frac{1}{2}$		9,70199
To cot. $\angle A$.	To cot. $\angle$ posit. = 73 09		9,48145

Whereby it appears, that to sail from A to B, or from B to A, the ship must first steer N. 73° 09' W. or E; and then gradually increase her course till she comes to I, where it will be due W. or E.; and from thence the course is to be gradually diminished again till she comes to the other port, where it will be 73° 09', the same as she set out with: But how these courses are to be thus altered, will be shewn hereafter.

105. CASE. IV. *When one place has latitude, and the other has none, or is under the equator.*

EXAMPLE. *What is the nearest distance between the Island of St. Thomas, in lat. 0° 00', and long. 1° 00' E., and Port St. Julian, in lat. 48° 51' S., and longitude 65° 10' W.?*

Port St. Julian, in lat. 48° 51' S. Long. 65° 10' W.
 Isl. St. Thomas, in lat. 0° 00'. Long. 1° 00' E.

Julian's co-lat. 41 09 Diff. long. 66 10

By CONSTRUCTION. Plate XIII. Fig. 26.

Let the point A be St. Thomas under the equator, whose poles are P and s.

Make AC, the measure of the angle ASC, equal to 66° 10' = diff. long. (IV. 74)

Then as Port St. Julian is in south lat., about s the south pole, at the distance of Julian's co-lat., describe the arc *aa*, cutting scp, the meridian of Julian, in B.

Through the points A, B, E, a great circle being described, then the arc AB is the distance sought.

The distance AB may be measured by Art. 70, Book IV.

And the angles of position at A and B are to be measured as directed in Art. 72, Book IV.

* For the complement of a complement, is the arc itself: Thus QA, or the distance from the equator, shews the lat. But QA is the comp. of AP; and AP of AQ.

COMPUTATION. In the quadrantal triangle ASB , there is

Given the co-lat. of St. Thomas, or $AS = 90^{\circ} 00'$ } The rest are
the co-lat. of Julian, $SB = 41^{\circ} 09'$ } found as in Art.
the diff. longitude, $\angle ASB = 66^{\circ} 10'$ } 162, Book IV.

Or, in the supplemental triangle ACB , right angled at C , there is

Given the lat. of St. Julian's, or the leg $CB = 48^{\circ} 51'$.
the diff. longitude, or the leg $CA = 66^{\circ} 10'$.

The solution falls under Article 140, Book IV.

To find the distance AB .

As Radius,	Or, As Radius	$= 90^{\circ} 00'$	10,00000
To cof. either leg ;	To cof. diff. long. $AC = 66^{\circ} 10'$		9,66646
So cof. other leg.	So cof. diff. lat. $CB = 48^{\circ} 51'$		9,81825
To cof. hyp.	To cof. dist. $AB = 74^{\circ} 35'$		9,42471

So that the dist. $AB = 74^{\circ} 35'$ is 4475 miles; which is less by 57 miles than the distance found by Mercator's sailing.

To find the angle of position at A . (IV. 139)

As Radius,	Or, As Radius	$= 90^{\circ} 00'$	10,00000
To fin. leg AC ;	To fin. diff. long. $AC = 66^{\circ} 10'$		9,96129
So cot. leg CB ,	So cot. diff. lat. $CB = 48^{\circ} 51'$		9,94146
To cot. op. $\angle A$.	To cot. $\angle$ posit. at $A = 53^{\circ} 22'$		9,90275

To find the angle of position at B .

As Radius,	Or, As Radius	$= 90^{\circ} 00'$	10,00000
To fin. leg CB ;	To fin. diff. lat. $CB = 48^{\circ} 51'$		9,87679
So cot. leg. AC ,	So cot. diff. long. $AC = 66^{\circ} 10'$		9,64517
To cot. op. $\angle B$.	To cot. $\angle$ posit. at $B = 71^{\circ} 36'$		9,52196

So that a ship sailing from the Island of St. Thomas must first shape her course $S. 51^{\circ} 22' W.$; and then by constantly altering her course towards the west, so as to arrive at Port St. Julian on a course $S. 71^{\circ} 36' W.$, she will have failed the shortest distance between those places.

106. CASE V. When the latitudes of the given places are both north, or both south.

EXAMPLE. What is the nearest distance between the Lizard and the Island of Bermudas; and also the angles of position at each place?

The Lizard in lat. $49^{\circ} 57' N.$ and long. $5^{\circ} 14' W.$
M. Bermudas in lat. $32^{\circ} 25' N.$ and long. $66^{\circ} 38' W.$

Diff. long. $61^{\circ} 24'$

By CONSTRUCTION. Pl. XIII. Fig. 27.

Make $PA = 57^{\circ} 35'$, the co-lat. of Bermudas; $Pa = 40^{\circ} 03'$, the co-lat. of the Lizard; and (IV. 68) with the tang. of Pa describe the arc aa' .
With

With the secant of $61^{\circ} 24'$, the diff. long. (IV. 76), arcs described from p and s , give the center of the circle PCs , the meridian of the Lizard; its intersection with aa gives B , the place of the Lizard.

A great circle described through A , B , D , (II. 72); the intercepted arc AB is the distance sought; and the angles PAB , PBA , are the positions required.

The distance AB may be measured by (IV. 70); and the positions by (IV. 72)

Had the eastern place, the Lizard, been put on the primitive circle, the great circle AB would have been difficult to describe; and therefore was the western place put on the primitive circle, it being a matter of indifference which of the places are so taken.

COMPUTATION. In the oblique spheric triangle APB .

Given Bermudas's co-lat., $PA = 57^{\circ} 35'$ } Required the rest.
Lizard's co-lat., $PB = 40^{\circ} 03'$

Diff. longitude, $\angle APB = 61^{\circ} 24'$
The solution falls under Art. 151, Book IV.

To find the distance AB.
As Radius, Or, As Radius $= 90^{\circ} 00'$ 10,00000
To cos. giv. $\angle$; To cos. diff. long. $\angle APB = 61^{\circ} 24'$ 9,68006
So tan. eith. side, So tan. Berm. co-lat. $PA = 57^{\circ} 35'$ 10,19721
To tang. M . To tang. fourth arc $= 37^{\circ} 01'$ 9,87727

Which take from Lizard co-lat. $= 40^{\circ} 03'$

Leaves a fifth arc $= 3^{\circ} 02' = N$.

As cos. M , As cos. fourth arc $= 37^{\circ} 01'$ 0,90775
To cos. N ; To cos. fifth arc $= 3^{\circ} 02'$ 9,99939
So cos. side 1st used, So sin. Bermudas lat. $= 32^{\circ} 25'$ 9,72922
To cos. side required. To cos. distance $AB = 47^{\circ} 54'$ 9,82636

To find the angle of position PBA. (IV. 150)

As this angle is opposite to the side used in the first proportion, therefore the fourth and fifth arcs here used, are the same as above.

As sin. N , As sin. fifth arc $= 3^{\circ} 02'$ 1,27641
To sin. M ; To sin. fourth arc $= 37^{\circ} 01'$ 9,779 3
So tan. given $\angle$, So tang. diff. long. $\angle APB = 61^{\circ} 24'$ 10,26343
To tang. req. $\angle$. To tang. $\angle$ posit. $\angle PBA = 87^{\circ} 15'$ 11,31947

To find the angle of position PAB.

The other parts being found, this angle may be found by the proportion between opposite sides and angles. Thus:

As sin. PA , Or, As sin. Bermudas's co-lat. $= 57^{\circ} 35'$ 0,97357
To sin. $\angle PBA$; To sin. $\angle$ pos. at Lizard $= 87^{\circ} 15'$ 9,99950
So sin. BP , So sin. Lizard's co-lat. $= 40^{\circ} 03'$ 9,80852
To sin. $\angle PAB$. To sin. $\angle$ pos. at Bermudas $= 49^{\circ} 35'$ 9,88159

R 2

Or,

Or working by Art. 150, Book IV. the same angle would be obtained.

Also, Had the two first proportions been worked with the Lizard's latitude instead of Bermudas's, the same distance would have been found.

So the shortest distance between the Lizard and Bermudas is $47^{\circ} 54'$, or 2874 sea miles; which is 178 miles less than the distance found by Mercator's sailing.

And a ship to run this shortest tract, must sail from the Lizard S. $87^{\circ} 15' W.$, and gradually to lessen the course, so as to arrive at Bermudas on the rhumb bearing S. $49^{\circ} 35' W.$: Whereas the direct course from one place to the other, as found by Mercator's sailing, is S. $69^{\circ} 02' W.$

107. CASE VI. *When one of the given places has north latitude, and the other has south latitude.*

EXAMPLE. *What is the nearest distance from the Island of St. Helena to the Island of Bermudas; and also the angles of position at each place?*

Island of Bermudas in lat. $32^{\circ} 25' N.$ and long. $66^{\circ} 38' W.$

Island of St. Helena in lat. $16^{\circ} 00' S.$ and long. $6^{\circ} 15' W.$

Diff. long. $60^{\circ} 23'$

By CONSTRUCTION. Pl. XIII. Fig. 28.

Make $QA = 60^{\circ} 00'$, the lat. of St. Helena; describe the arc aa about P (IV. 68) with the tangent of $Pa = 57^{\circ} 35'$, the co-lat. of Bermudas.

Arcs described from P, s, with the secant of $60^{\circ} 23'$, the diff. long., will give the centre of the circle pcs, the meridian of Bermudas; and its intersection B with aa , is the place of Bermudas.

Describe a great circle through A, B, D; the intercepted arc AB is the distance sought, which may be measured by IV. 70; and the angles PAB, ABS, are the positions required, which are to be measured as directed in Art. 72, Book IV.

COMPUTATION. In the oblique spheric triangle ABP.

Given $PA = 106^{\circ} 00'$, St. Helena's lat., added to 90° ; or its N. pol. dist.

$PB = 57^{\circ} 35'$, Bermudas's co-lat.

$\angle APB = 60^{\circ} 23'$, their difference of longitude.

To find the rest.

The solution falls under Art. 150, 151, Book IV.

To find the distance AB.

As Radius,	Or, As Radius	$= 90^{\circ} 00'$	10,00000
To cof. $\angle P$;	To cof. diff. long.	$= 60^{\circ} 23'$	9,69390
So tang. PB,	So tang. Berm. co-lat.	$= 57^{\circ} 35'$	10,19721
To tang. M.	To tang. fourth arc.	$= 37^{\circ} 54'$	9,89111
	Which taken from	$= 106^{\circ} 00' = \text{St. Helen's N.}$	[polar dist.]
	Leaves a fifth arc	$= 68^{\circ} 06' = N.$	

As cof. M,	Or, As cof. fourth arc	$= 37^{\circ} 54'$	0,10288
To cof. N;	To cof. fifth arc	$= 68^{\circ} 06'$	9,57169
So cof. PB,	So sin. Bermudas lat.	$= 57^{\circ} 35'$	9,72922
To cof. AB.	To cof. distance	$= 75^{\circ} 19'$	9,40379
			<u>70</u>

To find the angle of position PAB.

The fourth and fifth arcs being found as before.

As fin. N,	Or, As fin. fifth arc	=	68° 06'	0,03253
To fin. M;	To fin. fourth arc	=	37 54	9,78837
So tang. $\angle P$,	So tang. diff. long.	=	60 23	10,24530
To tang. $\angle A$.	To tang. $\angle$ position	=	49 20	10,06620

To find the angle of position ABP.

As radius,	Or, As Radius	=	90° 00'	10,00000
To cof. $\angle P$;	To cof. diff. long.	=	60 23	9,69390
So tang. P A,	Sotan. Helena's N. pol. dist.	=	106 00	10,54250
To tang. M.	So tang. fourth arc	=	59 53	10,23640

But by the determination, IV. 150, this fourth arc is to be like P A, or obtuse; therefore the supplement of $59^{\circ} 53'$, or $120^{\circ} 07'$, is to be the fourth arc: Then the difference between this fourth arc and P B, is $62^{\circ} 32'$ for N, or the fifth arc. And.

As fin. N,	Or, As fin. fifth arc	=	62° 32'	0,05194
To fin. M;	To fin. fourth arc	=	120 07	9,93702
So tang. $\angle P$.	So tang. diff. long.	=	60 23	10,24530
So tang. $\angle ABP$.	To tang. $\angle$ position	=	59 45	10,23426

But by the determination, IV. 150, the $\angle ABP$ is to be obtuse, and is therefore $120^{\circ} 15'$, which is the supplement of $59^{\circ} 45'$.

So that was a ship to sail from St. Helena to Bermudas on the arc of a great circle, she must first shape her course N. $49^{\circ} 20' W.$, and gradually deflect from the north, so as to arrive at Bermudas on a course N. $59^{\circ} 45' W.$, after having run $75^{\circ} 19'$, or 4519 sea miles.

The course found by Mercator's sailing is N. $50^{\circ} 05' W.$, and the distance is 4527 miles.

Whereby it appears, that when the places are one in N. latitude, and the other in S. latitude, there is but a small difference between the results found by Mercator's and great circle sailing: For near the equator the rhumb lines do not greatly differ from great circles.

From the solutions of the foregoing cases it is plain, that to sail on the arc of a great circle, the ship must continually alter her course: But as this is a difficulty too great to be admitted into the practice of Navigation, therefore it has been thought sufficiently exact to effect this business by a kind of approximation: that is, by a method which nearly approaches the sailing on a great circle.

108. The principle upon which this approximation is founded, is that in small arcs, the difference between the arc and its chord, or tangent, is so small, that they may be taken one for the other in any nautical operations.

R 3

Upon

Upon this principle, the great circles on the Earth are supposed to be made up of short right lines, each of which is a segment of a thumb line.

And on this supposition, the solution to the following Problem is deduced.

PROBLEM II.

109. Having given the latitudes and longitudes of the places sailed from and bound to: To find the successive latitudes on the arc of a great circle in those places where the alteration in longitude shall be a given quantity; together with the courses and distances between those places.

Solution.

- I. Find the angle of position at each place, and their distance by one of the preceding six cases.
- II. Find the greatest latitude the great circle runs through; that is, find the perpendicular from the pole to that circle; and also find the several angles at the pole, made by the given alterations of longitude between this perpendicular and the successive meridians come to.

III. With this perpendicular and the polar angles severally, find as many corresponding latitudes, by saying (IV. 148):

As Rad : tang, greatest lat. : : cof. 1st polar $\angle$: tang. 1st lat.
:: cof. 2d polar $\angle$: tang. 2d lat.

IV. Having the several latitudes passed through, and the difference of longitude between each, find, by Mercator's sailing (57), the courses and distances between those latitudes.

And these are the several courses and distances the ship must run, to keep nearly on the arc of a great circle.

Now the smaller the alterations in longitude are taken, the nearer will this method approach the truth: But it is sufficient to compute to every five degrees of difference of longitude; the length of an arc of five degrees differing from its chord or tangent only by 0,0002; as may be found from the Articles 25, 31, 33, Book III.

The examples to the third, fifth, and sixth of the preceding cases are hereafter resolved by this method, whereby the difference between the length of the great circular arc passing through the two places, and the sum of the several rhumbs failed on between those places, may be compared.

110. QUESTION I. *A ship being bound from the Island of St. Marks to Cape Henry, proposes to sail as near the arc of a great circle as she can, by altering her course at every five degrees difference of longitude: Required the latitude at each time of altering the course, and also the courses and distances between those several latitudes?*

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St. Marks lat. is $37^{\circ} 00' N.$ the long. $22^{\circ} 56' W.$

Cape Henry's lat. is $37^{\circ} 00' N.$ the long. $76^{\circ} 23' W.$

The diff. of long. $53^{\circ} 27'$

The figure being described, and the computation made as in Art. 104, the distance BA is found to be $42^{\circ} 06'$, and the $\angle A$ or $\angle B = 73^{\circ} 09'$, the angle of position. Plate XIII. Fig. 29.

Now the triangle APB being isosceles, the perpendicular PI falls in the middle of AB; and the latitudes, courses, and distances, being known in running the half BI, those in the half IA will also be known.

Let the points a, b, c, d , &c. be the places come to on each alteration of five degrees of longitude: then will the arcs Pa, Pb, Pc, Pd, &c. be the respective co-latitudes of those places, and are the hypothenuses of the right-angled spheric triangles PIA, PIB, PIC, PID, &c.

In the triangle PIB.			
Given	PB = $53^{\circ} 00'$	As Rad = $90^{\circ} 00'$	10,00000
	$\angle PBI = 73^{\circ} 09'$	To sin. PB = $53^{\circ} 00'$	9,90235
To find PI.		So sin. $\angle B = 73^{\circ} 09'$	9,98094
		To sin. PI = $49^{\circ} 51'$	9,88319

Now $\angle IPB = \left(\frac{53^{\circ} 27'}{2} \right) 26^{\circ} 43\frac{1}{2}'$; $\angle IPa = 21^{\circ} 43\frac{1}{2}'$; $\angle IPb = 16^{\circ} 43\frac{1}{2}'$;

$\angle IPc = 11^{\circ} 43\frac{1}{2}'$; $\angle IPd = 6^{\circ} 43\frac{1}{2}'$; are the several polar angles.

Then Rad = $90^{\circ} 00'$	To cot. PI = $49^{\circ} 51'$	So col. polar angle	$21^{\circ} 43\frac{1}{2}'$	$16^{\circ} 43\frac{1}{2}'$	$11^{\circ} 43\frac{1}{2}'$	$6^{\circ} 43\frac{1}{2}'$
			10,00000	10,00000	10,00000	10,00000
			9,92612	9,92612	9,92612	9,92612
			9,96800	9,98123	9,99084	9,99700
			9,89412	9,90735	9,91606	9,92312

Which are $38^{\circ} 05'$ $38^{\circ} 56'$ $39^{\circ} 33'$ $39^{\circ} 57'$
The degrees and min. set over each column, are the polar angles used in that proportion, and the corresponding latitudes stand at bottom.

111. The first term of these proportions being radius, and the second term constant, the operations may be very expeditiously performed thus.

On a slip of paper let the log. of the second or constant term be wrote, of the same size with the printed figures; apply this log. cot. successively to the log. co-sines of the polar angles: Then the sum of the two logs. being wrote down each time, will give the log. tangents of the several latitudes arrive l at.

By this method, each proportion will be worked by writing down only one line.

Hence it appears, the ship must first sail from the lat. $37^{\circ} 00' N.$ to lat. $38^{\circ} 05' N.$; thence to lat. $38^{\circ} 56' N.$; thence to lat. $39^{\circ} 33'$; thence to lat. $39^{\circ} 57' N.$; thence to lat. $40^{\circ} 09' N.$, which is the greatest latitude the must go to; and from thence she must proceed through the latitudes $39^{\circ} 57'$, $39^{\circ} 33'$, $38^{\circ} 56'$, $38^{\circ} 05'$, and so to $37^{\circ} 00'$, the parallel she set out from, and wherein she is to find Cape Henry.

R 4

Now

Now between these several latitudes, with the respective differences of longitude, find by Mercator's sailing, Art. 57, the courses and distances.

If the results of the several operations, in the questions of great circle sailing, be entered in such a table as the following, it will be found of some convenience to the operator.

Polar angles.	Success. longs.	Success. lats.	Diff. long.	Diff. lat.	Merid. parts.	Merid. d. lat.	Cour.	Diff.
$\angle IPB$ $26^{\circ} 43\frac{1}{2}'$	$22^{\circ} 56'$	$37^{\circ} 00'$	300	65	2392,6	82,0	—	—
$\angle IPa$ $21^{\circ} 43\frac{1}{2}'$	$7^{\circ} 56'$	$38^{\circ} 05'$	300	51	2474,6	65,2	$74^{\circ} 43'$	$246,6$
$\angle IPb$ $16^{\circ} 43\frac{1}{2}'$	$32^{\circ} 56'$	$38^{\circ} 56'$	300	37	2539,8	47,8	$77^{\circ} 44'$	$240,0$
$\angle IPc$ $11^{\circ} 43\frac{1}{2}'$	$37^{\circ} 56'$	$39^{\circ} 33'$	300	24	2587,6	31,2	$86^{\circ} 57'$	$235,2$
$\angle IPd$ $6^{\circ} 43\frac{1}{2}'$	$42^{\circ} 56'$	$39^{\circ} 57'$	300	12	2618,8	15,7	$84^{\circ} 04'$	$232,2$
	$49^{\circ} 39\frac{1}{2}'$	$40^{\circ} 09'$	$403,5$		$2634,5$		$87^{\circ} 46'$	$307,9$
								$1261,9$

In the first column are the angles at the pole contained between the perpendicular and the several meridians differing by 5° of longitude.

In the second column, the departed longitude $22^{\circ} 56'$ being increased by the differences of longitude, make the successive longitudes come to.

In the third column are the successive latitudes passed through, from St. Maries to the greatest latitude.

In the fourth and fifth columns, are the differences between the longitudes and latitudes in the second and third columns.

In the sixth column are the meridional parts to the successive latitudes; and in the seventh column are the meridional diff. latitudes.

The eighth and ninth columns contain the courses and distances between the places answering to the second and third columns.

The numbers in the third, eighth, and ninth columns, are found by working of the logarithmic proportions on a waste paper; but the work is here omitted, as it is so easily supplied.

Now the column of distances being summed up, amounts to 1261,9; which being doubled, gives 2523,8 miles for the distance between St. Maries and Cape Henry; differing from the distance found (104) in a great circle by 2,2 sea miles.

And the courses the ship must steer are, 1st. N. $74^{\circ} 43' W.$; 2d. N. $77^{\circ} 44' W.$; 3d. N. $80^{\circ} 57' W.$; 4th. N. $84^{\circ} 04' W.$; 5th. N. $87^{\circ} 46' W.$; 6th. S. $87^{\circ} 46' W.$; 7th. S. $84^{\circ} 04' W.$; 8th. S. $80^{\circ} 57' W.$; 9th. S. $77^{\circ} 44' W.$; 10th. S. $74^{\circ} 43' W.$: And on these courses she must run the respective distances standing against them.

112. QUESTION II. *A ship bound from the Lizard to the Island of Bermudas, proposes to sail on a great circle, and to alter her course at every five degrees of longitude: Required the latitudes through which the ship is to pass, and also the course and distance between each?*

The Lizard's lat. is $49^{\circ} 57' N.$ Long. $5^{\circ} 14' W.$ } Diff. long. $61^{\circ} 24'$
 Bermudas lat. is $32^{\circ} 25' N.$ Long. $66^{\circ} 38' W.$ }

Having described the figure, Plate XIII. Fig. 30, and found the $\angle PAB = 49^{\circ} 35'$, the $\angle PBA = 87^{\circ} 15'$, and the side $AB = 47^{\circ} 54'$, as in Art. 106.

Draw

Draw PI at right angles to ABD.

(IV. 72)

In the equator, lay off from the center the tangents of 5, 10, 15, 20, &c. to 55 degrees; and those points give the centers to the arcs of colat. to every 5° diff. longitude. (IV. 75)

(IV. 75)

<i>To find the perpend. PI.</i>		<i>To find the polar angle API.</i>	
As Rad	= 90° 00'	As Rad	= 90° 00'
To fin. PA	= 57 35	To tang. ∠ A	= 49 35
So fin. ∠ A	= 49 35	So cof. AP	= 32 25
To fin. PI	= 40 00	To cot. ∠ API	= 57 48
			9.799900

Now the polar angle API , or the diff. long. between the perpendicular and the meridian of Bermudas, $57^{\circ} 48'$, being taken from $61^{\circ} 24'$, the whole diff. long. leaves $3^{\circ} 36'$ for the diff. long. between the Lizard and the point 1: Also 5° , the proposed alteration of longitude, being subtracted as often as can be from $57^{\circ} 48'$, leaves the several polar angles; with which, and the perpendicular PI the several latitudes arrived at are found by Art. III.

Then having those latitudes, and the differences of longitude between them, find the successive courses and distances by Art. 57.

Let the several results be placed as in the following table; the logarithmic work being performed on a waste paper, is here omitted.

Also, suppose the small letters in the figure supplied.

Polar angles.	Successf. longgs.	Successf. lats.	Diff. long.	Diff. lats.	Merid. parts	Merid. d. lat.	Courses	Dists.
$\angle IPB$ 3° 36'	5° 14'	49° 57'	216	3	3469,8	4,7	88° 45'	137,5
$\angle IPA$ 2 48	8° 50	50 00	168	2	3474,5	3,1	88 56	107,4
$\angle IPB$ 7 48	11 38	49 58	300	14	3471,4	21,7	85 52	194,2
$\angle IPC$ 12 48	16 38	49 44	300	27	3449,7	41,6	82 06	196,4
$\angle IPD$ 17 48	21 38	49 17	300	40	3408,1	60,9	78 31	200,6
$\angle IPE$ 22 48	26 38	48 37	300	55	3347,2	82,5	74 37	207,3
$\angle IPF$ 27 48	31 38	47 42	300	71	3264,7	104,3	70 50	216,3
$\angle IPG$ 32 48	36 38	46 31	300	88	3160,4	126,2	67 10	226,8
$\angle IPH$ 37 48	41 38	45 03	300	106	3034,2	147,8	63 46	239,8
$\angle IPI$ 42 48	46 38	43 17	300	127	2886,4	171,5	60 15	255,9
$\angle IPJ$ 47 48	51 38	41 10	300	149	2714,9	194,4	57 03	273,9
$\angle IPK$ 52 48	56 38	38 41	300	175	2520,5	219,8	53 46	296,1
$\angle IPL$ 57 48	61 38	35 46	300	201	2300,7	242,7	51 02	319,6
	66 38	32 25			2058,0			2871,8

Now the sum of the several distances make 287,8 miles.

But the distance in the arc of a great circle is 2874 miles.

The difference of the two methods is 2,2 miles.

In this example it was judged necessary to keep the fractional place in the meridional parts, in order to have the distances more accurate.

(106)

II3. QUESTION III. *A ship being bound from the Island of St. Helena to the Island of Bermudas, it is proposed she should sail near the arc of a great circle passing through those places, and to alter her course at every 5° of longitude:*

note: Required the latitudes where those courses are to be altered, and also the course and distance between each place of alteration?

St. Helena's lat. $16^{\circ} 00' S$. Long. $6^{\circ} 15' W$. } Diff. long. $= 60^{\circ} 23'$
 Bermudas's lat. $32^{\circ} 25' N$. Long. $66^{\circ} 38' W$, }

Having described the figure, Plate XIII. Fig. 31, and found the $\angle PBA = 100^{\circ} 15'$; $\angle PAB = 49^{\circ} 20'$; and the distance $AB = 75^{\circ} 19'$; as in Art. 107.

In the equator QF , lay off from F the tangents of 5° , 10° , 15° , &c. to 55° ; and those points will give the centers to the meridians passing through every five degs. of long. between St. Helena and Bermudas.

To find the perpend.

As Rad.	$= 90^{\circ} 00'$	10,00000	As Rad.	$= 90^{\circ} 00'$	10,00000
To sin.	$PB = 57^{\circ} 35'$	9,92643	So cos.	$BP = 57^{\circ} 35'$	9,73922
So sin.	$\angle PBI = 59^{\circ} 45'$	9,93643	So tan.	$\angle PBI = 59^{\circ} 45'$	10,23419
To sin.	$PI = 46^{\circ} 49'$	9,86286	To cot.	$\angle PPI = 47^{\circ} 25'$	9,96341

To find the polar $\angle BPI$.

Now the several polar angles, longitudes, latitudes, diff. longs., diff. lats., mer. parts, mer. diff. lats., courses and distances, being computed on a waste paper, their results are to be entered in a table as follows.

Polar angles.	Success. longs.	Success. lats.	Diff. long.	Diff. lats.	Mer. parts.	Mer. d. lat.	Courses	Dist.
$\angle IP1 = 102^{\circ} 48'$	$6^{\circ} 15'$	$16^{\circ} 00' S$	300	255	973	264	$48^{\circ} 39'$	386,0
$\angle IPk = 97^{\circ} 48'$	11 15	11 45	300	269	709	272	47 48	400,5
$\angle IPi = 92^{\circ} 48'$	16 15	7 16	300	276	437	279	47 05	408,3
$\angle IPb = 87^{\circ} 48'$	21 15	2 38	300	282	158	282	46 46	411,7
$\angle IPg = 82^{\circ} 48'$	26 15	2 04 N.	300	279	124	280	46 58	408,8
$\angle IPf = 77^{\circ} 48'$	31 15	6 43	300	270	404	273	47 42	401,2
$\angle IPe = 72^{\circ} 48'$	36 15	11 13	300	258	677	266	48 26	388,8
$\angle IPd = 67^{\circ} 48'$	41 15	15 31	300	241	943	252	49 58	374,7
$\angle IPc = 62^{\circ} 48'$	46 15	19 32	300	221	1195	238	51 34	355,5
$\angle IPb = 57^{\circ} 48'$	51 15	23 13	300	201	1433	221	53 37	338,9
$\angle IPa = 52^{\circ} 48'$	56 15	26 34	300	180	1654	204	55 47	320,1
	61 15	29 34	300	171	1858	200	58 14	324,8
	66 38	32 25	323		2058			4519,3

Which is the same distance as found on a great circle by Art. 105.

114. The track of a ship when directed nearly in the arc of a great circle, as in the preceding articles, may be delineated on the Mercator's chart, by marking thereon, by the help of the latitudes and longitudes, the successive places where the ship is to alter her course; then those places or points being joined by right lines, will shew the path along which the ship is to sail under the proposed circumstances.

Thus in Plate XII. is drawn the route proposed in Question II, Art. 110, from the Lizard to Bermudas: And notwithstanding that this track on the chart appears much longer than the rhumb passing through those two places, yet it must be remembered, that these rhumb lines are only the representations of curves on the sphere whose absolute lengths are not hereby expressed.

SECTION

10

1911-12-21-1912-12-20

SECTION VIII.

Of the errors arising, in the cases of Sailing, upon the supposition that the Earth is not a Sphere.

115. The reasoning and conclusions in the preceding Sections of this Book, have been delivered as if the Earth was a sphere; but it has appeared from many observations made since the year 1672, that its figure is not that of a perfect sphere; its axis, or polar diameter, being shorter than the equatorial diameter.

This notion first arose from the observations made on pendulum clocks; which being fitted to beat seconds in the latitudes of Paris and London, were found to move slower as they approached the neighbourhood of the equator; and the pendulums were obliged to be shortened by about $\frac{1}{2}$ parts of an inch, to agree with the times of the stars passing the meridian (V. 264): This difference in the lengths of the pendulum appearing to those great men, *Huygens* and *Newton*, to be a much greater quantity than could arise from the alteration by heat only, they, separately, found that the Earth was flatted at the poles*; and Sir Isaac Newton has shewn in his *Principles of Natural Philosophy*, that this flatness is about $17\frac{1}{16}$ miles, and that the polar diameter is so the equatorial diameter, as 229 is to 230.

* To give the learner a notion how this conclusion came to be made, this note is annexed; deduced chiefly from Newton's Princip. Book III, Prop. 18, 19, 20.

CENTRIPETAL FORCE is, like *Gravitation*, that power by which heavy bodies tend towards a center.

CENTRIFUGAL FORCE is that by which bodies recede from a center; like as the particles of water and thumbs flying from a whirled mop.

The swifter the mop is whirled, the greater is the centrifugal force, and the particles fly off with greater velocity.

The diurnal rotation of the Earth gives to all its parts a centrifugal force, whereby the parts receding from the axe, endeavour to ascend about the equator; For was it not higher there, the seas by the centrifugal force subsiding about the poles, and rising towards the equator, would lay all things there under water.

Bodies acted on at the same time both by centripetal and centrifugal forces, have their power of gravitation lessened.

A falling body moves swifter, relative to the times in describing the spaces it has passed through, as it approaches the Earth.

Gravitating bodies move swifter as their gravity increases; and the contrary. A plummet at the end of a string, or a pendulum, when set a swinging, returns to its lowest point by its weight or gravity.

And the slower the vibration of the pendulum is, the less its gravitation. Consequently, the same pendulum vibrating slower than could arise from its lengthening by heat, shews a decrease of gravity in that pendulum; and as this decrease of gravity became gradually greater in approaching the equator, the Earth must be more prominent there than at the poles; that is, the Earth is a little flatted at the poles.

114. But

116. But notwithstanding the scientific deductions of this most accurate mathematician, many of the philosophers in France embraced the strange notion of the Earth's being of an oblong figure; the polar diameter being the longer: And as these different notions were supposed to retard the general progress of science in France, their KING resolved the affair should be determined by actual mensuration, at his own expence: And accordingly, about the year 1735, two companies of the most able mathematicians in their nation, were appointed to perform this business; one company to measure a degree of a meridian as near the equator as they could; and the other company to perform a like operation as near the north pole as could be conveniently attempted.

117. For, the Earth being supposed a spheroid, and consequently every meridian an ellipsis, it was known, as a mathematical problem, that by having the respective measures of a degree of a meridian in each of two known distant latitudes, the relation of the diameters could be found: Also, that if the measured lengths of those degrees increased towards the pole, the figure would be that of an oblate spheroid, such as *Newton* assigned; otherwise, its figure would be an oblong spheroid, such as insisted on by *Cassini*. Now the length of a degree of the meridian between Paris and Amiens, or which had its middle point in latitude $49^{\circ} 22'N$, was understood to be 57060 toises. It having been measured by *Picart*, in 1670, and been re-examined by *Cassini* in 1701. And this length was to be compared with the length of a degree as measured both at the polar circle and equator.

118. The gentlemen appointed for the northern expedition were Messieurs *Maupertuis*, *Clairaut*, *Camus*, *le Mannier*, and *Ouibier*, of France; and were joined by M. *Celsius*, an eminent astronomer in Sweden: The company proceeded to Swedish Lapland, and began their operations in July 1736, which they finished by the end of the following May; and among many other curious and useful particulars, they obtained the measure of a degree, the middle point of which was in the latitude of $66^{\circ} 20'N$, and found it to be 57438,9 toises, when reduced to the level of the sea; and that *the Earth is flattened at the Poles*; the proportion of the equatorial diameter to the polar diameter being as 1 to 0,9891.

119. The other company of gentlemen, who were ordered to the south, were Messieurs *Godin*, *Bouguer*, and *la Condamine*, of France; to whom were joined Don *Jorge Juan*, and Don *Antoine de Ulloa*, of Spain: These mathematicians departed from Europe about the middle of the year 1735, and began their operations, in the province of Quito in Peru, about October, 1736, and finished them about eight years after, they having met with many interruptions: Among the valuable pieces of knowledge resulting from their observations, it appears that *the Earth is flattened at the Poles*.

120. The Spanish gentlemen, who published their account separately, assign for the measure of a degree of the meridian at the equator 56767,8 toises; and that the equatorial diameter was to the polar diameter, as 266 to 265; or as 1 to 0,99624.

M. Bou-

*M. Bouguer gives for the measured length of a degree of the meridian at the equator 56753 toises, when reduced to the level of the sea, and makes the diameter of the equator to that at the poles, as 179 to 178; or as 1 to 0,99441.

M. De la Condamine gives for the measure of a degree under the equator, a length of 56749 toises.

M. De la Caille being at the *Cape of Good Hope* in 1752, found the length of a degree of the meridian in latitude $33^{\circ} 18\frac{1}{2}'$ S. to be 57037 toises.

In 1755, Father Boscovich found the length of a degree in latitude 43° N. to be 56972 toises, as measured between Rome and Rimini in Italy. In the year 1740, Messieurs Cassini (not him before mentioned) and De la Caille again examined the former measures in France; and after making all the necessary corrections, the measure of a degree, whereof the middle is in $49^{\circ} 22'$ N., they found to be 57074 toises; and in the latitude of 45° , the length of a degree of the meridian to be 57050 toises.

121. *M. de Maupertuis* has added to his narrative of the observations made by himself and associates in the north, a method for computing the proportion between the polar and equatorial diameters, from the measurements taken in any two different latitudes; though he solves the problem by an approximation only: but among the curious problems, communicated by Mr *Jones* to Mr. Gardiner, and published by him in his quarto edition of logarithmic tables, there is the result of a true solution of this problem, but without either analysis or demonstration. While I was preparing an investigation of this solution, Dr. *Pemberton* was pleased to put into my hands a very elegant solution of this problem by his friend Dr. *Letherland*, whereby the magnitude of the earth, as well as the proportion of its diameters, is determined; with leave to communicate it to the public; and I shall accompany it with such other problems, as are required for applying this to the purposes of navigation, composing together the series of problems following.

122.

PROBLEM I,

Supposing the Earth to be a spheroid, having its axis less than the equatorial diameter, to find the proportion between the axis and that diameter from measures taken of a degree of the meridian in two different latitudes.

ANALYSIS.

Let $apAp$ be an ellipsis representing a section of the Earth through its axis pp ; the equatorial diameter, or the greater axis of the ellipsis, being aa ; let E and F be two places, where the measure of a degree has been taken: these measures are proportional to the radii of curvature in the ellipsis at those places; and if cQ , cR are conjugates to the diameters, whose vertexes are E and F , cQ will be to cR in the subtriplicate ratio of the radius of curvature in E to the radius of curvature in F *, and therefore in a given ratio one to the other. Then QV

* See Milnes's Conic Sect. Cor. I. Prop. IV. Part. VI.

and

shall be to the tangent of either latitude, as the lesser axis to the greater, the measure of a degree in that latitude, will be to the measure of a degree at the equator, in the triplicate ratio of the co-sine of this angle to the co-sine of the latitude.

Thus E being one of the places measured, the tangent of the angle YCL , will be to the tangent of YQ , the latitude of the point E , as Y to YQ , that is, as the lesser axis to the greater; and $CQ : CL :: s, CH : s, CQY :: \cos. lcy : \cos. QCY$, QC being to CL (CP) as the radius of curvature in E to the radius of curvature in A , that is, as the measure of a degree in E to the measure of a degree in A .

And thus may be deduced both the figure and magnitude of the Earth from any two measurements given.

PROBLEM II.

126.

In the spheroidal figure of the Earth, to find for any given latitude the semi-diameter of the parallel to the equator in that latitude.

Let PDA be one fourth part of an elliptical meridian, where CA , the greater semi-axis, is the radius of the Earth's equator; and let BE be drawn to touch the ellipsis in D , and make with CA an angle CBD equal to the complement of the latitude given; then is D the place on the Earth having the given latitude; and the circular quadrant AF being described with the radius CA , let GDH be drawn parallel to the lesser semi-axis CP of the ellipsis; then BH , CH being joined, BH touches the arch AF in H , and is perpendicular to CH .

Now as $GD : GH$ (that is, $CP : CA$) $:: \text{tang. } \angle GBD : \text{tang. } \angle GBH :: \cot. \text{latitude} : \cot. \angle GCH$.

Hence the angle GCH is given; and radius is to the co-sine of the $\angle GCH$, as CH to CG . Whence CG is given, which is equal to the semi-diameter of the circle passing through D , parallel to the equator.

127.

PROBLEM III.

In the spheroidal figure of the Earth, to measure the elliptic arch corresponding with any latitude given.

M. Clairaut, one of the gentlemen engaged in the northern expedition, obliged me, when he was in London, in the year 1753, with the following series for measuring an elliptic arch.

"If in the preceding figure AC be denoted by a , CP by b , and $aa - bb$ by c ; then S denoting an arch in a circle, whose radius is unity,

" unity, which will measure the angle GDB, the latitude of the point D,
 " AD will be equal to

$$\text{" } \frac{b}{a} + \frac{3bbcc}{4a^3} + \frac{45bb^2c^2}{64a^5} \text{ \&c. } \times S - \frac{3bbcc}{8a^3} + \frac{15bb^2c^4}{32a^5} \times \sin. 2S + \frac{15bb^2c^6}{256a^7}$$

" $\times \sin. 4S$ &c.

" But in regard to the earth, all the powers of c above cc may be neglected, and the arch AD considered as equal

$$\text{" } \text{to } \frac{bb \times 4aa + 3cc}{4a^3} \times S - \frac{3bbcc}{8a^3} \times \sin. 2S.$$

$$\text{" } \text{Or to } \frac{bb \times 4aa + 3cc}{4a^3} \times S - \frac{3bbcc}{8a^3} \times \frac{\text{tabular sine of twice the lat.}}{\text{radius}}.$$

Whence if a circle be constituted, whose radius shall be $\frac{bb \times 4aa + 2cc}{4a^3}$,

then in this circle, the arch which measures the latitude, will exceed AD by $\frac{3bbcc}{8a^3} \times \frac{\text{radius}}{\text{tab. sine of twice the lat.}}$, that is, this excess will be to $3\frac{bbcc}{8a^3}$

as the sine of twice the latitude to radius.

COROLLARY.

When AD becomes the elliptical quadrant AP, the sine of twice the arch corresponding to the angle GDB vanishing, AP will be equal to the quadrantal arch of the said circle, whose radius is $\frac{bb \times 4aa + 3cc}{4a^3}$.

128.

PROBLEM IV.

To find the meridional parts in the chart of Mercator's form for any given latitude, the earth being a spheroid.

The late Mr. Mac Laurin, in his treatise on fluxions*, has given this rule for reducing the meridional parts of the sphere to the spheroid; to take an arch, whose sine shall be to the sine of the latitude, in the proportion of the distance of the foci in the generating ellipsis, to the greater axis; and to diminish the meridional parts in the sphere by a number, which shall be to the spherical meridional parts appertaining to this arch in the same proportion.

But in the Earth this arch will be so small, that its sine, the arch itself, and the spherical meridional parts, will not sensibly differ from each other: whence the number to be deducted from the spherical meridional parts appertaining to the latitude shall be to the number of minutes contained in the sine of the latitude in the duplicate ratio of the distance of the foci to the greater axis. Therefore in the preceding proposition, c being half the distance between the foci, the reduction of the meridional parts in the sphere to the spheroid, will be $\frac{cc}{aa} \times \frac{3437\frac{1}{2} \sin. \text{lat.}}{\text{radius}}$.

* Article 898. See also Phil. Transact. Vol. 41. page 808.

Or

Or in the figure to Proposition II. $\frac{\overline{CA}^2 - \overline{CP}^2}{\overline{CA}^2} \times \frac{3437\frac{1}{2} \text{ lin. lat.}}{\text{radius.}}$

Application of these problems to the several measurements above-mentioned.

127. For finding the ratio between the two axes of the Earth by Prob. I. Let E denote the greater, and F the lesser of the two latitudes, M and N the respective measures taken in each; and let o denote $\sqrt{\frac{M}{N}}$.

Then $\sqrt{\frac{ss, F - O^2 \times ss, E}{O^2 \times ss, E - ss, F}}$ is $= \frac{\text{lesser axis}}{\text{greater axis}}$.

Also if M represent the measure in a latitude denoted by E , and N the measure at the equator; let A denote an angle, whose measure is $\sqrt{\frac{M}{N}} \times i, E$.

Then $\frac{i, A}{i, E}$ is $= \frac{\text{lesser axis}}{\text{greater axis}}$.

129. The length of a degree of a meridian, obtained by actual mensuration in different latitudes, are the following.

Maupertuis and associates in lat. $66^{\circ}20'$ found	$M = 57438$ toises.
Cassini and De la Caille {	$M = 57074$
Boscovich {	$M = 57050$
De la Caille {	$M = 56972$
Juan and Ulloa {	$M = 57037$
Bouguer {	$M = 56768$
De la Condamine {	$M = 56753$
	$M = 56749$

Now by comparing the 1st with each of the following; the 2d with each of the following; and in like manner the 3d, 4th, and 5th, with each of the following, there will arise 25 results, each shewing the relation of those axes or diameters; which results are here annexed to be compared.

I. Comparing the 1st with each of the following.

Putting E for $66^{\circ} 20'$, and F for each of the following. Then log. of o will be $= 0,0009203$; or $0,0009812$; or $0,0011793$; or $0,0010142$; or $0,0016986$; or $0,0017368$; or $0,0017470$. Also $ss, E \times O^2 - ss, F = 0,2665109$; or $0,3426650$; or $0,3783127$; or $0,5413670$. $ss, F - ss, E \times O^2 = 0,2622636$; or $0,3381362$; or $0,3728667$; or $0,5366855$.

Also, at the equator, log. $i, A = 10,3562514$; or $10,3561943$; or $10,3561372$.

And the ratio of the greater axis to the lesser will be found to be as 1 to $0,9919994$; or to $0,99337$; or to $0,9927763$; or to $0,9945213$; or to $0,9954025$; or to $0,9952717$; or to $0,9951407$.

II. Putting E for $49^{\circ} 22'$, and F for each of the following. Then $\log.$ o will be $=0,0000609$; or $0,0002589$; or $0,0000939$; or $0,0007782$; or $0,0008165$; or $0,0008267$.

And the ratio of the greater axis to the lesser will be found to be as 1 to $0,9981542$; or to $0,9946330$; or to $0,9992120$; or to $0,9968662$; or to $0,9967689$; or to $0,9966710$.

III. Putting E for 45° , and F for each of the following. Then $\log.$ of o will be $=0,0001980$; or $0,0000330$; or $0,0007173$; or $0,0007556$; or $0,0007658$.

And the ratio of the greater axis to the lesser will be found to be as 1 to $0,9870030$; or to $0,9996164$; or to $0,9967087$; or to $0,9965152$; or to $0,996467$.

IV. Putting E for 43° , and comparing it with the equatorial measures (for I do not compare it with that of $33^{\circ} 18'$, because this comparison would require the Earth to be oblong, contrary to the tenor of the other observations,) and it will be found, that the ratio of the greater axis to the lesser is as 1 to $0,9974272$; or 1 to $0,9972334$; or 1 to $0,9971849$.

V. Putting E for $33^{\circ} 18'$, and comparing it with the equatorial measures, it will be found, that the ratio of the greater axis to the lesser is as 1 to $0,9949364$; or 1 to $0,9946205$; or 1 to $0,9945677$.

The arithmetical means of the several comparisons are $0,9940688$; $0,9946736$; $0,9952621$; $0,9972818$; $0,9947082$.
And the mean of all of them is $0,9951989$.

130.

REMARK.

The latitudes of $49^{\circ} 22'$ and of 45° fall within the meridian line drawn through France, the measures of which formerly taken have been re-examined, and corrected, since the northern and southern expeditions. If we compare these only with those two different measurements, that by Mr. Maupertuis and his associates in the north, and that by Bouguer at the equator, the ratio of the greater axis to the lesser will be reduced to these six, viz. as 1 to $0,9919994$; or 1 to $0,99337$; or 1 to $0,9952717$; or 1 to $0,9981542$; or 1 to $0,9967698$; or 1 to $0,9965152$.

The arithmetical mean of these ratios, viz. of 1 to $0,9953467$, may be taken for the ratio of the greater axis to the lesser; which is as 230 to $228,92974$, or very near the ratio assigned by Newton.

131. *The proportion of the axes being given, to find their magnitude from a measurement at the equator.*

The circular arch equal to the radius is $57,29578$ degrees; therefore this number multiplying the measure at the equator, gives the radius of curvature there; which is half the *latus rectum* appertaining to the greater axis; and this is to half the lesser axis in the ratio of the lesser axis to the greater: Hence first the lesser axis, and then the greater will be made known.

Thus if the greater axis is to the lesser as 1 to $0,9953467$, and the measure at the equator 56753 toises, as Bouguer has stated it, then the lesser axis will be 3266910 toises, and the greater 3282183 .

132. *To find the same from a measurement taken in any latitude.*

If *M* represents the measure in a latitude denoted by *E*, and *N* the measure at the equator, *N* may be found as follows.

By Corollary III. Problem 1. If an angle *A* be found, that *t*, *A* be

$$= \frac{\text{lesser axis}}{\text{greater axis}} \times t, E; N, \text{ the measure at the equator, will be } =$$

$$M \times \text{cube} \frac{N, E}{A}$$

And hence the axes may be known as above.

From the measures in the latitudes $66^{\circ} 20'$; $49^{\circ} 22'$; 45° ; and at the equator, six ratios between the axes were found (130); now each ratio being compared with both of its respective latitudes and measures, will give nine values of the axes. Thus

The greater axis = 3277500, or 3277484, or 3279655, or 3279702,
 or 3271718 , or 3271779 , or 3282937 , or 3273007 , or 3274479 .
 The lesser axis = 3251278 , or 3251263 , or 3257911 , or 3257958 .
 or 3265678 , or 3265704 , or 3267414 , or 3262430 , or 3263053 :

The respective means will be $\left\{ \begin{array}{l} \text{the greater axis } 3276473. \\ \text{the lesser axis } 3260299. \end{array} \right.$

And between these and those found at article 130, the means will be for the greater axis, 3279328, and for the lesser axis 3263604.

And these proportioned by the mean ratio of the axes, found art. 130, will give finally, for $\left\{ \begin{array}{l} \text{the greater axis } 3279328 \\ \text{the lesser axis } 3264068 \end{array} \right\}$ French toises.

133. Hence the English foot being to the French, as 107 to 114; the greater axis will be 3493863 English fathoms, and the lesser axis will be 3477606 English fathoms.

And an English mile consisting of 880 fathoms; the greater axis will be 3970,298 miles, and the lesser axis 3951,824 miles.

134. Since in Problem III. the excess of the arch of latitude, to the radius $\frac{bb \times 4aa + 3cc}{4a^3}$, above the corresponding elliptic arch, is to $\frac{3bcc}{8a^3}$;

as the sine of twice the latitude to radius; and the number of minutes in any arch is equal to the length of that arch multiplied by $3437\frac{1}{2}$ (the number of minutes in an arch equal to the radius) and divided by the radius of that arch, the number of minutes of the circle, whose radius is

$$\frac{bb \times 4aa + 3cc}{4a^3}, \text{ contained in this excess, will be } \frac{3bcc}{8a^3} \times 3437\frac{1}{2} \text{ divided}$$

by this radius, that is, to $\frac{\frac{3cc}{2}}{4aa + 3cc} \times 3437\frac{1}{2}$, as the sine of twice the latitude to radius. But *a* being equal to 230, and *b* = 228,92974,

$$\frac{\frac{3cc}{2}}{4aa + 3cc} \times 3437\frac{1}{2} \text{ will be equal to } 11,887 \text{ (nearly } 12).$$

Therefore the excess of the minutes of latitude above the minutes contained in the arch equal to *AD* of the circle, whose semidiameter is

$$\frac{bb \times 4aa + 3cc}{4a^3}, \text{ will be to } 11,887, \text{ as the sine of twice the latitude to radius.}$$

S 2

And

And thus will the arch AD be assigned, as measured by a log-line adjusted to the divisions of the elliptical quadrant into 60 times 90 parts, according to the length assigned to this quadrant in the corollary annex to Problem III.

135. This number to be deducted may also be found near enough by the traverse tables, thus.

Among the courses seek the double of the given latitude, when under 45° , or the double of the complement of the latitude, when above 45° , and against 12 in the column of distances take the corresponding departure for the number to be deducted.

In an oblique course, the distance run between any two latitudes will be to the arch of the meridian contained within those latitudes, as radius to the co-sine of the course.

136. In the next place, in regard to the ship's change in longitude on an oblique course, the difference between the meridional parts for the sphere and spheroid (by Problem IV.) is to $\frac{AC^2 - CP^2}{AC^2} \times 3437\frac{1}{2}$, as the sine of the latitude the ship arrives at from the equator, to radius.

Therefore, when the equatorial diameter of the earth is to the polar diameter, as 230 to 228,92974, the difference, of the spheroidal meridional parts from those of the sphere, will be to 31,919, as the sine of the latitude to radius. But by the constitution of the meridian line, the meridional parts express the change of longitude made by a ship, whether on the sphere or spheroid, on the rhumb of 45° .

And the change of longitude on this rhumb is to the change of longitude on any other rhumb, as the tangent of 45° , that is, the radius, to the tangent of that other rhumb.

137. Moreover, this reduction of the meridional parts from the sphere to the spheroid may be taken nearly from the traverse tables, thus.

With the given latitude among the courses, and against 32 in the column of distances, take the corresponding departure for the reduction.

138. In a course due east or west at any latitude, the ratio of the radius of that parallel run upon in the spheroid, to the semidiameter of the Earth's equator, is found by Problem II. And any distance run upon that parallel, as measured by the log, being reduced to minutes of the Earth's equator; those minutes will be to the change of longitude in minutes, as the radius of that parallel, to the semidiameter of the equator.

The preceding precepts will be sufficiently illustrated by the following examples.

139. EXAM. I. *Let it be proposed to find the distance, a ship shall run upon the supposed elliptical meridian, in increasing her latitude from 42 to 68 degrees.*

According to article 134, the log-line being adjusted to a sphere, whose semidiameter is $\frac{bb \times 4aa + 3cc}{4a^3}$, the reduction from this sphere to the spheroidal earth will be to 11,887, as the sine of twice the latitude to radius, and the calculation will proceed thus.

For

For the lat. 42° .		For the lat. 68° .	
As Rad.	10,00000	As Rad.	10,00000
To $1,84^{\circ}$ ($2 \times 42^{\circ}$)	9,99761	To $1,136^{\circ}$ ($2 \times 68^{\circ}$)	9,84177
So	11,887	So	1,07507
To the reduction 11,822		To the reduction 8,257	
Minutes in 42°		Minutes in 68°	
reduction = 2520,000		reduction = 4080,000	
= 11,822		= 8,257	
The elliptic arc for 42° = 2508,178		The elliptic arc for 68° = 4071,743	

The lesser number deducted from the greater gives 1563,564 for the divisions of the log-line adjusted to the measurement of the elliptic meridian, which are contained between the latitudes of 42° and 68° on the same side of the equator.

Moreover, in the traverse table, under twice 42° , over-against the number 12 in the column of distance, stands in the column of departure, the number 12 for the first correction; and under twice $90^{\circ} - 68^{\circ}$, that is, 44° , stands the number 8 for the second.

140. EXAM. II. Let it be proposed to find the course and distance from the Lizard, in the latitude of $49^{\circ} 57' N.$ to Barbadoes, in the latitude of $13^{\circ} N.$, the difference of longitude being $54^{\circ} 36' = 3276'$.

For the arch of the meridian, the reduction for the greater latitude is 11,71, and for the lesser 5,22. Therefore the latitudes reduced are $2985,29$, and $774,78$, the difference is 2210,51, and is the measure of the elliptic arch between the two latitudes in the measures of the log-line, adjusted as above.

In the next place the meridional parts are thus calculated.

For the latitude $49^{\circ} 57'$.		For the latitude $13^{\circ} 0'$.	
As Rad.	10,00000	As Rad.	10,00000
To $1, \text{lat.}$		To $1, \text{lat.}$	
So	$= 49^{\circ} 57'$	So	$13^{\circ} 0'$
	31,919		31,919
To the reduction 24,43		To the reduction 7,18	
1,38798		0,85614	
For the latitude of $49^{\circ} 57'$		For the latitude of 13°	
The spheroid merid. parts. = 3469,80		The spheroid merid. parts = 786,80	
reduction = 24,43		reduction = 7,18	
The spheroid merid. parts = 3445,37		The spheroid merid. parts = 779,62	

Also under each of the latitudes, in the traverse table the same reductions will be found nearly in the columns of departure over-against the number 32 in the column of distance.

The difference of the numbers 3445,37 and 779,62, being 2665,75, is the meridional parts in the spheroid for the difference of the latitudes. And the meridional diff. lat. in the sphere is 2683,0.

S 3

Hence

Hence for the course and distance.

In the spheroid.

Diff. long. = 3276.0
Mer. parts = 2665.75

But in the sphere.

Diff. long. = 3276.0
Mer. diff. lat. = 2683.0

Tan. course = $50^{\circ} 51' 50''$ 10,08952 Tang. course = $50^{\circ} 41' 0''$ 10,08672

For the distance.

In the spheroid.

Arch of the mer.
in min. or miles
of the log-line.

2210,51

In the sphere.

49° 57' - 13° 0' = 36° 57' = 2217 min.

Cof. of the course $50^{\circ} 51' 50''$ 9,80014 Proper diff. lat. = 2217 3,34577

Distance in the mea- } 3502 3,54435
sures of the log. } the sphere. } = 3499 3,54395

141. EXAM. III. To find the distance of Cape Henry in lat. 37° N. and long. $76^{\circ} 23' W.$ from St. Maries, nearly in the same latitude; with longitude $22^{\circ} 56' W.$

The difference of longitude $53^{\circ} 27' = 3207$ min.

In the spheroid.

$\frac{CA}{CP}$ (fig. Prob. II.) 0,00203

Cof. A

9,90307

In the sphere.

Cof. 37° 9,90235

Cot. latitude

10,12289

Diff. long.

3,50610

Diff. long. 3,50610

Cot. A

10,12492

2564½

3,40917

2561½

3,40845

Thus in the spheroid the parallel in the latitude of 37° contains for 3207 minutes of longitude $2564\frac{1}{2}$ minutes of the equator, and if the logarithm of $\frac{AF}{AP}$ in the figure of Prob. II. (= 0,00101) had been far-

ther added to the logarithm of $2564\frac{1}{2}$, the sum of these logarithms would have been 3,41018, and the arch of the parallel would be found to contain $2571\frac{1}{2}$ minutes, as measured by the log-line; whereas, in the sphere, this portion of the parallel contains $2561\frac{1}{2}$ minutes of the equator.

142.

REMARK.

The preceding reductions from the sphere to the spheroid shew, that the course of a ship on the earth, though flattened at the poles, is so near to that on a sphere, whose circuit should be equal to that of the earth through its poles, that with a log-line adjusted to such a sphere, an artist may safely trust to the single rules of globular sailing in his days works, even though his account of course and distance were much more certain than it is possible for them to be. It is therefore unnecessary here to enter into the question, whether the figure of the earth is a genuine spheroid, formed by the revolution of a real ellipsis about its axis, or of some more compounded figure, as Mr. Bouguer, presuming on the precise accuracy of the several measurements, contends, who has discussed this point at large

I43. A TABLE
OF
MERIDIONAL PARTS
To every DEGREE and MINUTE
OF THE
QUADRANT.

Calculated on the supposition of the spheric
figure of the EARTH.

Where D. l. stands for degree of lat. and M. P. for meridional parts.
Seek the degrees of latitude at the top or bottom, and the minutes in
the right or left-hand columns, and the corresponding meridional parts
will stand right against the minutes, and in the column signed with the
degree proposed.

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D. l.	0	1	2	3	4	5	6	7	8	9	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	090	1209	1809	2409	3004	3607	4211	4816	5422	6028	0
1	110	1210	1811	2412	3014	3617	4221	4826	5433	6034	1
2	210	1220	1821	2422	3024	3627	4231	4836	5443	6044	2
3	310	1230	1831	2432	3034	3637	4241	4846	5453	6054	3
4	410	1240	1841	2442	3044	3647	4251	4856	5463	6064	4
5	510	1250	1851	2452	3054	3657	4261	4866	5473	6074	5
6	610	1260	1861	2462	3064	3667	4271	4876	5483	6084	6
7	710	1270	1871	2472	3074	3677	4281	4886	5493	6094	7
8	810	1280	1881	2482	3084	3687	4291	4896	5503	6104	8
9	910	1290	1891	2492	3094	3697	4301	4907	5514	6114	9
10	1000	1300	1901	2502	3104	3707	4311	4917	5524	6124	10
11	1100	1310	1911	2512	3114	3717	4321	4927	5534	6134	11
12	1200	1320	1921	2522	3124	3727	4331	4937	5544	6144	12
13	1300	1330	1931	2532	3134	3737	4342	4947	5554	6154	13
14	1400	1340	1941	2542	3144	3747	4352	4957	5564	6164	14
15	1500	1350	1951	2552	3154	3757	4362	4967	5574	6174	15
16	1600	1360	1961	2562	3164	3768	4372	4977	5584	6184	16
17	1700	1370	1971	2572	3175	3778	4382	4987	5594	6194	17
18	1800	1380	1981	2582	3185	3788	4392	4998	5605	6204	18
19	1900	1390	1991	2592	3195	3798	4402	5008	5615	6214	19
20	2000	1400	2001	2602	3205	3808	4412	5018	5625	6224	20
21	2100	1410	2011	2613	3215	3818	4422	5028	5635	6234	21
22	2200	1420	2021	2623	3225	3828	4432	5038	5645	6244	22
23	2300	1430	2031	2633	3235	3838	4442	5048	5655	6254	23
24	2400	1440	2041	2643	3245	3848	4452	5058	5666	6264	24
25	2500	1450	2051	2653	3255	3858	4463	5068	5676	6274	25
26	2600	1460	2061	2663	3265	3868	4473	5078	5686	6284	26
27	2700	1470	2071	2673	3275	3878	4483	5089	5696	6294	27
28	2800	1480	2081	2683	3285	3888	4493	5099	5706	6304	28
29	2900	1490	2091	2693	3295	3898	4503	5109	5716	6314	29
30	3000	1500	2101	2703	3305	3908	4513	5119	5726	6324	30
31	3100	1510	2111	2713	3315	3918	4523	5129	5737	6334	31
32	3200	1520	2121	2723	3325	3929	4533	5139	5747	6344	32
33	3300	1531	2131	2733	3335	3939	4543	5149	5757	6354	33
34	3400	1541	2141	2743	3345	3949	4553	5159	5767	6364	34
35	3500	1551	2151	2753	3355	3959	4563	5169	5777	6374	35
36	3600	1561	2161	2763	3365	3969	4573	5180	5787	6384	36
37	3700	1571	2171	2773	3375	3979	4584	5190	5797	6394	37
38	3800	1581	2181	2783	3385	3989	4594	5200	5808	6404	38
39	3900	1591	2191	2793	3396	3999	4604	5210	5818	6414	39
40	4000	1601	2202	2803	3406	4009	4614	5220	5828	6424	40
41	4100	1611	2212	2813	3416	4019	4624	5230	5838	6434	41
42	4200	1621	2222	2823	3426	4029	4634	5240	5848	6444	42
43	4300	1631	2232	2833	3436	4039	4644	5250	5858	6454	43
44	4400	1641	2242	2843	3446	4049	4654	5260	5868	6464	44
45	4500	1651	2252	2853	3456	4059	4664	5271	5879	6474	45
46	4600	1661	2262	2863	3466	4070	4674	5281	5889	6484	46
47	4700	1671	2272	2873	3476	4080	4684	5291	5899	6494	47
48	4800	1681	2282	2883	3486	4090	4695	5301	5909	6504	48
49	4900	1691	2292	2893	3496	4100	4705	5311	5919	6514	49
50	5000	1701	2302	2903	3506	4110	4715	5321	5929	6524	50
51	5100	1711	2312	2913	3516	4120	4725	5331	5939	6534	51
52	5200	1721	2322	2924	3526	4130	4735	5341	5950	6544	52
53	5300	1731	2332	2934	3536	4140	4745	5351	5960	6554	53
54	5400	1741	2342	2944	3546	4150	4755	5362	5970	6564	54
55	5500	1751	2352	2954	3556	4160	4765	5372	5980	6574	55
56	5600	1761	2362	2964	3566	4170	4775	5382	5990	6584	56
57	5700	1771	2372	2974	3576	4180	4785	5392	6000	6594	57
58	5800	1781	2382	2984	3586	4190	4795	5402	6010	6604	58
59	5900	1791	2392	2994	3596	4200	4805	5412	6021	6614	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	0	1	2	3	4	5	6	7	8	9	D. l.

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D. I.	10	11	12	13	14	15	16	17	18	19	D. I.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	603.1	664.1	725.3	786.8	848.5	910.5	972.8	1035.3	1098.2	1161.5	0
1	604.1	665.1	726.4	787.8	849.5	911.5	973.8	1036.3	1099.3	1162.5	1
2	605.1	666.1	727.4	788.8	850.5	912.6	974.8	1037.4	1100.3	1163.6	2
3	606.1	667.1	728.4	789.9	851.6	913.6	975.9	1038.4	1101.4	1164.7	3
4	607.1	668.1	729.4	790.9	852.6	914.6	976.9	1039.5	1102.4	1165.7	4
5	608.2	669.2	730.5	791.9	853.6	915.7	977.0	1040.5	1103.5	1166.8	5
6	609.2	670.2	731.5	792.9	854.7	916.7	978.0	1041.6	1104.5	1167.8	6
7	610.2	671.2	732.5	794.0	855.7	917.7	980.0	1042.6	1105.6	1168.9	7
8	611.2	672.2	733.5	795.0	856.7	918.8	981.1	1043.7	1106.6	1170.0	8
9	612.2	673.2	734.5	796.0	857.8	919.8	982.1	1044.7	1107.7	1171.0	9
10	613.2	674.3	735.5	797.0	858.8	920.8	983.2	1045.8	1108.7	1172.1	10
11	614.2	675.3	736.6	798.1	859.8	921.9	984.2	1046.8	1109.8	1173.1	11
12	615.3	676.3	737.6	799.1	861.0	922.9	985.2	1047.9	1110.8	1174.2	12
13	616.3	677.3	738.6	800.2	862.0	923.9	986.3	1048.9	1111.9	1175.3	13
14	617.3	678.4	739.6	801.2	863.0	925.0	987.3	1049.9	1112.9	1176.3	14
15	618.3	679.4	740.7	802.2	864.0	926.0	988.4	1051.0	1114.0	1177.4	15
16	619.3	680.4	741.7	803.2	865.0	927.0	989.4	1052.0	1115.0	1178.4	16
17	620.3	681.4	742.7	804.2	866.0	928.1	990.4	1053.1	1116.1	1179.5	17
18	621.3	682.4	743.7	805.3	867.1	929.1	991.5	1054.1	1117.1	1180.5	18
19	622.4	683.4	744.8	806.3	868.1	930.1	992.5	1055.2	1118.2	1181.6	19
20	623.4	684.5	745.8	807.3	869.1	931.2	993.6	1056.2	1119.2	1182.7	20
21	624.4	685.5	746.8	808.4	870.1	932.2	994.6	1057.3	1120.3	1183.7	21
22	625.4	686.5	747.8	809.4	871.2	933.2	995.6	1058.3	1121.3	1184.8	22
23	626.4	687.5	748.9	810.4	872.2	934.3	996.7	1059.4	1122.4	1185.8	23
24	627.4	688.6	749.9	811.4	873.2	935.3	997.7	1060.4	1123.4	1186.9	24
25	628.5	689.6	750.9	812.5	874.3	936.3	998.8	1061.4	1124.5	1188.0	25
26	629.5	690.6	751.9	813.5	875.3	937.4	999.8	1062.5	1125.5	1189.0	26
27	630.5	691.6	753.0	814.5	876.3	938.4	1000.8	1063.5	1126.6	1190.1	27
28	631.5	692.6	754.0	815.5	877.4	939.4	1001.9	1064.6	1127.6	1191.1	28
29	632.5	693.6	755.0	816.6	878.4	940.4	1002.9	1065.6	1128.7	1192.2	29
30	633.5	694.7	756.0	817.6	879.4	941.5	1004.0	1066.7	1129.7	1193.2	30
31	634.6	695.7	757.1	818.6	880.5	942.6	1005.0	1067.7	1130.8	1194.3	31
32	635.6	696.7	758.1	819.6	881.5	943.6	1006.1	1068.8	1131.8	1195.4	32
33	636.6	697.7	759.1	820.7	882.5	944.6	1007.1	1069.8	1132.9	1196.4	33
34	637.6	698.7	760.1	821.7	883.6	945.6	1008.1	1070.9	1134.0	1197.5	34
35	638.6	699.8	761.1	822.7	884.6	946.7	1009.2	1072.0	1135.1	1198.5	35
36	639.6	700.8	762.2	823.8	885.6	947.7	1010.2	1073.0	1136.1	1199.6	36
37	640.7	701.8	763.2	824.8	886.7	948.7	1011.3	1074.1	1137.2	1200.7	37
38	641.7	702.8	764.2	825.8	887.7	949.8	1012.3	1075.1	1138.2	1201.7	38
39	642.7	703.8	765.2	826.8	888.7	950.8	1013.4	1076.2	1139.3	1202.8	39
40	643.7	704.9	766.3	827.9	889.8	951.9	1014.4	1077.2	1140.3	1203.9	40
41	644.7	705.9	767.3	828.9	890.8	952.9	1015.4	1078.3	1141.4	1204.9	41
42	645.8	706.9	768.3	829.9	891.8	953.9	1016.5	1079.3	1142.4	1206.0	42
43	646.8	707.9	769.3	830.9	892.9	955.0	1017.5	1080.4	1143.5	1207.1	43
44	647.8	709.0	770.4	832.0	893.9	956.0	1018.6	1081.4	1144.6	1208.1	44
45	648.8	710.0	771.4	833.0	894.9	957.1	1019.6	1082.5	1145.6	1209.2	45
46	649.8	711.0	772.4	834.1	896.0	958.1	1020.6	1083.5	1146.7	1210.3	46
47	650.8	712.0	773.4	835.1	897.0	959.2	1021.7	1084.6	1147.7	1211.3	47
48	651.9	713.0	774.5	836.1	898.0	960.2	1022.7	1085.6	1148.8	1212.4	48
49	652.9	714.1	775.5	837.2	899.1	961.3	1023.8	1086.7	1149.8	1213.4	49
50	653.9	715.1	776.5	838.2	900.1	962.3	1024.8	1087.7	1150.9	1214.5	50
51	654.9	716.1	777.5	839.2	901.1	963.4	1025.9	1088.8	1152.0	1215.5	51
52	655.9	717.1	778.6	840.2	902.2	964.4	1026.9	1089.8	1153.0	1216.6	52
53	657.0	718.2	779.6	841.3	903.2	965.5	1028.0	1090.9	1154.1	1217.7	53
54	658.0	719.2	780.6	842.3	904.3	966.5	1029.0	1091.9	1155.1	1218.7	54
55	659.0	720.2	781.7	843.4	905.3	967.6	1030.1	1093.0	1156.2	1219.8	55
56	660.0	721.2	782.7	844.4	906.3	968.6	1031.2	1094.0	1157.2	1220.9	56
57	661.0	722.3	783.7	845.4	907.4	969.6	1032.2	1095.1	1158.3	1221.9	57
58	662.1	723.3	784.7	846.5	908.4	970.7	1033.2	1096.1	1159.4	1223.0	58
59	663.1	724.3	785.8	847.5	909.4	971.7	1034.3	1097.2	1160.4	1224.1	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. I.	10	11	12	13	14	15	16	17	18	19	D. I.

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D. 1	30	31	32	33	34	35	36	37	38	39	D. 1
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	1888,4	1958,0	2028,4	2099,6	2171,5	2244,3	2318,0	2392,7	2468,3	2545,0	0
1	1889,5	1959,2	2029,6	2100,7	2172,7	2245,5	2319,3	2393,9	2469,6	2546,2	1
2	1890,7	1960,4	2030,7	2101,9	2173,9	2246,8	2320,5	2395,2	2470,8	2547,5	2
3	1891,9	1961,6	2031,9	2103,1	2175,1	2248,0	2321,7	2396,4	2472,1	2548,8	3
4	1893,0	1962,7	2033,1	2104,3	2176,3	2249,2	2323,0	2397,7	2473,4	2550,1	4
5	1894,1	1963,9	2034,3	2105,5	2177,5	2250,4	2324,2	2398,9	2474,6	2551,4	5
6	1895,3	1965,0	2035,5	2106,7	2178,7	2251,6	2325,4	2400,2	2475,9	2552,7	6
7	1896,5	1966,2	2036,7	2107,9	2180,0	2252,9	2326,7	2401,4	2477,1	2554,0	7
8	1897,6	1967,4	2037,8	2109,1	2181,2	2254,1	2327,9	2402,7	2478,5	2555,3	8
9	1898,8	1968,5	2039,0	2110,3	2182,4	2255,3	2329,2	2403,9	2479,7	2556,6	9
10	1899,9	1969,7	2040,2	2111,5	2183,6	2256,5	2330,4	2405,2	2481,0	2557,8	10
11	1901,1	1970,9	2041,4	2112,7	2184,8	2257,8	2331,6	2406,4	2482,3	2559,1	11
12	1902,3	1972,0	2042,6	2113,9	2186,0	2259,0	2332,9	2407,7	2483,5	2560,4	12
13	1903,4	1973,2	2043,8	2115,1	2187,2	2260,2	2334,1	2409,0	2484,8	2561,7	13
14	1904,6	1974,4	2044,9	2116,3	2188,4	2261,4	2335,3	2410,2	2486,1	2563,0	14
15	1905,7	1975,6	2046,1	2117,5	2189,6	2262,7	2336,6	2411,5	2487,4	2564,3	15
16	1906,9	1976,8	2047,3	2118,7	2190,8	2263,9	2337,8	2412,7	2488,6	2565,6	16
17	1908,1	1977,9	2048,5	2119,9	2192,0	2265,1	2339,0	2414,0	2489,9	2566,9	17
18	1909,2	1979,1	2049,7	2121,1	2193,3	2266,3	2340,3	2415,2	2491,2	2568,2	18
19	1910,4	1980,3	2050,8	2122,2	2194,5	2267,6	2341,5	2416,5	2492,5	2569,5	19
20	1911,5	1981,4	2052,0	2123,4	2195,7	2268,8	2342,8	2417,8	2493,7	2570,7	20
21	1912,7	1982,6	2053,2	2124,6	2196,9	2270,0	2344,0	2419,0	2495,0	2572,0	21
22	1913,8	1983,7	2054,4	2125,8	2198,1	2271,2	2345,3	2420,3	2496,3	2573,3	22
23	1915,0	1984,9	2055,6	2127,0	2199,3	2272,5	2346,5	2421,5	2497,5	2574,6	23
24	1916,2	1986,1	2056,8	2128,2	2200,5	2273,7	2347,8	2422,8	2498,8	2575,9	24
25	1917,3	1987,3	2058,0	2129,4	2201,7	2274,9	2349,0	2424,0	2500,1	2577,2	25
26	1918,5	1988,5	2059,1	2130,6	2203,0	2276,1	2350,2	2425,3	2501,4	2578,5	26
27	1919,6	1989,6	2060,3	2131,8	2204,2	2277,4	2351,5	2426,5	2502,7	2579,8	27
28	1920,8	1990,8	2061,5	2133,0	2205,4	2278,6	2352,7	2427,8	2503,9	2581,1	28
29	1921,9	1992,0	2062,7	2134,2	2206,6	2279,8	2354,0	2429,1	2505,2	2582,4	29
30	1923,1	1993,1	2063,9	2135,4	2207,8	2281,0	2355,2	2430,3	2506,5	2583,7	30
31	1924,3	1994,3	2065,1	2136,6	2209,0	2282,3	2356,5	2431,6	2507,8	2585,0	31
32	1925,4	1995,5	2066,2	2137,8	2210,2	2283,5	2357,7	2432,9	2509,0	2586,3	32
33	1926,6	1996,6	2067,4	2139,0	2211,4	2284,7	2358,9	2434,1	2510,3	2587,6	33
34	1927,8	1997,8	2068,6	2140,2	2212,7	2286,0	2360,2	2435,4	2511,6	2588,9	34
35	1928,9	1999,0	2069,8	2141,4	2213,9	2287,2	2361,4	2436,7	2512,9	2590,2	35
36	1930,1	2000,2	2071,0	2142,6	2215,1	2288,4	2362,7	2437,9	2514,2	2591,5	36
37	1931,3	2001,3	2072,2	2143,8	2216,3	2289,7	2363,9	2439,2	2515,4	2592,8	37
38	1932,4	2002,5	2073,4	2145,0	2217,5	2290,9	2365,2	2440,4	2516,7	2594,1	38
39	1933,6	2003,7	2074,6	2146,2	2218,7	2292,1	2366,4	2441,7	2518,0	2595,4	39
40	1934,7	2004,9	2075,7	2147,4	2219,9	2293,3	2367,7	2443,0	2519,3	2596,7	40
41	1935,9	2006,0	2076,9	2148,6	2221,2	2294,6	2368,9	2444,2	2520,6	2598,0	41
42	1937,1	2007,2	2078,1	2149,8	2222,4	2295,8	2370,2	2445,5	2521,8	2599,3	42
43	1938,2	2008,4	2079,3	2151,0	2223,6	2297,0	2371,4	2446,8	2523,1	2600,6	43
44	1939,4	2009,6	2080,5	2152,2	2224,8	2298,3	2372,7	2448,0	2524,4	2601,9	44
45	1940,5	2010,7	2081,7	2153,4	2226,0	2299,5	2373,9	2449,3	2525,7	2603,2	45
46	1941,7	2011,9	2082,9	2154,6	2227,2	2300,7	2375,2	2450,6	2527,0	2604,5	46
47	1942,9	2013,1	2084,1	2155,8	2228,5	2302,0	2376,4	2451,8	2528,3	2605,8	47
48	1944,0	2014,3	2085,3	2157,0	2229,7	2303,2	2377,7	2453,1	2529,6	2607,1	48
49	1945,2	2015,4	2086,5	2158,2	2230,9	2304,4	2378,9	2454,3	2530,8	2608,4	49
50	1946,4	2016,6	2087,7	2159,4	2232,1	2305,7	2380,1	2455,6	2532,1	2609,7	50
51	1947,5	2017,8	2088,9	2160,7	2233,3	2306,9	2381,4	2456,9	2533,4	2611,0	51
52	1948,7	2019,0	2090,1	2161,9	2234,6	2308,1	2382,6	2458,1	2534,7	2612,3	52
53	1949,9	2020,2	2091,3	2163,1	2235,8	2309,4	2383,9	2459,4	2536,0	2613,6	53
54	1951,0	2021,3	2092,5	2164,3	2237,0	2310,6	2385,1	2460,7	2537,2	2614,9	54
55	1952,2	2022,5	2093,7	2165,5	2238,2	2311,8	2386,4	2461,9	2538,5	2616,2	55
56	1953,4	2023,7	2094,9	2166,7	2239,4	2313,1	2387,6	2463,2	2539,8	2617,5	56
57	1954,5	2024,9	2096,1	2167,9	2240,7	2314,3	2388,9	2464,5	2541,1	2618,8	57
58	1955,7	2026,0	2097,3	2169,1	2241,9	2315,5	2390,2	2465,8	2542,4	2620,1	58
59	1956,9	2027,2	2098,5	2170,3	2243,1	2316,7	2391,4	2467,0	2543,7	2621,4	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. 1	30	31	32	33	34	35	36	37	38	39	D. 1.

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D. I.	40	41	42	43	44	45	46	47	48	49	D. I.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
1	2622.7	2701.6	2781.7	2863.1	2945.7	3030.0	3115.6	3202.8	3291.6	3382.1	2
2	2624.0	2702.9	2783.1	2864.5	2947.2	3031.4	3117.0	3204.2	3293.1	3383.6	3
3	2625.3	2704.3	2784.4	2865.8	2948.6	3032.8	3118.5	3205.7	3294.6	3385.2	4
4	2626.6	2705.6	2785.8	2867.2	2950.0	3034.2	3119.9	3207.2	3296.1	3386.7	5
5	2627.9	2706.9	2787.1	2868.5	2951.4	3035.6	3121.4	3208.6	3297.5	3388.2	6
6	2629.2	2708.2	2788.4	2870.0	2952.8	3037.0	3122.8	3210.1	3299.0	3389.7	7
7	2630.5	2709.6	2789.8	2871.3	2954.2	3038.4	3124.2	3211.6	3300.5	3391.3	8
8	2631.9	2710.9	2791.2	2872.7	2955.6	3039.8	3125.7	3213.0	3302.0	3392.8	9
9	2633.2	2712.2	2792.5	2874.1	2957.0	3041.3	3127.1	3214.5	3303.5	3394.3	10
10	2634.5	2713.6	2793.8	2875.4	2958.4	3042.7	3128.6	3216.0	3305.0	3395.9	11
11	2635.8	2714.9	2795.1	2876.8	2959.8	3044.1	3130.0	3217.4	3306.5	3397.4	12
12	2637.1	2716.2	2796.5	2878.2	2961.1	3045.5	3131.5	3218.9	3308.0	3398.9	13
13	2638.4	2717.5	2797.9	2879.5	2962.5	3047.0	3132.9	3220.4	3309.5	3400.4	14
14	2639.7	2718.9	2799.3	2880.9	2963.9	3048.4	3134.3	3221.9	3311.0	3402.0	15
15	2641.0	2720.2	2800.6	2882.3	2965.3	3049.8	3135.8	3223.3	3312.5	3403.5	16
16	2642.3	2721.5	2802.0	2883.7	2966.7	3051.2	3137.2	3224.8	3314.0	3405.0	17
17	2643.6	2722.9	2803.3	2885.0	2968.1	3052.6	3138.7	3226.3	3315.5	3406.6	18
18	2644.9	2724.2	2804.7	2886.4	2969.5	3054.1	3140.1	3227.7	3317.0	3408.1	19
19	2646.2	2725.5	2806.0	2887.8	2970.9	3055.5	3141.6	3229.2	3318.5	3409.6	20
20	2647.5	2726.9	2807.4	2889.2	2972.3	3056.9	3143.0	3230.7	3320.0	3411.2	21
21	2648.8	2728.2	2808.8	2890.5	2973.7	3058.3	3144.5	3232.2	3321.5	3412.7	22
22	2650.1	2729.5	2810.1	2891.9	2975.1	3059.7	3145.9	3233.6	3323.1	3414.2	23
23	2651.4	2730.8	2811.4	2893.3	2976.5	3061.2	3147.4	3235.1	3324.6	3415.8	24
24	2652.7	2732.2	2812.8	2894.7	2977.9	3062.6	3148.8	3236.6	3326.1	3417.3	25
25	2654.0	2733.5	2814.1	2896.0	2979.3	3064.0	3150.3	3238.1	3327.6	3418.8	26
26	2655.3	2734.8	2815.5	2897.4	2980.7	3065.4	3151.7	3239.5	3329.1	3420.4	27
27	2656.6	2736.2	2816.8	2898.8	2982.1	3066.9	3153.2	3241.0	3330.6	3421.9	28
28	2657.9	2737.5	2818.2	2900.2	2983.5	3068.3	3154.6	3242.5	3332.1	3423.5	29
29	2659.2	2738.8	2819.5	2901.5	2984.9	3069.7	3156.1	3244.0	3333.6	3425.0	30
30	2660.5	2740.2	2820.9	2902.9	2986.3	3071.1	3157.5	3245.5	3335.1	3426.5	31
31	2661.8	2741.5	2822.3	2904.3	2987.7	3072.6	3159.0	3246.9	3336.6	3428.1	32
32	2663.1	2742.9	2823.6	2905.7	2989.1	3074.0	3160.4	3248.4	3338.1	3429.6	33
33	2664.4	2744.2	2825.0	2907.1	2990.5	3075.4	3161.9	3249.9	3339.6	3431.2	34
34	2665.7	2745.5	2826.3	2908.4	2991.9	3076.9	3163.3	3251.4	3341.1	3432.7	35
35	2667.0	2746.9	2827.7	2909.8	2993.3	3078.3	3164.8	3252.9	3342.7	3434.2	36
36	2668.3	2748.2	2829.0	2911.2	2994.7	3079.7	3166.2	3254.4	3344.2	3435.8	37
37	2669.6	2749.5	2830.4	2912.6	2996.1	3081.1	3167.7	3255.8	3345.7	3437.3	38
38	2670.9	2750.9	2831.8	2914.0	2997.5	3082.6	3169.1	3257.3	3347.2	3438.9	39
39	2672.2	2752.2	2833.1	2915.5	2998.9	3084.0	3170.6	3258.8	3348.7	3440.4	40
40	2673.5	2753.5	2834.5	2916.7	3000.3	3085.4	3172.1	3260.3	3350.1	3442.0	41
41	2674.8	2754.9	2835.8	2918.1	3001.8	3086.9	3173.5	3261.8	3351.7	3443.5	42
42	2676.1	2756.2	2837.2	2919.5	3003.2	3088.3	3175.0	3263.3	3353.2	3445.0	43
43	2677.4	2757.6	2838.6	2920.9	3004.6	3089.7	3176.4	3264.7	3354.8	3446.6	44
44	2678.7	2758.9	2839.9	2922.3	3006.0	3091.2	3177.9	3266.2	3356.3	3448.1	45
45	2680.0	2760.2	2841.3	2923.6	3007.4	3092.6	3179.3	3267.7	3357.8	3449.7	46
46	2681.3	2761.5	2842.6	2925.0	3008.8	3094.0	3180.8	3269.2	3359.3	3451.2	47
47	2682.6	2762.9	2844.0	2926.4	3010.2	3095.5	3182.3	3270.7	3360.8	3452.8	48
48	2683.9	2764.3	2845.4	2927.8	3011.6	3096.9	3183.7	3272.2	3362.3	3454.3	49
49	2685.2	2765.6	2846.7	2929.2	3013.0	3098.3	3185.2	3273.7	3363.9	3455.9	50
50	2686.5	2766.9	2848.1	2930.6	3014.4	3099.8	3186.6	3275.2	3365.4	3457.4	51
51	2687.8	2768.3	2849.5	2932.0	3015.8	3101.2	3188.1	3276.6	3366.9	3459.0	52
52	2689.1	2769.6	2850.8	2933.3	3017.2	3102.6	3189.6	3278.1	3368.4	3460.5	53
53	2690.4	2771.0	2852.2	2934.7	3018.7	3104.1	3191.0	3279.6	3369.9	3462.1	54
54	2691.7	2772.3	2853.6	2936.1	3020.1	3105.6	3192.5	3281.1	3371.5	3463.6	55
55	2693.0	2773.7	2854.9	2937.5	3021.5	3107.0	3194.0	3282.6	3373.0	3465.2	56
56	2694.3	2775.0	2856.3	2938.9	3022.9	3108.4	3195.4	3284.1	3374.5	3466.7	57
57	2695.6	2776.4	2857.7	2940.3	3024.3	3109.8	3196.9	3285.6	3376.0	3468.3	58
58	2696.9	2777.7	2859.1	2941.7	3025.7	3111.2	3198.4	3287.1	3377.5	3469.8	59
59	2698.2	2779.0	2860.5	2943.1	3027.1	3112.7	3199.8	3288.6	3379.1	3471.4	60
60	2699.5	2780.4	2861.8	2944.4	3028.5	3114.1	3201.3	3290.1	3380.6	3473.0	61
n.n.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.

MÉRIDIONAL PARTS. 143. 269

D. l.	50	51	52	53	54	55	56	57	58	59	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	3474.5	3568.8	3665.2	3763.8	3864.7	3968.0	4073.9	4182.6	4294.3	4409.2	0
1	3476.1	3570.4	3666.9	3765.5	3866.4	3969.7	4075.7	4184.5	4296.2	4411.1	1
2	3477.6	3572.0	3668.5	3767.1	3868.1	3971.5	4077.5	4186.3	4298.1	4413.1	2
3	3479.2	3573.6	3670.1	3768.8	3869.8	3973.2	4079.3	4188.2	4300.0	4415.0	3
4	3480.7	3575.2	3671.7	3770.4	3871.5	3975.0	4081.1	4190.0	4301.9	4417.0	4
5	3482.3	3576.8	3673.4	3772.1	3873.2	3976.7	4082.9	4191.8	4303.8	4418.9	5
6	3483.9	3578.4	3675.0	3773.8	3874.9	3978.4	4084.7	4193.7	4305.7	4420.8	6
7	3485.4	3580.0	3676.6	3775.4	3876.6	3980.2	4086.5	4195.5	4307.6	4422.8	7
8	3487.0	3581.6	3678.2	3777.1	3878.3	3982.0	4088.3	4197.4	4309.5	4424.7	8
9	3488.5	3583.2	3679.9	3778.8	3880.0	3983.7	4090.1	4199.2	4311.4	4426.7	9
10	3490.1	3584.8	3681.5	3780.4	3881.7	3985.5	4091.9	4201.1	4313.2	4428.6	10
11	3491.7	3586.4	3683.1	3782.1	3883.4	3987.2	4093.7	4202.9	4315.1	4430.6	11
12	3493.2	3588.0	3684.8	3783.8	3885.1	3989.0	4095.5	4204.7	4317.0	4432.5	12
13	3494.8	3589.5	3686.4	3785.5	3886.8	3990.7	4097.3	4206.6	4318.9	4434.5	13
14	3496.3	3591.1	3688.0	3787.1	3888.6	3992.5	4099.1	4208.4	4320.8	4436.4	14
15	3497.9	3592.7	3689.7	3788.8	3890.3	3994.2	4100.9	4210.3	4322.7	4438.4	15
16	3499.5	3594.3	3691.3	3790.5	3892.0	3996.0	4102.7	4212.1	4324.6	4440.4	16
17	3501.0	3595.9	3692.9	3792.1	3893.7	3997.7	4104.5	4214.0	4326.5	4442.3	17
18	3502.6	3597.5	3694.6	3793.8	3895.4	3999.5	4106.3	4215.8	4328.4	4444.3	18
19	3504.2	3599.1	3696.2	3795.5	3897.1	4001.3	4108.1	4217.7	4330.3	4446.2	19
20	3505.7	3600.7	3697.8	3797.2	3898.8	4003.0	4109.9	4219.5	4332.2	4448.2	20
21	3507.3	3602.3	3699.5	3798.8	3900.5	4004.8	4111.7	4221.4	4334.2	4450.2	21
22	3508.9	3603.9	3701.1	3800.5	3902.3	4006.5	4113.5	4223.2	4336.1	4452.1	22
23	3510.5	3605.5	3702.7	3802.2	3904.0	4008.3	4115.3	4225.1	4338.0	4454.1	23
24	3512.0	3607.1	3704.4	3803.9	3905.7	4010.0	4117.1	4227.0	4339.9	4456.0	24
25	3513.6	3608.7	3706.0	3805.5	3907.4	4011.8	4118.9	4228.8	4341.8	4458.0	25
26	3515.1	3610.3	3707.7	3807.2	3909.1	4013.6	4120.7	4230.7	4343.7	4460.0	26
27	3516.7	3611.9	3709.3	3808.9	3910.9	4015.3	4122.5	4232.5	4345.6	4461.9	27
28	3518.3	3613.6	3710.9	3810.6	3912.6	4017.1	4124.3	4234.4	4347.5	4463.9	28
29	3519.8	3615.2	3712.6	3812.3	3914.3	4018.9	4126.1	4236.2	4349.4	4465.9	29
30	3521.4	3616.8	3714.2	3813.9	3916.0	4020.6	4127.9	4238.1	4351.3	4467.8	30
31	3523.0	3618.4	3715.9	3815.6	3917.7	4022.4	4129.7	4240.0	4353.3	4469.8	31
32	3524.6	3620.0	3717.5	3817.3	3919.5	4024.2	4131.6	4241.8	4355.2	4471.8	32
33	3526.1	3621.6	3719.2	3819.0	3921.2	4025.9	4133.4	4243.7	4357.1	4473.8	33
34	3527.7	3623.2	3720.8	3820.7	3922.9	4027.7	4135.2	4245.6	4359.0	4475.7	34
35	3529.3	3624.8	3722.4	3822.3	3924.6	4029.5	4137.0	4247.4	4360.9	4477.7	35
36	3530.9	3626.4	3724.1	3824.0	3926.4	4031.2	4138.8	4249.3	4362.8	4479.7	36
37	3532.4	3628.0	3725.7	3825.7	3928.1	4033.0	4140.6	4251.2	4364.8	4481.7	37
38	3534.0	3629.6	3727.4	3827.4	3929.8	4034.8	4142.5	4253.0	4366.7	4483.6	38
39	3535.6	3631.3	3729.0	3829.1	3931.5	4036.6	4144.3	4254.9	4368.6	4485.6	39
40	3537.2	3632.9	3730.7	3830.8	3933.3	4038.3	4146.1	4256.8	4370.5	4487.6	40
41	3538.8	3634.5	3732.3	3832.5	3935.0	4040.1	4147.9	4258.6	4372.5	4489.6	41
42	3540.3	3636.1	3734.0	3834.2	3936.7	4041.9	4149.7	4260.5	4374.4	4491.6	42
43	3541.9	3637.7	3735.6	3835.8	3938.5	4043.6	4151.6	4262.4	4376.3	4493.5	43
44	3543.5	3639.3	3737.3	3837.5	3940.2	4045.4	4153.4	4264.3	4378.2	4495.5	44
45	3545.1	3640.9	3738.9	3839.2	3941.9	4047.2	4155.2	4266.1	4380.1	4497.5	45
46	3546.7	3642.5	3740.6	3840.9	3943.7	4049.0	4157.0	4268.0	4382.1	4499.5	46
47	3548.2	3644.2	3742.2	3842.6	3945.4	4050.8	4158.8	4269.9	4384.0	4501.5	47
48	3549.8	3645.8	3743.9	3844.3	3947.1	4052.5	4160.7	4271.8	4385.9	4503.5	48
49	3551.4	3647.4	3745.6	3846.0	3948.9	4054.3	4162.5	4273.6	4387.9	4505.5	49
50	3553.0	3649.0	3747.2	3847.7	3950.6	4056.1	4164.3	4275.5	4389.8	4507.5	50
51	3554.6	3650.6	3748.9	3849.4	3952.3	4057.9	4166.2	4277.4	4391.7	4509.4	51
52	3556.1	3652.3	3750.5	3851.1	3954.1	4059.7	4168.0	4279.3	4393.7	4511.4	52
53	3557.7	3653.9	3752.2	3852.8	3955.8	4061.4	4169.8	4281.1	4395.6	4513.4	53
54	3559.3	3655.5	3753.8	3854.5	3957.6	4063.2	4171.7	4283.0	4397.5	4515.4	54
55	3560.9	3657.1	3755.5	3856.2	3959.3	4065.0	4173.5	4284.9	4399.5	4517.4	55
56	3562.5	3658.7	3757.2	3857.9	3961.0	4066.8	4175.3	4286.8	4401.4	4519.4	56
57	3564.1	3660.4	3758.8	3859.6	3962.8	4068.6	4177.2	4288.7	4403.4	4521.4	57
58	3565.7	3662.0	3760.5	3861.3	3964.5	4070.4	4179.0	4290.6	4405.3	4523.4	58
59	3567.3	3663.6	3762.2	3863.0	3966.3	4072.2	4180.8	4292.5	4407.2	4525.4	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	50	51	52	53	54	55	56	57	58	59	D. l.

MERIDIONAL PARTS. 143. 269

D. l.	50	51	52	53	54	55	56	57	58	59	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	3474.5	3568.8	3665.2	3763.8	3864.7	3968.0	4073.9	4182.6	4294.5	4409.2	0
1	3476.1	3570.4	3666.9	3765.5	3866.4	3969.7	4075.7	4184.5	4296.2	4411.1	1
2	3477.6	3572.0	3668.5	3767.1	3868.1	3971.5	4077.5	4186.3	4298.1	4413.1	2
3	3479.2	3573.6	3670.1	3768.8	3869.8	3973.2	4079.3	4188.2	4300.0	4415.0	3
4	3480.7	3575.2	3671.7	3770.4	3871.5	3975.0	4081.1	4190.0	4301.9	4417.0	4
5	3482.3	3576.8	3673.4	3772.1	3873.2	3976.7	4082.9	4191.8	4303.8	4418.9	5
6	3483.9	3578.4	3675.0	3773.8	3874.9	3978.4	4084.7	4193.7	4305.7	4420.8	6
7	3485.4	3580.0	3676.6	3775.4	3876.6	3980.2	4086.5	4195.5	4307.6	4422.8	7
8	3487.0	3581.6	3678.2	3777.1	3878.3	3982.0	4088.3	4197.4	4309.5	4424.7	8
9	3488.5	3583.2	3679.9	3778.8	3880.0	3983.7	4090.1	4199.2	4311.4	4426.7	9
10	3490.1	3584.8	3681.5	3780.4	3881.7	3985.5	4091.9	4201.1	4313.2	4428.6	10
11	3491.7	3586.4	3683.1	3782.1	3883.4	3987.2	4093.7	4202.9	4315.1	4430.6	11
12	3493.2	3588.0	3684.8	3783.8	3885.1	3989.0	4095.5	4204.7	4317.0	4432.5	12
13	3494.8	3589.5	3686.4	3785.5	3886.8	3990.7	4097.3	4206.6	4318.9	4434.5	13
14	3496.3	3591.1	3688.0	3787.1	3888.6	3992.5	4099.1	4208.4	4320.8	4436.4	14
15	3497.9	3592.7	3689.7	3788.8	3890.3	3994.2	4100.9	4210.3	4322.7	4438.4	15
16	3499.5	3594.3	3691.3	3790.5	3892.0	3996.0	4102.7	4212.1	4324.6	4440.4	16
17	3501.0	3595.9	3692.9	3792.1	3893.7	3997.7	4104.5	4214.0	4326.5	4442.3	17
18	3502.6	3597.5	3694.6	3793.8	3895.4	3999.5	4106.3	4215.8	4328.4	4444.3	18
19	3504.2	3599.1	3696.2	3795.5	3897.1	4001.3	4108.1	4217.7	4330.3	4446.2	19
20	3505.7	3600.7	3697.8	3797.2	3898.8	4003.0	4109.9	4219.5	4332.2	4448.2	20
21	3507.3	3602.3	3699.5	3798.8	3900.5	4004.8	4111.7	4221.4	4334.2	4450.2	21
22	3508.9	3603.9	3701.1	3800.5	3902.3	4006.5	4113.5	4223.2	4336.1	4452.1	22
23	3510.5	3605.5	3702.7	3802.2	3904.0	4008.3	4115.3	4225.1	4338.0	4454.1	23
24	3512.0	3607.1	3704.4	3803.9	3905.7	4010.0	4117.1	4227.0	4339.9	4456.0	24
25	3513.6	3608.7	3706.0	3805.5	3907.4	4011.8	4118.9	4228.8	4341.8	4458.0	25
26	3515.1	3610.3	3707.7	3807.2	3909.1	4013.6	4120.7	4230.7	4343.7	4460.0	26
27	3516.7	3611.9	3709.3	3808.9	3910.9	4015.3	4122.5	4232.5	4345.6	4461.9	27
28	3518.3	3613.6	3710.9	3810.6	3912.6	4017.1	4124.3	4234.4	4347.5	4463.9	28
29	3519.8	3615.2	3712.6	3812.3	3914.3	4018.9	4126.1	4236.2	4349.4	4465.9	29
30	3521.4	3616.8	3714.2	3813.9	3916.0	4020.6	4127.9	4238.1	4351.3	4467.8	30
31	3523.0	3618.4	3715.9	3815.6	3917.7	4022.4	4129.7	4240.0	4353.3	4469.8	31
32	3524.6	3620.0	3717.5	3817.3	3919.5	4024.2	4131.6	4241.8	4355.2	4471.8	32
33	3526.1	3621.6	3719.2	3819.0	3921.2	4025.9	4133.4	4243.7	4357.1	4473.8	33
34	3527.7	3623.2	3720.8	3820.7	3922.9	4027.7	4135.2	4245.6	4359.0	4475.7	34
35	3529.3	3624.8	3722.4	3822.3	3924.6	4029.5	4137.0	4247.4	4360.9	4477.7	35
36	3530.9	3626.4	3724.1	3824.0	3926.4	4031.2	4138.8	4249.3	4362.8	4479.7	36
37	3532.4	3628.0	3725.7	3825.7	3928.1	4033.0	4140.6	4251.2	4364.8	4481.7	37
38	3534.0	3629.6	3727.4	3827.4	3929.8	4034.8	4142.5	4253.0	4366.7	4483.6	38
39	3535.6	3631.2	3729.0	3829.1	3931.5	4036.6	4144.3	4254.9	4368.6	4485.6	39
40	3537.2	3632.9	3730.7	3830.8	3933.3	4038.3	4146.1	4256.8	4370.5	4487.6	40
41	3538.8	3634.5	3732.3	3832.5	3935.0	4040.1	4147.9	4258.6	4372.5	4489.6	41
42	3540.3	3636.1	3734.0	3834.2	3936.7	4041.9	4149.7	4260.5	4374.4	4491.6	42
43	3541.9	3637.7	3735.6	3835.8	3938.5	4043.6	4151.6	4262.4	4376.3	4493.5	43
44	3543.5	3639.3	3737.5	3837.5	3940.2	4045.4	4153.4	4264.3	4378.2	4495.5	44
45	3545.1	3640.9	3738.9	3839.2	3941.9	4047.2	4155.2	4266.1	4380.1	4497.5	45
46	3546.7	3642.5	3740.6	3840.0	3943.7	4049.0	4157.0	4268.0	4382.1	4499.5	46
47	3548.2	3644.2	3742.2	3842.6	3945.4	4050.8	4158.8	4269.9	4384.0	4501.5	47
48	3549.8	3645.8	3743.9	3844.3	3947.1	4052.5	4160.7	4271.8	4385.9	4503.5	48
49	3551.4	3647.4	3745.6	3846.0	3948.9	4054.3	4162.5	4273.6	4387.9	4505.5	49
50	3553.0	3649.0	3747.2	3847.7	3950.6	4056.1	4164.3	4275.5	4389.8	4507.5	50
51	3554.6	3650.6	3748.9	3849.4	3952.3	4057.9	4166.2	4277.4	4391.7	4509.4	51
52	3556.1	3652.3	3750.5	3851.1	3954.1	4059.7	4168.0	4279.3	4393.7	4511.4	52
53	3557.7	3653.9	3752.2	3852.8	3955.8	4061.4	4169.8	4281.1	4395.6	4513.4	53
54	3559.3	3655.5	3753.8	3854.5	3957.6	4063.2	4171.7	4283.0	4397.5	4515.4	54
55	3560.9	3657.1	3755.5	3856.2	3959.3	4065.0	4173.5	4284.9	4399.5	4517.4	55
56	3562.5	3658.7	3757.2	3857.9	3961.0	4066.8	4175.3	4286.8	4401.4	4519.4	56
57	3564.1	3660.4	3758.8	3859.6	3962.8	4068.6	4177.2	4288.7	4403.4	4521.4	57
58	3565.7	3662.0	3760.5	3861.3	3964.5	4070.4	4179.0	4290.6	4405.3	4523.4	58
59	3567.3	3663.6	3762.2	3863.0	3966.3	4072.1	4180.8	4292.5	4407.2	4525.4	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	50	51	52	53	54	55	56	57	58	59	D. l.

270 MERIDIONAL PARTS. 143.

D. 1.	60	61	62	63	64	65	66	67	68	69	D. 1.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	4527.6	4699.2	4775.0	4805.0	4839.4	511.8	522.5	5474.0	5509.9	5714.0	0
1	4529.4	4631.3	4777.1	4907.2	5041.0	513.2	526.5	5476.8	5533.5	5791.4	1
2	4531.4	4623.4	4779.3	4909.4	5044.0	515.6	528.5	5479.2	5536.2	5800.2	2
3	4533.4	4625.5	4781.4	4911.6	5046.3	518.0	530.9	5481.7	5538.9	5809.0	3
4	4535.4	4627.5	4783.5	4913.8	5048.6	518.5	533.4	5484.3	5541.5	5809.5	4
5	4537.4	4629.6	4785.7	4916.0	5050.8	519.7	535.9	5486.9	5544.2	5808.6	5
6	4539.4	4631.7	4787.8	4918.2	5053.2	519.3	538.3	5489.4	5546.8	5811.4	6
7	4541.4	4633.7	4790.0	4920.4	5055.5	519.5	540.5	5492.0	5549.0	5814.2	7
8	4543.4	4635.8	4792.1	4922.6	5057.7	519.8	543.3	5494.6	5552.3	5817.0	8
9	4545.4	4637.9	4794.2	4924.8	5060.0	520.2	545.7	5497.1	5555.0	5819.8	9
10	4547.5	4639.9	4796.4	4927.1	5062.3	520.6	548.2	5499.7	5557.6	5822.6	10
11	4549.5	4642.0	4798.5	4929.3	5064.6	520.9	550.7	5502.3	5560.3	5825.4	11
12	4551.5	4644.1	4800.7	4931.5	5066.9	520.7	553.2	5504.9	5563.0	5828.2	12
13	4553.5	4646.2	4802.8	4933.7	5069.2	520.7	555.6	5507.4	5565.7	5831.0	13
14	4555.5	4648.2	4804.9	4935.9	5071.5	521.2	558.1	5510.0	5568.4	5833.9	14
15	4557.5	4650.3	4807.1	4938.1	5073.8	521.5	560.6	5512.6	5571.1	5836.7	15
16	4559.5	4652.4	4809.2	4940.4	5076.1	521.9	563.1	5515.2	5573.8	5839.5	16
17	4561.5	4654.5	4811.4	4942.6	5078.4	521.9	565.6	5517.8	5576.5	5842.3	17
18	4563.6	4656.6	4813.5	4944.8	5080.7	522.1	568.1	5520.4	5579.2	5845.1	18
19	4565.6	4658.6	4815.7	4947.0	5083.0	522.4	570.5	5523.0	5581.9	5848.0	19
20	4567.6	4660.7	4817.8	4949.1	5085.3	522.6	573.0	5525.6	5584.6	5850.8	20
21	4569.6	4662.8	4820.0	4951.5	5087.7	522.9	575.3	5528.2	5587.3	5853.7	21
22	4571.6	4664.9	4822.2	4953.7	5090.0	523.1	577.8	5530.8	5590.0	5856.5	22
23	4573.7	4667.0	4824.3	4956.0	5092.3	523.7	580.3	5533.4	5592.8	5859.3	23
24	4575.7	4669.1	4826.5	4958.2	5094.6	523.1	583.0	5536.0	5595.5	5862.2	24
25	4577.7	4671.2	4828.6	4960.4	5096.9	523.5	585.5	5538.6	5598.2	5865.0	25
26	4579.7	4673.3	4830.8	4962.7	5099.2	524.0	588.0	5541.2	5600.9	5867.9	26
27	4581.8	4675.4	4832.9	4964.9	5101.5	524.3	590.5	5543.8	5603.6	5870.7	27
28	4583.8	4677.4	4835.1	4967.1	5103.9	524.7	593.0	5546.4	5606.4	5873.5	28
29	4585.8	4679.5	4837.3	4969.4	5106.2	524.8	595.5	5549.0	5609.1	5876.4	29
30	4587.8	4681.6	4839.4	4971.6	5108.5	525.0	598.0	5551.6	5611.8	5879.2	30
31	4589.9	4683.7	4841.6	4973.9	5110.8	525.2	598.3	5554.2	5614.5	5882.1	31
32	4591.9	4685.7	4843.8	4976.1	5113.1	525.5	598.6	5556.8	5617.2	5885.0	32
33	4593.9	4687.9	4845.9	4978.3	5115.5	525.7	598.6	5559.3	5620.0	5887.8	33
34	4596.0	4690.0	4848.1	4980.6	5117.8	526.1	598.1	5561.8	5622.7	5890.7	34
35	4598.0	4692.1	4850.3	4982.8	5120.1	526.6	598.0	5564.7	5625.5	5893.6	35
36	4600.1	4694.2	4852.5	4985.1	5122.5	526.5	598.0	5567.3	5628.2	5896.4	36
37	4602.1	4696.3	4854.6	4987.3	5124.8	526.7	598.0	5569.9	5631.0	5899.3	37
38	4604.1	4698.4	4856.8	4989.6	5127.1	526.8	598.1	5572.6	5633.7	5902.2	38
39	4606.2	4699.5	4859.0	4991.8	5129.5	527.3	598.7	5575.2	5636.4	5905.0	39
40	4608.2	4701.6	4861.2	4994.1	5131.8	527.7	598.9	5577.8	5639.2	5907.9	40
41	4610.3	4703.7	4863.3	4996.3	5134.1	527.7	598.9	5580.5	5641.9	5910.8	41
42	4612.3	4705.9	4865.5	4998.6	5136.5	527.5	598.9	5583.1	5644.7	5913.7	42
43	4614.3	4708.0	4867.7	5000.9	5138.8	528.0	598.8	5585.7	5647.5	5916.6	43
44	4616.4	4710.1	4869.9	5003.1	5141.2	528.4	598.3	5588.4	5650.2	5919.5	44
45	4618.4	4712.2	4872.1	5005.4	5143.5	528.8	598.3	5591.0	5653.0	5922.4	45
46	4620.5	4714.3	4874.3	5007.6	5145.9	528.9	598.4	5593.7	5655.7	5925.2	46
47	4622.5	4716.4	4876.4	5009.9	5148.2	529.1	598.9	5596.3	5658.5	5928.1	47
48	4624.6	4718.5	4878.6	5012.2	5150.6	529.4	598.5	5598.9	5661.3	5931.0	48
49	4626.6	4720.6	4880.8	5014.4	5153.0	529.6	598.6	5601.6	5664.0	5933.9	49
50	4628.7	4722.7	4883.0	5016.7	5155.3	529.9	598.2	5604.2	5666.8	5936.8	50
51	4630.7	4724.8	4885.2	5019.0	5157.6	530.1	598.1	5606.9	5669.6	5939.7	51
52	4632.8	4726.9	4887.4	5021.2	5160.0	530.3	598.1	5609.6	5672.3	5942.6	52
53	4634.8	4729.0	4889.6	5023.5	5162.3	530.4	598.1	5612.2	5675.1	5945.5	53
54	4636.9	4731.1	4891.8	5025.8	5164.7	530.8	598.7	5614.9	5677.9	5948.4	54
55	4639.0	4733.2	4894.0	5028.1	5167.0	531.1	598.3	5617.5	5680.7	5951.3	55
56	4641.0	4735.3	4896.2	5030.3	5169.4	531.7	598.3	5620.2	5683.5	5954.2	56
57	4643.1	4737.4	4898.4	5032.6	5171.8	531.6	598.4	5622.9	5686.2	5957.1	57
58	4645.1	4739.5	4900.6	5034.9	5174.1	531.8	598.6	5625.6	5689.0	5960.0	58
59	4647.2	4741.6	4902.8	5037.2	5176.5	532.1	598.6	5628.2	5691.8	5962.9	59
60	4649.2	4743.7	4905.0	5039.5	5178.8	532.3	598.9	5630.9	5694.6	5965.8	60

MERIDIONAL PARTS. 143. 271

D. 1.	70	71	72	73	74	75	76	77	78	79	D. 2.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	59690	61457	63243	65043	66857	68685	70527	72383	74253	76137	0
1	59689	61456	63242	65042	66856	68684	70526	72382	74252	76136	1
2	59688	61455	63241	65041	66855	68683	70525	72381	74251	76135	2
3	59687	61454	63240	65040	66854	68682	70524	72380	74250	76134	3
4	59686	61453	63239	65039	66853	68681	70523	72379	74249	76133	4
5	59685	61452	63238	65038	66852	68680	70522	72378	74248	76132	5
6	59684	61451	63237	65037	66851	68679	70521	72377	74247	76131	6
7	59683	61450	63236	65036	66850	68678	70520	72376	74246	76130	7
8	59682	61449	63235	65035	66849	68677	70519	72375	74245	76129	8
9	59681	61448	63234	65034	66848	68676	70518	72374	74244	76128	9
10	59680	61447	63233	65033	66847	68675	70517	72373	74243	76127	10
11	59679	61446	63232	65032	66846	68674	70516	72372	74242	76126	11
12	59678	61445	63231	65031	66845	68673	70515	72371	74241	76125	12
13	59677	61444	63230	65030	66844	68672	70514	72370	74240	76124	13
14	59676	61443	63229	65029	66843	68671	70513	72369	74239	76123	14
15	59675	61442	63228	65028	66842	68670	70512	72368	74238	76122	15
16	59674	61441	63227	65027	66841	68669	70511	72367	74237	76121	16
17	59673	61440	63226	65026	66840	68668	70510	72366	74236	76120	17
18	59672	61439	63225	65025	66839	68667	70509	72365	74235	76119	18
19	59671	61438	63224	65024	66838	68666	70508	72364	74234	76118	19
20	59670	61437	63223	65023	66837	68665	70507	72363	74233	76117	20
21	59669	61436	63222	65022	66836	68664	70506	72362	74232	76116	21
22	59668	61435	63221	65021	66835	68663	70505	72361	74231	76115	22
23	59667	61434	63220	65020	66834	68662	70504	72360	74230	76114	23
24	59666	61433	63219	65019	66833	68661	70503	72359	74229	76113	24
25	59665	61432	63218	65018	66832	68660	70502	72358	74228	76112	25
26	59664	61431	63217	65017	66831	68659	70501	72357	74227	76111	26
27	59663	61430	63216	65016	66830	68658	70500	72356	74226	76110	27
28	59662	61429	63215	65015	66829	68657	70499	72355	74225	76109	28
29	59661	61428	63214	65014	66828	68656	70498	72354	74224	76108	29
30	59660	61427	63213	65013	66827	68655	70497	72353	74223	76107	30
31	59659	61426	63212	65012	66826	68654	70496	72352	74222	76106	31
32	59658	61425	63211	65011	66825	68653	70495	72351	74221	76105	32
33	59657	61424	63210	65010	66824	68652	70494	72350	74220	76104	33
34	59656	61423	63209	65009	66823	68651	70493	72349	74219	76103	34
35	59655	61422	63208	65008	66822	68650	70492	72348	74218	76102	35
36	59654	61421	63207	65007	66821	68649	70491	72347	74217	76101	36
37	59653	61420	63206	65006	66820	68648	70490	72346	74216	76100	37
38	59652	61419	63205	65005	66819	68647	70489	72345	74215	76099	38
39	59651	61418	63204	65004	66818	68646	70488	72344	74214	76098	39
40	59650	61417	63203	65003	66817	68645	70487	72343	74213	76097	40
41	59649	61416	63202	65002	66816	68644	70486	72342	74212	76096	41
42	59648	61415	63201	65001	66815	68643	70485	72341	74211	76095	42
43	59647	61414	63200	65000	66814	68642	70484	72340	74210	76094	43
44	59646	61413	63199	64999	66813	68641	70483	72339	74209	76093	44
45	59645	61412	63198	64998	66812	68640	70482	72338	74208	76092	45
46	59644	61411	63197	64997	66811	68639	70481	72337	74207	76091	46
47	59643	61410	63196	64996	66810	68638	70480	72336	74206	76090	47
48	59642	61409	63195	64995	66809	68637	70479	72335	74205	76089	48
49	59641	61408	63194	64994	66808	68636	70478	72334	74204	76088	49
50	59640	61407	63193	64993	66807	68635	70477	72333	74203	76087	50
51	59639	61406	63192	64992	66806	68634	70476	72332	74202	76086	51
52	59638	61405	63191	64991	66805	68633	70475	72331	74201	76085	52
53	59637	61404	63190	64990	66804	68632	70474	72330	74200	76084	53
54	59636	61403	63189	64989	66803	68631	70473	72329	74199	76083	54
55	59635	61402	63188	64988	66802	68630	70472	72328	74198	76082	55
56	59634	61401	63187	64987	66801	68629	70471	72327	74197	76081	56
57	59633	61400	63186	64986	66800	68628	70470	72326	74196	76080	57
58	59632	61399	63185	64985	66799	68627	70469	72325	74195	76079	58
59	59631	61398	63184	64984	66798	68626	70468	72324	74194	76078	59
60	59630	61397	63183	64983	66797	68625	70467	72323	74193	76077	60

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D. l.	80	81	82	83	84	85	86	87	88	89	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	8375,3	8739,1	9145,5	9605,9	10137,0	10764,7	11532,6	12522,3	13916,6	16999,8	0
1	8381,0	8745,5	9152,7	9612,1	10143,6	10776,2	11547,0	12541,4	13945,4	16997,5	1
2	8386,8	8751,9	9159,9	9618,4	10149,1	10782,7	11553,1	12547,5	13951,5	16995,2	2
3	8392,6	8758,3	9167,2	9624,7	10155,2	10789,0	11559,2	12553,6	13957,6	16992,9	3
4	8398,3	8764,8	9174,4	9631,0	10161,3	10795,3	11565,3	12559,7	13963,7	16990,6	4
5	8404,1	8771,2	9181,6	9637,2	10167,4	10801,6	11571,4	12565,8	13969,8	16988,3	5
6	8409,9	8777,7	9188,9	9643,5	10173,5	10807,9	11577,5	12571,9	13975,9	16986,0	6
7	8415,8	8784,1	9196,2	9649,8	10179,6	10814,2	11583,6	12578,0	13982,0	16983,7	7
8	8421,6	8790,6	9203,5	9656,1	10185,7	10820,5	11589,7	12584,1	13988,1	16981,4	8
9	8427,4	8797,1	9210,8	9662,4	10191,8	10826,8	11595,8	12590,2	13994,2	16979,1	9
10	8433,3	8803,6	9218,1	9668,7	10197,9	10833,1	11601,9	12596,3	13999,3	16976,8	10
11	8439,1	8810,1	9225,4	9675,0	10204,0	10839,4	11608,0	12602,4	14005,4	16974,5	11
12	8445,0	8816,6	9232,7	9681,3	10210,1	10845,7	11614,1	12608,5	14011,5	16972,2	12
13	8450,9	8823,2	9240,0	9687,6	10216,2	10852,0	11620,2	12614,6	14017,6	16969,9	13
14	8456,8	8829,7	9247,3	9693,9	10222,3	10858,3	11626,3	12620,7	14023,7	16967,6	14
15	8462,6	8836,3	9254,6	9699,2	10228,4	10864,6	11632,4	12626,8	14029,8	16965,3	15
16	8468,5	8842,8	9261,9	9705,5	10234,5	10870,9	11638,5	12632,9	14035,9	16963,0	16
17	8474,4	8849,4	9269,2	9711,8	10240,6	10877,2	11644,6	12639,0	14042,0	16960,7	17
18	8480,3	8856,0	9276,5	9718,1	10246,7	10883,5	11650,7	12645,1	14048,1	16958,4	18
19	8486,2	8862,6	9283,8	9724,4	10252,8	10889,8	11656,8	12651,2	14054,2	16956,1	19
20	8492,1	8869,2	9291,1	9730,7	10258,9	10896,1	11662,9	12657,3	14060,3	16953,8	20
21	8498,0	8875,8	9298,4	9737,0	10265,0	10902,4	11669,0	12663,4	14066,4	16951,5	21
22	8504,0	8882,4	9305,7	9743,3	10271,1	10908,7	11675,1	12669,5	14072,5	16949,2	22
23	8510,0	8889,0	9313,0	9749,6	10277,2	10915,0	11681,2	12675,6	14078,6	16946,9	23
24	8516,0	8895,6	9320,3	9755,9	10283,3	10921,3	11687,3	12681,7	14084,7	16944,6	24
25	8522,0	8902,2	9327,6	9762,2	10289,4	10927,6	11693,4	12687,8	14090,8	16942,3	25
26	8528,0	8908,8	9334,9	9768,5	10295,5	10933,9	11699,5	12693,9	14096,9	16940,0	26
27	8534,0	8915,4	9342,2	9774,8	10301,6	10940,2	11705,6	12699,0	14103,0	16937,7	27
28	8540,0	8922,0	9349,5	9781,1	10307,7	10946,5	11711,7	12705,1	14109,1	16935,4	28
29	8546,0	8928,6	9356,8	9787,4	10313,8	10952,8	11717,8	12711,2	14115,2	16933,1	29
30	8552,0	8935,2	9364,1	9793,7	10319,9	10959,1	11723,9	12717,3	14121,3	16930,8	30
31	8558,0	8941,8	9371,4	9799,0	10326,0	10965,4	11730,0	12723,4	14127,4	16928,5	31
32	8564,0	8948,4	9378,7	9805,3	10332,1	10971,7	11736,1	12729,5	14133,5	16926,2	32
33	8570,0	8955,0	9386,0	9811,6	10338,2	10978,0	11742,2	12735,6	14139,6	16923,9	33
34	8576,0	8961,6	9393,3	9817,9	10344,3	10984,3	11748,3	12741,7	14145,7	16921,6	34
35	8582,0	8968,2	9400,6	9824,2	10350,4	10990,6	11754,4	12747,8	14151,8	16919,3	35
36	8588,0	8974,8	9407,9	9830,5	10356,5	10996,9	11760,5	12753,9	14157,9	16917,0	36
37	8594,0	8981,4	9415,2	9836,8	10362,6	10993,2	11766,6	12759,0	14164,0	16914,7	37
38	8600,0	8988,0	9422,5	9843,1	10368,7	10999,5	11772,7	12765,1	14170,1	16912,4	38
39	8606,0	8994,6	9429,8	9849,4	10374,8	11005,8	11778,8	12771,2	14176,2	16910,1	39
40	8612,0	9001,2	9437,1	9855,7	10380,9	11012,1	11784,9	12777,3	14182,3	16907,8	40
41	8618,0	9007,8	9444,4	9862,0	10386,0	11018,4	11791,0	12783,4	14188,4	16905,5	41
42	8624,0	9014,4	9451,7	9868,3	10392,1	11024,7	11797,1	12789,5	14194,5	16903,2	42
43	8630,0	9021,0	9459,0	9874,6	10398,2	11031,0	11803,2	12795,6	14200,6	16900,9	43
44	8636,0	9027,6	9466,3	9880,9	10404,3	11037,3	11809,3	12801,7	14206,7	16898,6	44
45	8642,0	9034,2	9473,6	9887,2	10410,4	11043,6	11815,4	12807,8	14212,8	16896,3	45
46	8648,0	9040,8	9480,9	9893,5	10416,5	11049,9	11821,5	12813,9	14218,9	16894,0	46
47	8654,0	9047,4	9488,2	9899,8	10422,6	11056,2	11827,6	12819,0	14225,0	16891,7	47
48	8660,0	9054,0	9495,5	9906,1	10428,7	11062,5	11833,7	12825,1	14231,1	16889,4	48
49	8666,0	9060,6	9502,8	9912,4	10434,8	11068,8	11839,8	12831,2	14237,2	16887,1	49
50	8672,0	9067,2	9510,1	9918,7	10440,9	11075,1	11845,9	12837,3	14243,3	16884,8	50
51	8678,0	9073,8	9517,4	9925,0	10447,0	11081,4	11852,0	12843,4	14249,4	16882,5	51
52	8684,0	9080,4	9524,7	9931,3	10453,1	11087,7	11858,1	12849,5	14255,5	16880,2	52
53	8690,0	9087,0	9532,0	9937,6	10459,2	11094,0	11864,2	12855,6	14261,6	16877,9	53
54	8696,0	9093,6	9539,3	9943,9	10465,3	11100,3	11870,3	12861,7	14267,7	16875,6	54
55	8702,0	9100,2	9546,6	9950,2	10471,4	11106,6	11876,4	12867,8	14273,8	16873,3	55
56	8708,0	9106,8	9553,9	9956,5	10477,5	11112,9	11882,5	12873,9	14279,9	16871,0	56
57	8714,0	9113,4	9561,2	9962,8	10483,6	11119,2	11888,6	12879,0	14286,0	16868,7	57
58	8720,0	9120,0	9568,5	9969,1	10489,7	11125,5	11894,7	12885,1	14292,1	16866,4	58
59	8726,0	9126,6	9575,8	9975,4	10495,8	11131,8	11900,8	12891,2	14298,2	16864,1	59
60	8732,0	9133,2	9583,1	9981,7	10501,9	11138,1	11906,9	12897,3	14304,3	16861,8	60

END OF BOOK VIII.



THE
ELEMENTS
OF
NAVIGATION.

BOOK IX,
OF DAYS WORKS.

I. **B**Y DAYS WORKS is here meant the practical methods of finding a ship's place every day at noon, and settling the course and distance she has then to fail.

The navigator requires for this purpose the following elements, or necessary things, to be known.

- I. The means of measuring and correcting the run of the ship.
- II. The nature and use of the sea and azimuth compasses.
- III. How to find the Sun's amplitude and azimuth.
- IV. How to find the variation of the compass, and thereby to correct the courses the ship has sailed.
- V. How to find the ship's lee-way, and thence correct the courses.
- VI. The nature and use of the instruments proper for taking the altitudes and distances of celestial objects.
- VII. To find the latitude and longitude by celestial observations.
- VIII. How to correct the daily account of latitude and longitude.

SECTION I.

2. *Of the measuring a Ship's run.*

A great variety of methods have been proposed to measure the rate at which a ship sails, but that mostly in use at this time, especially among the English, is by the *log* and *half minute glass*.

The *LOG* is a piece of thin board of a sectoral or quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water: To this is fastened a line of about 150 fathoms long, called the *LOG-LINE*, which is divided into certain spaces, called knots; and the log-line is wound on a reel which turns very easily.

The *HALF MINUTE GLASS* is of the make of an *hour glass*, and contains such a quantity of fine sand as will run through the hole in its neck in half a minute of time.

The making of the experiment to find the velocity of the ship, is called *HEAVING THE LOG*, which is thus performed.

Two men hold the reel between them, and another holds the half minute glass; an officer of the watch throws the log over the ship's stern, on the lee side, and when he observes the stray line is run off the reel (which is about 10 fathoms, this distance being usually allowed to carry the log out of the eddy of the ship's wake), and the first mark is going off, he cries *turn!* the glass-holder answers *done!* who watching the glass, the moment it is run out says *stop!* the reel being immediately stopt, the last mark run off shews the number of knots, and the distance of that mark from the reel is estimated in fathoms: Then the knots and fathoms together shew the distance the ship has run the preceding hour, if the wind has been constant.

In the King's ships, India ships, and some others, it is usual to heave the log every hour: But coasters, and those using short voyages, heave the log once in two hours only.

3. This practice of measuring a ship's rate of sailing, is founded upon the following principle.

That the length of each knot, ought to be the same part of a sea mile; as half a minute is, of an hour.

Therefore the length of each knot should be either

$$\left(\frac{1}{2} \times \frac{1}{60} \right) = \frac{1}{120} \text{ of a nautical mile.}$$

$$\text{Or } \left(\frac{1}{2} \times \frac{1}{60} \times \frac{1}{60} \right) = \frac{1}{7200} \text{ of a degree.}$$

$$\text{Or } \left(\frac{1}{2} \times \frac{1}{60} \times \frac{1}{60} \times \frac{1}{3600} \right) = \frac{1}{2592000} \text{ of a great circle.}$$

By Mr. Richard Norwood's experiment (see V. 73) it appears that a degree of a great circle on the Earth contains 367200 English feet and $\frac{1}{80}$ th thereof, or 6120 feet, are in one nautical mile.

Hence $\frac{1}{120}$ th of 6120, or 51 feet, should be the length of each knot. But because it is safer to have the reckoning rather before the ship than after it, therefore 50 feet may be taken as the proper length of each knot.

And as most men can conveniently fathom with their arms about five feet; so a knot may be supposed to consist of 10 fathoms, each of five feet.

The

The length of the knots in the log-line used formerly, were only of 42 feet; and it is much to be wished that not any line so divided was now in use, but custom in many things prevails over reason: This way of dividing the log-line was founded on the supposition that 60 miles, each of 5000 English feet, made a degree; for $\frac{1}{120}$ of 5000 is $41\frac{2}{3}$; or in round numbers 42 feet. And although many mariners find by experience that this length of the knot is too short, yet rather than quit the old way, they use glasses for half minute ones that run but 24 or 25 seconds.

The method usually recommended to try if the glasses are accurate, is: On a round nail or peg, hang a string that has a musket-ball fixed to one end, carefully measuring between the center of the ball and the string's loop over the peg 39 $\frac{1}{2}$ inches: Then swing it, and count one for every time it passes under the peg, beginning at the second time it passes, and the number of swings made during the time the glass is running out, shews the seconds it contains.

For experience shews, that the length of a second pendulum is about 39 $\frac{1}{2}$ inches.

The distance given by the log may be wrong on three accounts, namely, by there being an error either in the glass, or log, or in both.

4. CASE I. *When the log-line is truly divided, and the glass is faulty.*

RULE. As the seconds run by the glass, is to 30 seconds;

So is the distance run by the log, to the true distance.

EXAM. Suppose a ship sails at the rate of $6\frac{1}{2}$ knots while a glass runs out, which is but 25 seconds: What is the true rate of sailing?

As 25 : 30 :: 6,5 : 7,8 miles, the true distance sailed in an hour.

5. CASE II. *When the glass is true, and the log-line is faulty.*

RULE. As 50 feet, is to the distance measured between knot and knot;

So is the distance run by the log, to the true distance.

EXAM. Suppose a ship sails at the rate of $6\frac{1}{2}$ knots an hour, by a log-line which is only 44 feet to a knot: What is the true rate of sailing?

As 50 : 44 :: 6,5 : 5,72 miles, the true distance sailed in an hour.

6. CASE III. *When both log-line and glass are faulty.*

RULE. Multiply thrice the measured length of a knot by the distance run per log; the product divided by five times the measured time of the glass, will give the true distance run.

EXAM. Suppose a ship runs 5 knots an hour, by a log-line of 45 feet to a knot, and a glass of 25 seconds: What is the true rate of sailing?

Then $3 \times 45 \times 5$, and divided by 5×25 , gives 5,4 miles = true dist. For suppose K = meas. length of a knot; G = seconds per glass; D = run per log.

Then $G : 30 :: D : 30 \times D \div *G$ = dist. run by a correct glass. (4)

And $50 : K :: 30 \times D \div G : 30 \times D \times K \div G \times 50 = 3 \times D \times K \div 5 \times G$.

* The mark $\div$ signifies divided by.

As the log is subject to drive with the motion which the water may have at its surface, and thereby give an erroneous rate of sailing (for the experiment supposes the *log* as fixed in the place where it is, when the commencement of the knots goes off the *reel*); therefore many methods have been^d proposed to remove, or at least to lessen this error; and among them, that seemed worthy of notice, which was proposed by the late M. *Bouguer**, in his *Treatise of Navigation*, published at Paris in the year 1753, and which was composed by the order of the Minister of State; and again reprinted there in the year 1760, with some alterations by the late Abbé de la Caille, especially as both of those gentlemen had been separately engaged in very long voyages, and consequently had much marine experience, therefore the substance of this method will be here given, though somewhat differently drawn up.

7. Take for the *Log* a conical piece of wood, which fix to the log-line (passed through or along its axis), at about 40, 50, 60, or more feet from one end; and to this end fix the *DIVER*, which is a body formed of two equal square pieces of tin, or of thin iron plate, fixed at right angles to one another along their diagonals, and its size so fitted to that of the cone, that the whole may float.

A cone of three inches diameter in the base, and of six inches in the flant height, is proposed by M. *Bouguer*, to suit a diver made of plates about $9\frac{1}{4}$ inches square: The intersection of the diagonals is joined to the log-line; and the *loop* and *peg* fixed as in the common log.

Now when this compound log is hove overboard, the diver will sink too deep to be much affected by the current, or motion of the water at the surface; and the log will thereby keep more steadily in the place where it first fell; and consequently, the knots run off the reel will shew more accurately the ship's rate of sailing.

As the common log is affected by the whole motion of the current, so this compound log will feel only a part thereof; viz. such a part nearly, as the resistance of the *cone* is of the resistance of the *diver*: thus the resistances of the above cone and diver are about as 1 to 5; and consequently this log will drive but $\frac{1}{5}$ part of what the common log would; and so the ship's true run will be affected by $\frac{4}{5}$ part only of the motion of the waters.

8. To obtain the true rate of sailing, it will be proper to heave alternately, hour and hour, the common log and this compound log: Then the difference of their knots run off, augmented by its $\frac{4}{5}$ th part, is the correction; which applied to the knots of the common log, will give the ship's true rate of sailing, at the middle time between the hours when those logs were hove. The correction is additive when the compound log's run is greatest; otherwise it is subtractive.

9. To find the *course made good*. Increase the observed angle between the log-lines by one fourth part; and this gives the correction to be applied to the apparent course, or the opposite of that shewn by the common log.

* The French Academicians concerned in measuring a degree of a meridian in Peru.

The correction is to be applied to the $\left\{ \begin{smallmatrix} \text{right} \\ \text{left} \end{smallmatrix} \right\}$ of the apparent course, when the bearing of the common log is to the $\left\{ \begin{smallmatrix} \text{left} \\ \text{right} \end{smallmatrix} \right\}$ of the comp. log.

10. Or thus. The lengths run off both logs, together with their bearings, being known: Then on a card or compass apply the knots run off (taken from a scale of equal parts) along their respective bearings, from the center; join the ends, and in this line produced, on the side next the compound log's length, take $\frac{1}{2}$ of the interval; then a line drawn from the end, thus produced, to the center of the card, will shew the true course and distance made good.

Or. The true course and distance may be found by computation; by working in the two triangles formed by the above construction.

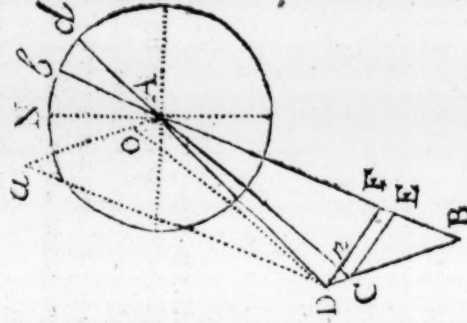
Ex. I. *The common log having run 6,7 knots, bore N. 42° E. at 9 h. 30 m. A M. and at 11 h. 30 m. A M. the compound log having run off 8,3 knots, it bore N. 37° E. Required the correct course and rate of sailing at 10 h. A. M.?*

Ex. II. *At 5 h. P. M. the common log having run off 7,6 knots, it bore S. 18° W.; and at 7 h. P. M. the compound log having run off 5,2 knots, it bore S. 26° W. Required the true course and rate at 6 h. P. M.?*

Comm. log.	Run 6,7 k.	Bear. N. 42° E.
Comp. log.	8,3 k.	N. 37° E.
Diff.	1,6	5
$\frac{1}{2}$ part	0,4	$1\frac{1}{4}$
Correction	2,0	$6\frac{1}{4}$
True run	8,7	Bear. S. $35\frac{1}{4}^{\circ}$ W.

Comm. log.	Run 7,6 k.	Bear. S. 18° W.
Comp. log.	5,2	S. 26° W.
Diff.	2,4	8
$\frac{1}{2}$ part	0,6	2
Correction	3,0	10
True run	4,6	Bear. N. 28° E.

II. The following explanation of Ex. II. will shew the reason of these corrections. Suppose the line AB to represent the run and bearing of the ship by the common log; and AC the run and bearing by the compound log; and let the ship be supposed at a when the logs were hove: Then her apparent run by the wind, with the common log, is the line Da, or its parallel BA, while the log is drove by the current along the line DB; so that the ship's real run is along the line DA (VII. 48). Also, by the compound log, the ship's apparent run by the wind is the line Do, or its parallel CA; while the log drives from D to C, which is only $\frac{1}{2}$ th part of DB; so the real run is DA. Take $EB = AB \oslash AC$; draw CE and its parallel DF; then is $EF = \frac{1}{2}EB$ (II. 165), AF is very nearly equal to AD, and $\angle DAC$ is very nearly equal to $\frac{1}{4}\angle BAC$.



For $AN = AF$ (II. 165); and AN in the time of making the experiment cannot differ much from AD ; neither can DN differ much from $\frac{1}{4}FN$; consequently the $\angle DAN$ is very nearly equal to $\frac{1}{4}\angle FAN$.

Therefore AF being known, gives the run DA : Also the bearing AB being known by observation, or its opposite AB ; and the $\angle ABD$ found by correction, the bearing DAd , or the real course, becomes known.

Or. Computing in the triangle BAC , with the two sides, and the included angle, find $\angle BCA$ and BC : Then in the triangle ACD , knowing AC , $CD (= \frac{1}{4}BC)$, and $\angle ACD$; find $\angle CAD$ and AD .

12.

REMARK.

I. There seems to be a mistake in the dimensions of the cone, and diver, proposed by M. Bouguer, and followed by M. La Caille; for no kind of wood used in British dock-yards, when formed into a cone of those dimensions, will float a diver made of stout tin plates, one side of the square being $9\frac{3}{4}$ inches. Such a diver, weighing $1\frac{1}{2}$ lb. ayerdupoise, required to float it, a cone of 5 inches diameter, and 12 inches on the slant side, so as the point of the cone which was made of light fir, should just appear above the water. Now supposing one side of such a square tin diver to be about 10 inches, and made of plates only $\frac{2}{3}$ of the thickness of the former (for it is suspected there is no tin of half its thickness), such a diver would weigh, with its folder, about 20 ounces, and can be floated by a light fir cone of 4 inches diameter in the base, and 10 inches in the slant height, or length; and such a compound log might, perhaps, be found on trial to be affected by about as much again as that proposed by M. Bouguer, and consequently the difference between the numbers given by the common log and compound logs, must be augmented by $\frac{2}{3}$ of itself, to produce the necessary correction.

13. II. When a current, such as a tide, runs to any depth, the velocity of that current may be much better ascertained by the compound log than by the common one, provided the diver does not descend lower than the run of the current; for as those ships which are deepest immersed, drive fastest with the tide; so the diver by being acted on below, as well as the log on the surface, their joint motion will give the total effort of the current's motion, better than what could be derived from the motion at the surface only. Also, by such a compound log, the depth to which any current runs, may be easily tried.

SECTION

SECTION II.
Of the Mariner's Compass.

14. The MARINER'S COMPASS is an instrument used at sea to shew, or point out the way a ship does, or is to, go; and consists of three parts, namely, the box, the card, or fly, and the needle.

The Box, which contains the card and needle, is a circular brass box hung within a square wooden one, by two concentric rings, called *jimbals*, so fixed by cross centers to the two boxes, that the inner one, or compass box, shall retain an horizontal position in all motions of the ship.

The CARD, or FLY, is a circle of stiff paper representing the horizon; and is divided into points and quarter-points of the compass.

The NEEDLE is a straight piece of steel made magnetical; whereby it has the property of pointing one of its ends towards the north pole of the world.

The needle is fixed to the under side of the card, and in the center is placed a conical socket, which is put on an upright pointed pin, fixed in the bottom of the box; so that the card hanging on the pin, turns freely round its center; one of the points being, by the property of the needle, always directed toward the north pole*.

The top of the box is covered with a glass pane, whereby the card in its motion is not disturbed by the wind. And this apparatus together, is called the *Compass*: And the compass box is to be so placed in the ship, that the middle section of the box, parallel to its sides, is to be parallel to the middle section of the ship, along its keel.

15. Before the invention of the compass, which was towards the latter end of the 13th century, the navigating of ships was a very tedious and precarious operation, and seldom out of sight of land: But the compass enables the mariner to hold his course over the seas in as direct and true a tract, as the land carrier directs his carriage in a well beaten road. Hence it might be reasonably imagined, that no necessary expence or care should ever be wanting in the construction of this most useful instrument: But it has so happened, that some years ago scarce one sea-compass in ten was fit for the use for which it was made; and this happened from their being fabricated by unskilful and ignorant workmen for the wholesale dealers in the shipping way: such dealers generally pay no more regard to the construction of this instrument, whereon the success of the voyage, and the lives of the men, in a great measure depend, than they do to any indifferent thing of the same price.

* If a magnet, or a piece of iron made magnetical, be suspended by a thread, or floated on a piece of light wood or cork in a vessel of water; then let the body be turned round either eastward or westward, one and the same point will always be turned towards the north when the body becomes at rest.

There are however compasses used in the Royal Navy, India ships, and some few trading ships, which are constructed with more care, and on better principles; and are therefore vastly preferable to those commonly used in the merchant's service; which, from their construction, seem as though they had been contrived purposely to vary from what was to be expected of them: Such as joining the outside box together with iron nails; making the needle of soft wire, and disposing it in the form of a rhombus, in expectation that the magnetical forces of the sides would conspire to act in the diagonal: Making the pin and socket so badly, as to prevent the traversing of the card, &c. &c.

But it must be observed, that it is only within a very few years that sea-compasses have been made free from the multitude of inconveniences which usually attended the best constructed of them: Such as, first, needles having several poles, occasioned by their irregular shape, the best kind being straight bars with flat ends. 2d. The needles being made of such a temper as was neither susceptible of receiving, or of retaining half the virtue it was possible to give them, and consequently they were not with due strength and perseverance directed towards the poles of the world, 3d. The want of proper means to restore their loss of magnetism while on a voyage. 4th. The troublesome and inaccurate methods of rectifying a damaged pin on which the card turned; and also the expence attending the use of agate caps or sockets, than which however none are so proper. 5th. The want of proper contrivances to hinder the card from being greatly affected by the various motions of the ship. These, and several other imperfections, have been happily removed by the labours of the truly celebrated Dr. *Gauss* Knight, F. R. S. whose admirable invention of giving magnetism to steel bars, greatly superior to any power they could derive from the natural loadstones, joined to a multitude of experiments which he has made for the marine service, have produced the means of constructing sea compasses so perfect, that there seems nothing farther to be wished for, as necessary to this instrument. It may be reasonably expected that such correct compasses will readily come into general use, as well in merchant ships as in ships of war; for Knight's compasses are now used in almost all the royal navy.

16. After the discovery of that most useful property of the magnet, or loadstone, namely, the giving hardened iron or steel a polarity, the compass was for many years used without knowing that its direction in any wise deviated from the poles of the world: And about the middle of the 16th century, so certain were some of its inflexibly pointing to the north, that they treated with contempt the notion of the variation, which about that time began to be suspected. However, careful observations soon discovered that in England, and its neighbourhood, the needle pointed to the eastward of the true north: But the quantity of this deviation being known, mariners became as well satisfied as if the compass had none; because they imagined that the true course could be obtained by making allowance for the true variation.

From successive observations made afterwards, it was found that the deviation of the needle from the north was not a constant quantity; but that it gradually diminished, and at last, namely about the year 1657, it was

was found at London that the needle pointed due north, and has ever since been getting to the westward, and now the variation is near 22 degrees to the westward of the north: So that in any one place it may be suspected the variation has a kind of libratory motion, traversing through the north to unknown limits eastward and westward. But the settling of this point must be left to time.

During the time of the said observations it was also discovered, that the variation of the needle was different in different parts of the world, it being west in some places when it was east in others; and in places where the variation was of the same name, yet the quantity of it greatly differed. It was therefore found necessary that mariners should, every day, or as often as they had opportunity, make, during their voyage, proper observations for an amplitude or azimuth, whereby they might be enabled to find the variation of the compass in their present place, and thence correct their courses.

17. Beside the common sea compass, there is another called the AZIMUTH COMPASS, whose use is to take the bearing of any celestial objects either when it is at, or above the horizon. This compass in no respect differs from the other, only the circumference of the card or box is divided into degrees; and there is fitted to the box an index with two sights, which are upright pieces of brass placed diametrically opposite, having slits down their middles, through which the sun or star is to be viewed at the time of observation.

18.

To make Artificial Magnets.

As there are many accidents at sea, which make it very convenient, and even necessary, for mariners to know how to restore the magnetic virtue to their damaged needles, it was judged proper to shew how, independent of a loadstone, magnetism might be given to steel bars; and with these bars to touch the compass needles.

The following method is extracted from a paper of Mr. John Canton's, F. R. S. in the Philosophical Transactions for the year 1750.

Procure of soft steel, six bars of about 3 inches long, $\frac{1}{4}$ inch broad, and $\frac{1}{16}$ thick; and of the hardest steel, other six, each of double those dimensions; and let each of the 12 bars be marked with a line round them at one end, which call the north end; and the other, the south end. Also of soft iron, four bars, called conductors; two, of the same thickness and breadth, but only half the length, of the soft bars; the other two, alike proportioned to the hard bars; have ready also an iron poker and tongs that have been long in use.

Let the poker be held nearly upright, with its point downwards; and let one of the soft steel bars, with its N. end downwards, be held tightly against the middle of the poker by a thread; then with the lower end of the tongs, held also nearly upright, stroke the steel bar from the N. end upwards, about 10 times, on both sides: And in this manner serve four of the soft steel bars; and thereby each will be able to suspend, by its N. end, a small key.

Lay

Lay the other two soft steel bars in a parallel position, at the distance of $\frac{1}{4}$ inch, with a N. and S. end to touch each of their proper conductors : Then placing two of the touched steel bars together, breadth to breadth, the N. end of one to the S. end of the other ; and the other two put in like manner against these, one on each side, so as to have two north ends together, and also two south ends ; separate with a large pin the two north points from the two south points, at one end ; and to the middle of one of the two soft bars apply the open end of the compound bar, held upright, with the two norths towards the south end of the lying bar, and in this position slide the compound bar backwards and forwards four times the whole length of the lying bar, ending at the middle ; from whence remove the compound bar to the middle of the other lying one, which rub in like manner : then turn both bars, the other side upwards, and repeat the former operation : Let the two lying bars make the inner two of the compound one, to rub the former outside two, when placed between the conductors ; and repeat this process till each pair of bars has been rubbed four times ; and then their former magnetic power will be much increased.

Now putting three contiguous N. ends to three S. ends ; with this compound bar, the lower ends being opened with a pin, as before, touch two pairs of the hard bars, placed at half an inch distance between their proper conductors, in like manner as aforesaid ; and with these four touched bars make a compound one to touch the other pair ; observing that the lower ends be parted near a quarter of an inch, after being set on the middle of the lying bar, and closed when taken off ; and touch each pair of hard bars three times over, as before said, and these bars will become strongly magnetical ; and to make them more so, let each pair, lying between their conductors, be touched twice over by two of them, held as nearly flat as can be, one in each hand : by drawing, at the same time, the N. end of one from the middle of the lying bar over its S. end ; and the S. end of the other from the middle over the N. end : Then bringing them to the middle again, without touching the flat of the lying bar, give it four of these horizontal strokes along each edge or side : These operations will give as much magnetic strength to the hard bars as they seem capable of receiving ; whereby each bar, if well hardened, will lift about two pounds weight avoirdupois.

19.

To preserve these Artificial Magnets.

The six hard bars, with their two conductors put end to end as one bar, should be kept in a case ; the bars being put face to face, having a N. and S. end together alternately ; whereby they will retain their virtue a long time, and be ready for any experiments : and should their power be found to fail, it may be restored, as in the last parts of the operation, in a few minutes.

20.

To give Magnetism to Compass Needles.

Lay two needles of equal length, about an inch asunder, with the N. end of one and S. end of the other pointing the same way, and apply two conductors in contact with their ends: Then with two magnetic hard bars, one in each hand, and held as nearly horizontal as can be, with the upper ends, of contrary names, turned outwards to the right and left, let a needle be stroked or rubbed from the middle to both ends at the same time, for 10 or 12 times, a N. end of a bar going over the S. end of the needle, and the S. end of a bar going over the N. end of a needle: Then, without moving from the place, change hands with the bars, or, in the same hands, turn the other ends downwards, and stroke the other needle in like manner, and they will both be magnetic: But to make them still stronger, repeat the operation three or four times from needle to needle; and at last turn the lower side of each needle upwards, and repeat the operations of rubbing them, as on the other sides.

In the rubbings or touchings to procure magnetism, the hand should not return directly back the way it came, but should return in a kind of oval figure, carrying the hand about six or eight inches beyond the point, where the touch ended; but not beyond, on the side where the touch begins.

It may be proper to observe that the needle, which before it was made magnetic, hung level on its pivot, does afterwards incline to the horizon; which inclination is called its *Dipping*: The horizontality is to be restored by filing away, by little and little, that end which appears to be heaviest.

P R O B L E M.

To make an observation with the Azimuth compass.

21.

I. FOR AN AMPLITUDE*.

Move the box about till the four cardinal points of the card coincide with the four lines in the sides of the compass box; keep the box steady, and turn the index till the center of the sun, at the rising or setting, when its lower edge just touches the horizon, is seen through both the sights: Then the degrees intercepted between the north point of the compass and that end of the index pointing to the sun, will be the magnetic amplitude sought.

22.

II. FOR AN AZIMUTH†.

Set the box so, that the four cardinal points of the card stand opposite to the four lines in the compass box; hold the box in this position, and move the index till (the sights being directed to the center of the sun) the

* Amplitudes, are from observations made on objects when in the horizon. See Book V. 112, 147 to 152.

† Azimuths, are from observations made on objects when above the horizon. See Book V. 111, 152.

Shadow

the shadow of the string joining the sights falls directly along the middle line of the index : Then the degrees in the arc, intercepted between that end of the index next the sun, and north point of the card, will give the magnetic azimuth required.

If the sun does not shine strong enough to give a distinct shadow, look through one of the sights, and move the index till the string cuts the sun's center ; and then the intercepted arc, as before, shews the sun's azimuth.

What is here said of the sun is in like manner to be applied to the stars.

23. When there is a rough sea, and the ship rolls much, the observation is best performed by two persons, thus :

Let one look through the sights, and turn them till the thread cuts the center of the sun, and be sure to keep a constant sight of it, notwithstanding the motion of the ship, which being signified to the assistant by any word, let him that instant observe what degree of the card is under the middle line of the sights, and the arc intercepted between that point and the meridian will shew the quantity of the magnetic amplitude or azimuth. If the card should vibrate very much, let him take the middle degree between the limits to which the vibration reaches.

When the magnetic azimuth of any object is observed, its altitude must also be taken at the same time.

This instrument is also useful in setting the ship's wake, in order to find the leeway ; and also to find the bearings of headlands and other objects.

SECTION III.

24. *To Work an Amplitude.*

The true amplitude of a celestial object, is its distance from one of the cardinal points of the horizon, at the rising or setting of that object.

The amplitude is reckoned eastward in the morning ; but it is named westward in the evening.

The customary way of accounting the amplitude is, in the morning to reckon it so much to the southward or northward of the east ; and in the evening so much to the southward or northward of the west : But as bearings are generally taken from the meridian, therefore in the following pages the amplitudes will be chiefly estimated from the north point of the horizon.

The finding of the true amplitude at sea is of considerable importance in navigation ; for the true courses the ship has steered is partly ascertained thereby.

PROBLEM.

25. *Given the latitude of the place, and the declination of a celestial object: Required the true amplitude of that object?*

SOLUTION.

Say: As the co-sine of the latitude, to radius

So sine of the given decl. to the sine of an horizontal arc ^{to}
Or, Add 10 to the index of the log. sine of the declin. and from that sum ~~take~~ the log. co-sine of the latitude, ^{and} ~~then~~ the log. sine of the horiz. arc. — *striking off 10 from the Index of the whole*

Which horizontal arc taken from 90 degrees, if the declination is north, will give the sun's true amplitude from the north point of the horizon: But if the declination is south, that horizontal arc added to 90 degrees, gives the amplitude from the north.

When the amplitude is reckoned from the east or west points of the horizon; then the said horizontal arc is the true amplitude, and is to be taken to the northward of those points, when the declination is north: But to the southward, when the declination is south.

26. SOLUTION BY THE TRAVERSE TABLE.

With the declination among the courses, find the departure to the distance 100.

With the given latitude among the courses, and said departure in the column of latitude, find the distance.

In a column of departure, find the said distance against 100 in the column of distance, and the degrees in the corresponding course will give the horizontal arc sought.

This rule will always give the degrees as truly as by logarithms: And as the magnetic amplitude can be observed to degrees only by the compass, the true amplitude will be had sufficiently correct by this rule.

EXAM. I. *In the latitude of $38^{\circ} 25' N$. What is the Sun's true amplitude when his declination is $18^{\circ} 59' N$?*

By Logarithms.

As cos. lat.	$= 38^{\circ} 25'$	0,10595
To radius	$= R$	10,00000
So sin. decl.	$= 18 \quad 59$	9,51227
To sin. hor. arc	$= 24 \quad 32$	9,61822
Subtract from	90 00 for N. dec.	
Leavestruamp.	$= 65 \quad 28$	from N.

By the Traverse Table.

To decl. $19^{\circ} 00'$, and dist. 100, the	
dep. is $32,56$	
To lat. $38^{\circ} 25'$, and diff. lat. 32 , the	
dist. is 41 .	
To dist. 100, and dep. 41 , the course	
is $24^{\circ} 45'$. Therefore the horizontal arc is $24^{\circ} 45'$.	

In this and the following examples, the nearest degree and minute to the declination that can be found in the Traverse Table, is taken for the declination.

EXAM.

EXAM. II. Required the Sun's true amplitude in the lat. of $51^{\circ} 32' N$, when the Sun's declination is $20^{\circ} 56' S$.?

By Logarithms.

As cof. lat.	= $51^{\circ} 32'$	0.20617
To Radius	= R	10.00000
So sin. decl.	= $20 \ 56$	9.55301
To fin. hor. arc	= $35 \ 03$	19.75918
Add to	$90 \ 00$ for S. decl.	
Gives true amp.	= $125 \ 03$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 00'$, and dist. 100, the dep. is 35.8 .
To lat. $51^{\circ} 30'$, and diff. lat. 35.8 , the dist. is 57 .
To dist. 100, and dep. 57 , the course is $34^{\circ} 45'$. Therefore 35° may be taken for the horizontal arc sought.

EXAM. III. When the Sun's declination was $20^{\circ} 10' S$. what was his amplitude at the Cape of Good Hope, in lat. $34^{\circ} 15' S$.?

By Logarithms.

As cof. lat.	= $34^{\circ} 15'$	0.8271
To Radius	= R	10.00000
So sin. decl.	= $20 \ 10$	9.53751
To fin. hor. arc	= $24 \ 39$	19.62022
Which add to	$90 \ 00$ for S. decl.	
Gives true amp.	= $114 \ 39$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 15'$, and dist. 100, the dep. is 34.6 .
To lat. $34^{\circ} 15'$, and diff. lat. 34.6 , the dist. is 42 .
To dist. 100, and dep. 42 , the course is $24^{\circ} 45'$. Therefore 25° may be taken for the horizontal arc sought.

EXAM. IV. On what point of the horizon does the star Arcturus rise and set at Buenos Ayres in lat. $34^{\circ} 35' S$.?

By Logarithms.

As cof. lat.	= $34^{\circ} 35'$	0.8444
To Radius	= R	10.00000
So sin. decl.	= $20 \ 29$	9.54399
To fin. hor. arc	= $25 \ 09$	19.62843
Which add to	$90 \ 00$ for S. dec.	
Gives true amp.	= $115 \ 09$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 30'$, and dist. 100, the dep. is 35.0 .
To lat $34^{\circ} 30'$, and diff. lat. 35.0 , the dist. is 42.5 .
To dist. 100, and dep. 42.5 the course is $25^{\circ} 15'$. Therefore the horizontal arc may be taken as 25 degrees.

SECTION

SECTION VI.

To work an Azimuth.

27.

The true azimuth of a celestial object, is its bearing, when above the horizon, from the north or south points thereof.

In the forenoon the azimuth is reckoned eastward, and in the afternoon westward.

The azimuth is usually estimated from the south, when to the southward of the east or west points of the horizon; and when to the northward of those points, it is reckoned from the north: But it may be more convenient to account it always from the north.

The use of the azimuth is to find the variation of the compass, in order to correct the courses the ship has steered.

28.

P R O B L E M.

The latitude of the place, the declination of the object, and its altitude being known: Required the true azimuth of that object?

S O L U T I O N.

When the latitude and declination are of the same name, let the co-declination be called A.

But when of contrary names, add the declination to 90 degrees, and call the sum A.

Let the difference between the co-lat. and co-alt. be called D.

Also, find the half sum and half difference of A and D.

Then write these four logarithms under one another.

Namely: The arith. comp. of the log. sine of the co-latitude.

The arith. comp. of the log. sine of the co altitude.

The logarithm sine of the foresaid half sum of A and D.

The logarithm sine of the foresaid half diff. of A and D.

Add the four logarithms together, and take half their sum, which seek among the sines for the corresponding degrees and minutes, and these being doubled will give the true azimuth required.

And is to be reckoned from the north, in north latitude.

But in south latitude, take those degrees from 180°, and the remainder will give the true azimuth from the north.

The working of an azimuth by logarithms, although as short as any numeral solution of this problem can be expected, has been thought one of the most tedious elements that enter into a common day's work: But by Gunter's scale, it may be readily done, sufficiently correct.

29.

To work an azimuth by the Gunter's Scale.

Find the co-lat., co alt., the half sum, and half difference, as above.

Then, On the scale of log. sines, set one point of the compasses on the half sum, extend the other point to the co-lat., observing whether the second point falls to the right or left of the first, and take the compasses off, without altering their extent.

Set one point on the co-alt., and let the second point fall where it will (the same way it did before); there fix it, and open the compasses till the other

EXAM. II. Required the Sun's true amplitude in the lat. of $51^{\circ} 32' N$, when the Sun's declination is $20^{\circ} 56' S$.?

By Logarithms.

As cos. lat.	$= 51^{\circ} 32'$	0.20617
To Radius	$= R$	10.00000
So sin. decl.	$= 20 \ 56$	9.55301
To fin. hor. arc	$= 35 \ 03$	<u>19.75918</u>
Add to	$90 \ 00$ for S. decl.	
Gives true amp.	$= 125 \ 03$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 00'$, and dist. 100, the dep. is 35.8.
To lat. $51^{\circ} 30'$, and diff. lat. 35.8, the dist. is 57.
To dist. 100, and dep. 57, the course is $34^{\circ} 45'$. Therefore 35° may be taken for the horizontal arc sought.

EXAM. III. When the Sun's declination was $20^{\circ} 10' S$. what was his amplitude at the Cape of Good Hope, in lat. $34^{\circ} 15' S$.?

By Logarithms.

As cos. lat.	$= 34^{\circ} 15'$	0.8271
To Radius	$= R$	10.00000
So sin. decl.	$= 20 \ 10$	9.53751
To fin. hor. arc	$= 24 \ 39$	<u>19.62022</u>
Which add to	$90 \ 00$ for S. decl.	
Gives true amp.	$= 114 \ 39$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 15'$, and dist. 100, the dep. is 34.6.
To lat. $34^{\circ} 15'$, and diff. lat. 34.6, the dist. is 42.
To dist. 100, and dep. 42, the course is $24^{\circ} 45'$. Therefore 25° may be taken for the horizontal arc sought.

EXAM. IV. On what point of the horizon does the star Arcturus rise and set at Buenos Ayres in lat. $34^{\circ} 35' S$.?

By Logarithms.

As cos. lat.	$= 34^{\circ} 35'$	0.8444
To Radius	$= R$	10.00000
So sin. decl.	$= 20 \ 29$	9.54399
To fin. hor. arc	$= 25 \ 09$	<u>19.62843</u>
Which add to	$90 \ 00$ for S. dec.	
Gives true amp.	$= 115 \ 09$ fr. the N.	

By the Traverse Table.

To decl. $20^{\circ} 30'$, and dist. 100, the dep. is 35.0.
To lat $34^{\circ} 30'$, and diff. lat. 35.0, the dist. is 42.5.
To dist. 100, and dep. 42.5 the course is $25^{\circ} 15'$. Therefore the horizontal arc may be taken as 25 degrees.

SECTION

SECTION VI.

27. *To work an Azimuth.*

The true azimuth of a celestial object, is its bearing, when above the horizon, from the north or south points thereof.

In the forenoon the azimuth is reckoned eastward, and in the afternoon westward.

The azimuth is usually estimated from the south, when to the southward of the east or west points of the horizon; and when to the northward of those points, it is reckoned from the north: But it may be more convenient to account it always from the north.

The use of the azimuth is to find the variation of the compass, in order to correct the courses the ship has steered.

28. PROBLEM.

The latitude of the place, the declination of the object, and its altitude being known: Required the true azimuth of that object?

SOLUTION.

When the latitude and declination are of the same name, let the co-declination be called A.

But when of contrary names, add the declination to 90 degrees, and call the sum A.

Let the difference between the co-lat. and co-alt. be called D.

Also, find the half sum and half difference of A and D.

Then write these four logarithms under one another.

Namely: The arith. comp. of the log. sine of the co-latitude.

The arith. comp. of the log. sine of the co altitude.

The logarithm sine of the foresaid half sum of A and D.

The logarithm sine of the foresaid half diff. of A and D.

Add the four logarithms together, and take half their sum, which seek among the sines for the corresponding degrees and minutes, and these being doubled will give the true azimuth required.

And is to be reckoned from the north, in north latitude.

But in south latitude, take those degrees from 180°, and the remainder will give the true azimuth from the north.

The working of an azimuth by logarithms, although as short as any numeral solution of this problem can be expected, has been thought one of the most tedious elements that enter into a common day's work: But by Gunter's scale, it may be readily done, sufficiently correct.

29. *To work an azimuth by the Gunter's Scale.*

Find the co-lat., co alt., the half sum, and half difference, as above.

Then, On the scale of log. sines, set one point of the compasses on the half sum, extend the other point to the co-lat., observing whether the second point falls to the right or left of the first, and take the compasses off, without altering their extent.

Set one point on the co-alt., and let the second point fall where it will (the same way it did before); there fix it, and open the compasses till the other

other point falls on the half difference, then take them off without altering their extent.

Now set one point of the compasses on the brass pin at the beginning, or right hand end of the scale of versed fines (marked V. S.), turn the other point to the left, and observe what degrees and minutes in the versed fines it falls on; those degrees and minutes will be the azimuth, if the latitude is south: But if north, take them from 180° , and the remainder is the azimuth sought.

If the extent, when applied from the co-alt. to a fourth, should reach beyond the right-hand end of the scale, as it will sometimes happen, then hold the Gunter flat on a board, and thereon, in a line with the fines, let the over point rest, and thence take the extent to the half difference, and work as before.

These directions, with those in Book III. 84, 85, will remove all the seeming difficulties that beginners may meet with in working an azimuth by the Gunter's scale.

EXAM. I. *Being at sea, in latitude $40^{\circ} 38' N.$, in the afternoon the Sun's altitude was observed to be $20^{\circ} 46'$, when his declination was $17^{\circ} 10' S.$: Required the Sun's true azimuth at that time?*

By LOGARITHMS.

The co-alt. is $69^{\circ} 14'$	Ar. co-log. fin. co-alt. = $49^{\circ} 22'$	0,11982
The co-lat. is $49^{\circ} 22'$	Ar. co-log. fin. co-lat. = $69^{\circ} 14'$	0,03917
The diff. $D = 19^{\circ} 52'$	Log. fin. $\frac{1}{2}$ sum = $63^{\circ} 31'$	9,95185
Now $107^{\circ} 10' = A = 90^{\circ}$ & decl.	Log. fin. $\frac{1}{2}$ diff. = $43^{\circ} 39'$	9,83901
And $19^{\circ} 52' = D.$	The sum of the four logs	<u>19,93985</u>
$127 \quad 02 \quad 63^{\circ} 31' = \frac{1}{2}$ sum.	Their $\frac{1}{2}$ sum gives $38^{\circ} 55'$	9,96992
2) $87 \quad 18 \quad 43 \quad 39 = \frac{1}{2}$ diff.	The double is, $137^{\circ} 50'$, which is the Sun's true azim. from the N. point of the horizon	

By GUNTER'S SCALE.

On the fines, the extent from $63^{\circ} 31'$ to $49^{\circ} 22'$, will reach from $69^{\circ} 14'$ to a little more than 52° : Let the point rest there, and extend the other to $43^{\circ} 39'$; this extent will reach from the beginning of the versed fines to about 42° , which taken from 180 leaves 138° for the azimuth, and is very near what was found by the logarithms.

EXAM. II. *What is the Sun's true azimuth in the latitude of $26^{\circ} 30' N.$, when his altitude in the forenoon is $24^{\circ} 28'$, and his decl. is $22^{\circ} 40' N.$?*

By LOGARITHMS.

The co-alt. is $65^{\circ} 32'$	Ar. co-log. fine co-lat. = $63^{\circ} 30'$	0,04821
The co-lat. is $63^{\circ} 30'$	Ar. co-log. fin. co-alt. = $65^{\circ} 32'$	0,04086
The diff. $D = 2^{\circ} 02'$	Log. fine $\frac{1}{2}$ sum = $34^{\circ} 41'$	9,75514
Now $67^{\circ} 20' = A$, is 90° less decl.	Log. fine $\frac{1}{2}$ diff. = $32^{\circ} 39'$	<u>0,73200</u>
And $2^{\circ} 02' = D$	The sum of the four logs	<u>19,57921</u>
$69 \quad 22 \quad 34^{\circ} 41' = \frac{1}{2}$ sum.	Their $\frac{1}{2}$ sum, gives $37^{\circ} 52'$	9,78810
2) $65 \quad 18 \quad 32 \quad 39 = \frac{1}{2}$ diff.	The double is, $75^{\circ} 44'$, which is the Sun's true azim. from the N. point of the compass.	

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By GUNTER'S SCALE.

On the log. fines, the extent from $34^{\circ} 41'$ to $63^{\circ} 30'$, will reach from $65^{\circ} 32'$ to some point beyond the right-hand end of the scale; let the point rest there, and extend the other point to the fine of $32^{\circ} 39'$, and this extent will, on the verfed fines, reach from the beginning, or 0° deg., to $104^{\circ} 20'$, whose supplement $75^{\circ} 40'$ is the azimuth, nearly, as found by logarithms.

EXAM. III. *At the island of St. Helena, in lat. $16^{\circ} 00'$ S., the Sun's altitude was observed in the forenoon to be $30^{\circ} 22'$ when his declination was $22^{\circ} 58' S.$: What was the Sun's azimuth at that time?*

The co-lat. is $74^{\circ} 00'$
The co-alt. is $59^{\circ} 38'$

The co-alt. is 59 38

The diff. D = 14 22

Now $67^{\circ}02' = A$, is 90° less decl.
And $14\ 22 = D$.

$$\begin{array}{r} 81 \ 24 \ 40^{\circ} \ 42' = \frac{1}{2} \text{ sum.} \\ \hline 2) \\ \hline 52 \ 40 \ 26 \ 20 = \frac{1}{2} \text{ diff.} \end{array}$$

Or thus: The extent from $40^{\circ} 42'$ to $74^{\circ} 00'$ on the fines, will reach from the fine of $59^{\circ} 38'$ to a point beyond the limits of the scale; then the extent from that point to the fine of $26^{\circ} 20'$ will, on the verfed fines, reach from 0 deg. to about $107^{\circ} 40'$, the azimuth from the north.

EXAM. IV. On what point of the horizon, at the Cape of Good Hope, does the star Aldebaran bear, when its altitude is $22^{\circ} 25'$?

The co-alt. is $67^{\circ} 35'$

The co-lat. is 55 45

The diff. p = 11 50

Now $105^{\circ} 59' = A$, is 90, & decl.
And $11^{\circ} 50' = D$.

$$\begin{array}{r} 117\ 49 \\ 2) \overline{) 58^{\circ}\ 54'} = \frac{1}{2} \text{ sum.} \\ \underline{234\ 98} \\ 94\ 00\ 47\ 05 = \frac{1}{2} \text{ diff.} \end{array}$$

Or thus: The extent from about 59° to about $67\frac{1}{2}^\circ$ on the fines reaches from about $55\frac{1}{2}^\circ$ to about 63° : Then the extent from about 63° to about 47° , reaches from the beginning of the verfed fines, to about 50° for the azimuth fought.

These examples sufficiently shew, that an azimuth may be worked by the Gunter's scale, not only with great readiness, but also as accurately as the magnetic azimuth can be taken by the compass.

The manner of working amplitudes and azimuths have been pretty largely insisted on, in order to render them familiar to mariners; who, for the generality, esteem these operations as very troublesome; and it is on this account that many writers have contrived various tables to remove the complained of trouble: But from a due consideration of what is here done, it is apprehended such tables will not hereafter be so much wanted.

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SECTION

Ar. co-log. fine co-lat. = 74° 00' 0,01716
 Ar. co-log. fine co-alt. = 59 38 0,06409
 Log. fine $\frac{1}{2}$ sum = 40 42 9,81431
 Log. fine $\frac{1}{2}$ diff. = 26 20 9,64698

Ar. co-log. fine co-alt. = 59 38 0.06409

Log. fine $\frac{1}{2}$ fum = 40 42 9,81431

Log. fine $\frac{1}{2}$ diff. = 26 20 9,64698

The sum of the four logs.

19,54254

Their sum, gives $36^{\circ} 12'$

9,77127

Which doubled, is 72 24, for the Sun's true azim. from the south, or 107° 36' from north.

Ar. co-log. fin. co-lat. = $55^{\circ} 45' 0.08271$

Ar. co-log. fin. co-alt. = 67 35 0,03412

Log. fin. $\frac{1}{2}$ fin. = 58 54 9,93261

Log. fin. $\frac{1}{2}$ diff. = 47 05 9,86472

The sum of the four logs.

19,91416

Their sum gives $64^{\circ} 57'$

9,95708

Which doubled, is 12954, for the Sun's true azim. from the S. or 50° 6' from the N.

SECTION V.

30. *Of the Variation of the Compass.*

The *magnetic poles*, are those points of the world, towards which the north and south ends of the needle in the sea compass are directed.

Magnetical meridians, are imaginary lines, passing through the magnetic poles.

The *variation of the compass*, is the difference between the bearings of the *true* and *magnetic* north poles; and is measured by an arc of the horizon contained between the true and magnetic points thereof.

The variation is named *east*, when the magnetic north point is to the eastward of the true north: But when to the westward, it is named *west* variation.

The magnetic amplitude of a celestial object, is its bearing by the compass when in the horizon.

The magnetic azimuth of a celestial object, is its bearing by the compass when above the horizon.

The variation is found by comparing together the true and magnetic amplitudes or azimuths of celestial objects.

The use of the variation is to correct the courses a ship has steered by the compass.

31.

PROBLEM.

Given the true and magnetic amplitudes, or azimuths, of a celestial object: Required the variation of the compass?

SOLUTION.

Let the true and magnetic amplitudes, or azimuths, be reckoned from the north; and named east when the observation is made in the forenoon; but when made in the afternoon, let it be named west.

Then the difference between the true and magnetic amplitudes, or azimuths, is the variation of the compass.

And is of the same name as the true amplitude, or azimuth, if the true is greater than the magnetic: But when the true is less than the magnetic, the variation is of a contrary name to the true amplitude, or azimuth.

For, let *N* represent the true north point of the horizon, and the arc *NWO* the true amplitude or azimuth: Now it is evident, that if the arc *OWN*, representing the magnetic amplitude or azimuth, be less than *OWN*; then the magnetic north point of the compass will fall on the same side of the true north, that the amplitude or azimuth is on: But when greater, *n* falls on the contrary side. Plate XIV. Fig. 8.

EXAM.

EXAM. I. *The Sun was observed to rise 70° from the north by the sea-compass, when his true amplitude was N. $81^{\circ} 45'$ E.: What was the variation, and which way?*

True amplitude = $81^{\circ} 45'$ E.
Magnetic amplitude = $70^{\circ} 00'$

The variation = $11^{\circ} 45'$ E.

EXAM. III. *Suppose the Sun's true azimuth to be N. $84^{\circ} 40'$ W., when by the compass his bearing was W. b. S.: Required the quantity and quality of the variation?*

True azimuth = $84^{\circ} 40'$ W.
Mag. azimuth = $101^{\circ} 15'$ = 9 pts.

The variation = $16^{\circ} 35'$ E.

The manner of finding the variation being known, the courses may be corrected by the following rules.

PROBLEM.

32. *Given the course set, and the variation of the compass: Required the true course the ship steers?*

SOLUTION.

If the variation is west, call the NW. quarter the 1st, the SW. the 2d, the SE. the 3d, and the NE. the 4th: But if the variation be east, call the NE. quarter the 1st, the SE. the 2d, the SW. the 3d, and the NW. the 4th.

Then, if the course set be in the first or third quarters, add the variation to the points or degrees in the given course: But if in the second or fourth quarters, subtract, and the sum in the former, or difference in the latter case, will be the course corrected by the variation *.

EXAM. I. *Suppose the variation is $1\frac{1}{2}$ point W.; to correct these courses NNW.; SW. $\frac{1}{2}$ W.; SE. b. S. $\frac{1}{2}$ E.; ENE. $\frac{1}{2}$ E.*

Courses set	NNW. 2 pts.	SW. $\frac{1}{2}$ W. $4\frac{1}{2}$ pts.	SE. b. S. $\frac{1}{2}$ E. $3\frac{1}{2}$ pts.	ENE. $\frac{1}{2}$ E. $6\frac{1}{2}$ pts.
Variation	Add $1\frac{1}{2}$ pts.	Subtr. $1\frac{1}{2}$ pts.	Add $1\frac{1}{2}$ pts.	Subtract $1\frac{1}{2}$ pts.

Cor.co. NW. b. N. $\frac{1}{2}$ W. $3\frac{1}{2}$ pts. SW. b. S. 3 SE. b. E. $\frac{1}{2}$ E. $5\frac{1}{2}$ pts. NE. $\frac{1}{2}$ E. $4\frac{1}{2}$ pts.

EXAM. II. *The variation is 18° E.; to correct the following courses.*

Courses set	NE. b. N. $33^{\circ} 45'$	S. b. E. $11^{\circ} 15'$	SW. b. W. $56^{\circ} 15'$	NW. $45^{\circ} 00'$
Variation	Add $18^{\circ} 00'$	Subtr. $18^{\circ} 00'$	Add $18^{\circ} 00'$	Subtr. $18^{\circ} 00'$

Cor. cour. N. $51^{\circ} 45'$ E. S. $6^{\circ} 45'$ W. S. $74^{\circ} 15'$ W. N. $27^{\circ} 00'$

* For where the compass has a variation, the course set is not so far removed from the true north as it should be, by the quantity of the variation: But the quantity of the variation being added in the 1st and 3d quadrants, and subtracted in the 2d and 4th, puts the course so much farther from the true north, counting from thence, the same way with the variation, round the compass.

S E C T I O N VI.

33. *Of finding the Ship's drift, or leeway.*

When a ship turns to windward, or is clofe hauled on a wind, ſhe generally runs to the leeward of her intended courſe.

LEEWAY is the angle which the ſhip's real courſe makes with her intended courſe, occaſioned by contrary winds, and a rough ſea.

The leeway made by different ſhips, under the ſame circumſtances of wind and ſails, will be different; and even the ſame ſhip, with different lading, and having more or leſs ſail abroad, will make more or leſs leeway. However, the following precepts for allowing for the leeway have been generally given by moſt Engliſh writers on this ſubject within the preſent century; they were given by Mr. John Buckler to the late Mr. William Jones, who firſt publiſhed them about the year 1702.

34. *Allowances for Leeway.*

- 1ſt. When a ſhip is cloſe hauled, has all her ſails ſet, the water ſmooth, and a moderate gale of wind, ſhe is then ſuppoſed to make little or no leeway.
 - 2d. Allow one point, when it blows ſo freſh that the ſmall ſails are taken in.
 - 3d. Allow two points, when the top-fail muſt be cloſe reefed.
 - 4th. Allow two and a half points, when one top-fail muſt be handed.
 - 5th. Allow three and a half points, when both top-fails are to be taken in.
 - 6th. Allow four points, when the fore courſe is handed.
 - 7th. Allow five points, when trying under the main-fail only.
 - 8th. Allow fix points, when both main and fore courſes are taken in.
 - 9th. Allow ſeven points, when the ſhip tries a hull, or all ſails handed.
- When the wind has blown hard in either quarter, and ſhifts acroſs the meridian into the next quarter, the leeway will be leſſened.
- But in all theſe caſes, reſpect muſt be had to the roughneſs of the ſea, with the trim of the ſhip; and hence the mariner will be able to correct his courſe.

35. P R O B L E M.

Given the place of the wind, the courſe ſet, and the leeway : Required the true courſe the ſhip makes ?

RULE. Count the neareſt way of the compaſs from the wind to the courſe ſet, and as much beyond as is the leeway, and it gives the point for the true courſe.

EXAM. I. *The wind NW. b. N.; the courſe ſet NE. b. N., leeway 1 point: What is the courſe made good ?*

The wind NW. b. N.

Courſe ſet NE. b. N. = 3 points.

Add the leeway = 1 point.

True courſe NE. = 4 points.

Here, counting the neareſt way in the compaſs from NW. b. N. to NE. b. N., the leeway going beyond it towards the eaſt muſt be added to drive it ſo much farther from the wind.

EXAM. II. *The wind SE. b. S., a ſhip is cloſe hauled with her ſtarboard tacks aboard, and both top-fails handed: What is her true courſe ?*

The wind SE. b. S.

Courſe ſet E. b. N. = 7 points.

Subtraſt leeway = $3\frac{1}{2}$ points.

True co. NE. b. N. $\frac{1}{2}$ E. = $3\frac{1}{2}$ points.

The ſhip being cloſe hauled, is ſuppoſed 6 pts. from the wind, and muſt be in the NE. quarter, being on the ſtarb. tacks: Then counting 6 points from SE. b. S., gives E. b. N. 7 pts; from which the leeway is to be taken

S E C T I O N

SECTION VII.

Of Instruments for taking of Altitudes.

The altitude of the Sun, or any other celestial object, may be taken by instruments contrived after various ways; those only, best adapted for use at sea, will be here considered.

Of DAVIS'S QUADRANT.

36. *The description and use.* Plate XIV. Fig. 9.

The instrument most in use at sea, till about the year 1740, by us called DAVIS'S QUADRANT, and by foreigners the ENGLISH QUADRANT, consists of two arches (making together ninety degrees, and therefore called a quadrant), fixed and braced, the one above, the other below, a straight bar, whose length is equal to the radius of the lower arch, and is about thrice the length of the radius of the upper arch.

The upper, or as it is commonly called, the GREATER ARCH, usually contains 65 degrees, not subdivided into smaller parts; and the LESSER ARCH contain 25 degrees, which, on account of their larger size, are subdivided into smaller parts, such as $\frac{1}{3}$ or $\frac{1}{2}$ parts of degrees; and these, by the help of diagonal divisions, and 11 concentric circles, are usually subdivided so, as to shew every one or two minutes.

There are three pieces called vanes, so fitted to the instrument as to stand at right angles to its plane.

First, The HORIZON VANE is fitted to slip on the end of the bar or radius, close to the center of the two arches: In this vane there is a long slit, through which is to be seen the horizon, that is, the line where the sky and water apparently meet.

Secondly, The SIGHT VANE, which is so fitted as to slide along the lesser, or 25 arch, having a sharp edge to cut the division it stands at; and in a line with this edge is a small hole, through which the horizon is to be seen through the slit in the horizon vane.

Thirdly, The SHADE VANE, which is fitted to slide on the greater, or 65 arch: The upper line of the shade cast by this vane is to fall on the edge of the slit in the horizon vane during the time of observation.

Instead of the shade vane there is generally used one called the GLASS VANE, it having a convex lens (or, as usually called, a burning glass) fitted to it, which collects the Sun's rays into a bright spot on the horizon vane.

AB, is the bar or radius. BC. the lesser or 25 arch. DE, the greater or 65 arch. *ab, eg, hi*, are braces which fix the arches to the radius and strengthen the instrument. G the horizon vane. H the sight vane. F the shade or glass vane. © FA a ray of the Sun. ∠ DAC a right angle, or 90°.

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37. *To observe the Sun's altitude or zenith distance by Davis's quadrant.*
See Plate XIV. Figure 9.

First, Put on the horizon vane close to the shoulder or notch at A, in the bar AB: Set the shade or glass vane on the arch DE to an even degree, about 10 or 15 degrees less than what the zenith distance may be then guessed at; and put the sight vane H about the middle of the arch BC: observing that the shade and sight vanes be thrust close to the backs of the arches.

Secondly, Turn your back to the Sun, hold the plane of the instrument in an upright position, look through the hole in the sight vane, and raise or lower the quadrant until the bright spot, cast by the Sun through the glass vane, falls on the $\odot$ in the line drawn through the slit in the horizon vane; then if the horizon is seen through the slit in the horizon vane, even with the $\odot$, the quadrant will give the Sun's altitude or zenith distance at that time.

Thirdly, But if the sky appears through the slit instead of the horizon, slide the sight vane H higher, till the horizon coincides with the bright spot falling on $\odot$; and if the sea appears instead of the horizon, slide the side vane H lower till the spot and horizon come together, and the observation at that time is made.

Fourthly, To the degrees between D and F add those between C and H, their sum gives the zenith distance: But the degrees between B and H added to those between F and E, shew the altitude.

38. Now to find the latitude, the *meridian* altitude, or meridian zenith distance of the Sun is wanted: Therefore begin some time before noon, and having taken an observation as above directed, wait a minute or two and observe again; and if the sea appears through the horizon vane, the Sun has got higher, therefore slide the sight vane lower, till the solar spot and horizon coincide; and thus continue, until the sky appearing instead of the sea, shews the Sun has past the meridian; therefore let the vane stand as at the last alteration, and add the degrees in the greater arc to the degrees and minutes in the lesser, their sum gives the meridional zenith distance.

If instead of the bright solar spot, the upper border of the shade is made to cut the horizon line, at the time the horizon is abreast that line; then the sum of the arcs gives the zenith distance of the Sun's upper edge or limb, to which add 16 minutes, and the Sum is the distance of the Sun's center from the zenith when he is upon the meridian. (See the appendix to this book.) In most quadrants there are on the back of the greater arch a scale of degrees, where the Sun's semidiameter is allowed for.

39. One great objection against this instrument, is the trouble and time lost in sliding the sight vane up or down, which sometimes cannot be conveniently done without taking the quadrant from the eye, whereby an opportunity may be lost of making the observation: But this defect is easily removed by having a long index or ruler fitted to the quadrant; one end moving round the center to which the horizon vane is fixed, and having the sight vane fixed to the other end. By this contrivance

the sight vane may be readily raised higher, or lowered, by the motion of the index about its center, which may be done without taking the instrument from the eye.

40. There is another instrument, called a **FORE STAFF, or CROSS STAFF**, that has been a long time in use for taking the altitudes of celestial objects, particularly the stars. With this instrument, the observer holds one end of the staff, which is a three feet rod, close to one side of his eye, and slides a transum or cross piece, which is in an upright position, along the staff till one end of the cross cuts the star, and the other end at the same time cuts the horizon; then the division of the staff, which the cross stands against, gives the altitude.

But the great trouble in the use of this instrument, joined to other inconveniences attending it, make it now of little note, especially since the use of Hadley's quadrant is become more general; which far surpasses all others for the taking of altitudes or zenith distances of celestial objects at sea, not only for the accuracy which it shews the altitude, but also for the readiness and ease with which they are taken.

Of HADLEY'S QUADRANT.

The description and use. Pl. XIV. Fig. II.

41.

This instrument consists of the following particulars.

- I. The arch ABC being an octant, or eighth part of a circle; but is more useful when a sextant, or sixth part of a circle.
- II. The index D, with its Vernier's scale.
- III. The speculum E.
- IV. Two horizon glasses, F and G, with their adjusters.
- V. Two skreens, K, K.
- VI. Two sight vanes, H, I.

I. Of the Octant.

The octant consists of the two radius's or bars AB, AC; the arch or limb BC; also of the two braces, L, M, which are to strengthen and prevent it from warping. The arch contains only the eighth part of the circumference of a circle, or 45 degrees, but is divided into 90 primary divisions, each of which stands for degrees, and are numbered C, 10, 20, 30, &c. to 90, beginning at each end of the arch for the convenience of numbering both ways, either for altitudes or zenith distances. Every degree is subdivided into two or three parts, and these either by the method of diagonals, or by (another which is much better, and called the) *Vernier's* division, or scale, are so divided as to shew one or two minutes.

II. Of the Index.

The index D is a flat rod or bar, moveable round the center of the instrument; that part of the index which slides over the graduated arch BC, has either a sharp edge to cut the diagonal divisions, when so divided, or has *Vernier's* scale on it.

U 4

From

From the bottom of the index turn it up against the back of the instrument a piece of brass, with a screw in it, serving to fasten the index against any division.

The index, when moved along the arch, should be taken hold on by the bottom part, and not by the middle.

The principles upon which the Vernier scale depends, were shewn in Book V. art. 219. To which it may be proper to add, in this place some general directions, whereby learners may be enabled to read the degrees and minutes shewn by the index to this scale, in whatever part of the arch it stands against.

The generality of Hadley's quadrants being of 18 inches radius, have each degree on its limb divided into three parts, of 20 minutes each; and the breadth of the Vernier's scale, equal to 7 degrees; and as these 7 degrees are divided into 21 parts on the limb, each of 20 minutes; the scale is divided into 20 equal parts: consequently each division on the scale is larger than each division on the limb by $\frac{1}{20}$ th part of a division on the limb, or $\frac{1}{20}$ th of 20 minutes; that is, each division on the scale exceeds each on the limb by 1 minute of a degree: consequently in whatever place the Vernier's scale stands, one of its divisions will always stand against, or close to a division on the limb.

Now the middle line of the index D, which is the moveable radius of the quadrant, gives the index, or pointer, of the Vernier scale, which is usually its middle line; having 10 divisions, or minutes, on each side, and numbered 5, 10, to the right, and 15, 10, to the left; that is, the first 10 minutes of the scale is reckoned in order from the middle line or index to the right; and the latter 10 minutes is to be reckoned from the left-hand end of the scale towards the right, and ends at the middle line.

Now suppose the middle line or index of the Vernier stood against 48° 1 division. and less than $\frac{1}{2}$ of another division; and on examining the right-hand side of the scale, the 7th division stood against a division on the limb: Then is the index reckoned to stand against 48° 27'.

Again. Suppose the index of the scale stood against 34° , two divisions and more than half of another; and on examining the left-hand side of the scale, it was found that the 16th division thereof stood against a division on the limb; it is to be read 34° 56'; and the like of others.

The exact coincidence of the divisions on the limb and scale, is best discovered by a magnifying glass, or double convex lens.

III. Of the Speculum.

This is a piece of flat glass, quicksilvered on one side (like common looking-glass), and set in a brass box, with the surface of the speculum perpendicular to the plane of the instrument, against whose center stands the middle of the glass, its surface coinciding with the middle line of the index. Now as the index slides along the arch, the position of the speculum is altered, it being fixed to the index.

The use of the speculum is, to receive the rays from the object observed, and reflect them on the horizon glasses.

IV. Of

IV. *Of the Horizon Glasses.*

These are small pieces of looking-glasses placed on one of the radius's, with their faces turned obliquely to the speculum, from whence they receive the reflected rays of observed objects.

The glass *F* has only its lower part quicksilvered, and set in brass work; the upper part being left transparent to see the horizon through it.

In the middle of the glass *G* is a transparent slip, through which the horizon is to be seen.

Both of these glasses are so mounted, as to have their positions set truly by their adjusters at the back, should the frame of the instrument be warped.

V. *Of the Screens.*

These are two pieces of coloured glass set in frames, and are to prevent the strength of the Sun's rays from hurting the eye during the time of observation: When they are used with the glass *F*, they are to be where the figure shews them; but when used with the glass *G*, they are to be set at the hole *r* above *G*.

VI. *Of the Sight Vanes.*

That at *H* is used with the glass *G* in back observations: And the sight vane *I*, which has two holes, is used with the glass *F* in fore observations.

For with this instrument an observation may be made with the face either turned from the object, and called a back observation; or with the face turned towards the object, and called a fore observation: But before any observation is attempted, the instrument should be examined to see if it is fit for use.

*To rectify, or adjust, Hadley's Quadrant.*I. *For the fore Observation.*

42.

Bring the index *D* close to the button *b*, that the middle of the Vernier's scale stand against 0 degrees: Hold the plane of the instrument vertical with the arch downwards; look through the right-hand hole in the vane *I*, and direct the sight through the transparent part of the glass *F* to observe the horizon: Now if the horizon line, seen both in the quicksilvered part, and through the transparent part, should coincide, or make one straight line, then is the glass *F* truly adjusted: But if one of the horizon lines stands above the other, slacken the screw in the middle of the lever backwards, or forwards, as there is occasion, until the horizon lines coincide; fasten the screw in the middle of the lever, and then is the horizon glass adjusted.

II. *For the back Observation.*

43.

Turn the button *b* on one side, and set the middle line of the index *D* as many degrees before 0 degrees, as is twice the dip of the horizon; on your height above the water (found in the table following): Hold the plane of the instrument vertically with the arch downwards, look through the hole in the vane *H*, and if the horizon line, seen through the transparent slit in the glass *G*, coincides with the image of the horizon seen in the quicksilvered part of the same glass, then is the glass *G* in

in its proper position: If not, slacken the screw pin in the middle of the lever behind the glass *G*, and, looking through the vane *H* as before directed, turn the screw at the end of the lever backwards, or forwards, as it is wanted, until the horizon line coincides; then tighten the middle screw, and the glass *G* is adjusted. In setting this glass by the opposite horizon, the head should be held a little backwards, not to intercept the light from behind. The horizon seen from behind will be inverted; that is, the water will appear above, and the sky below: And if the two horizon lines cross one another, the instrument is not held upright.

To take the Sun's altitude with Hadley's Quadrant.

I. *By the fore Observation.*

44.

Fix the screens *K* above the horizon glass *F*, using either, or both of them, according to the strength of the Sun's rays, by turning one or both of the frames of these glasses close against the plane or face of the instrument; then the face being turned towards the Sun, hold the quadrant by the braces *L*, *M*, or by either radius, as is found most convenient, so as to be in a vertical position with the arch downwards; put the eye close to the right-hand hole in the vane *I*; look at the horizon through the transparent part of the horizon glass *F*; at the same time, move the index *D* with the left hand until the image of the Sun, seen in the quicksilver part, falls in with the line of the horizon, taking either the upper or under edge of the solar image; Swing your body gently from side to side, and if the edge of the Sun used be observed not to cut, but to touch the horizon line, like a tangent, the observation is well made: Then shall the degrees on the arch, reckoned from that end next your body, give the altitude of that edge of the Sun which was brought to the horizon.

If the lower edge was observed, then 16 minutes added to the said degrees, gives the altitude of the Sun's center; but if the upper edge was used, the 16 minutes must be subtracted.

II. *By the back Observation.*

45.

Put the stem of the screens *K* into the hole next the horizon glass *G*, using them as before said, according to the strength of the Sun's rays: Then the back being turned to the Sun, hold the instrument by the radius *C* and brace *L*, in a vertical position with the arch downwards; put the eye close to the hole in the vane *H*, look for the horizon through the transparent slit in the glass *G*; with the right hand move the index *D* until the image of the Sun, seen in the quicksilver part of the glass *G*, stands in the horizon line, seen through the transparent slit, using either the upper or under edge of the Sun: Swing your body gently to the right and left, to try if the Sun's edge run's along the horizon; if it does, the observation is well made, and the degrees reckoned from that end of the arch farthest from your body, will give the altitude of that part of the Sun which was observed.

If the Sun's lower edge was observed, then 16 minutes subtracted from the before found degrees, will give the altitude of the Sun's center; but if the upper edge was observed, then the 16 minutes are to be added. In

In either of these observations, if the altitude of the center could be observed, there would then be no need of using the 16 minutes.

46. To find the Sun more easily in the horizon; turn your back to the Sun, and look through the vane *i* and glass *F*, down on the middle of the shadow of your head; move the index forward, till the reflected image of the horizon before you is brought down to the place you look on, in the glass *F*, and the index will then be set pretty near to the altitude, and so fitted to find the Sun more readily, either in the fore or back observation.

The fore observation is rather most convenient, especially in great altitudes, because there is a much larger scope above and below the altitude wanted, than there is in the back observation, which, on account of the obliquity of the speculum and horizon glasses, is more contracted in its use: And indeed the back observations need never be used for the Sun, when there is a clear horizon forwards, or under the Sun; when this is hazy and ill defined, then it is best to use the back observation, if the horizon is clear that way; and therefore it is proper that the horizon glass *G* should be always in readiness, by having it well adjusted; which because the former method is not readily attained by beginners, we shall annex another, which is somewhat more convenient.

47. *Another way to adjust for the back Observation.*

Take the altitude of the lower edge of the Sun by the fore observation, as near to noon as can be; then put the screens into the hole near the vane *H*, and turning your back to the Sun, and holding the instrument properly, taking care not to move the index, look for the horizon line through the transparent slit of the glass *G*, and if the horizon line touches the upper edge of the Sun's image in the glass *G*, it is properly adjusted: If they do not touch, turn the glass by the lever behind it, till they do.

This operation must be done quickly, before the Sun sensibly alters in altitude; and may be frequently repeated to make the fore and back observations agree.

To take the altitude of a star by Hadley's Quadrant.

48. I. *By the fore Observation.*

Look directly up at the star through the vane *i* and transparent part of the glass *H*, the index *D* being close to the button *b*; then will the image of the star, by reflection, be seen in the silvered part right against the star, seen through the other part: Move the index forward, and as the image descends, let the center of the quadrant descend also to keep it in the silvered part, till it comes down on a line with the horizon, seen through the transparent part; and the observation is made.

49. II. *By the back Observation.*

Through the vane *H*, and the transparent slit in the glass *G*, look directly at the star; at the same time move the index till the image of the horizon behind you, being reflected by the great speculum, is seen in the

the quicksilver part of *G*, and meets the star; and the index will then shew the degrees of altitude.

If the altitude of the star is pretty great, some other stars, beside that wanted, may appear by the light passing by the great speculum, and falling on the glass *G*; the light of these will dance with the motion of the instrument, but the true star will remain about the middle of the glass at rest, and so the others cannot be mistaken for the true one.

For the principles of the construction see the appendix.

50. Hadley's Quadrants within a few years past, having been applied to take the necessary observations for finding the longitude at sea, it has been found that such observations required a degree of accuracy, of which the instruments, constructed in the common way, were not capable. And expert mariners having complained of these defects; different workmen have applied sundry articles to the quadrants to remedy the inconveniences complained of; some of which are,

1st. A screw to the lower end of the index, to regulate its motion when brought by the hand to a division nearly shewing the contact required; and a magnifying glass to read off the divisions between the Limb and the Vernier.

2d. A small tube, or telescope, instead of the sight-vanes, to direct the line of sight in a position parallel to the plane of the quadrant; which tube was screwed into a ring fixed to a square stem that slid in the socket made for the sight-vane, and by the help of a screw in its bottom (at the back of the quadrant) the stem could be raised or lowered so as to move the axis of the telescope to point to any part of the horizon glass judged fittest for the observation.

3d. The extending the arch from an Octant, to a Sextant, or sixth part of a circle; whereby angles exceeding 90 degrees may be observed; as is sometimes wanted in taking the angular distance between the Moon and Sun, or Moon and Star: Also, for the convenience of holding the instrument easily in such observations, a stout handle was fixed to the back, which was a very necessary addition.

But other errors or inconveniences still attended these instruments, namely,

1st. The speculum being liable, in the ordinary way of fitting it, to be bent; and consequently the same observation would have different measures, according as the object happened to be reflected from different parts of the speculum: This has lately been rectified by a new manner of setting the speculum in its frame.

2d. The adjustment of the horizon glasses.

Both these glasses are to stand perpendicular to the plane of the instrument; and when *o* on the index stands against *o* on the arch, the plane of the fore horizon glass is to be parallel to the plane of the speculum; and the plane of the back horizon glass is to be at right angles to the speculum. A new method of adjusting these glasses has been lately discovered, very accurate in principle, and ready in practice: Particularly for the back observation, which hitherto has been but little used on account of the difficulty of adjusting its horizon glass; which has lately been happily removed by the ingenious Mr. *Peter Dollond*; Optician

to his Majesty; and he also thought of the method of preventing the index speculum from being bent in its frame. By his adjustments, angles of any magnitude under 180 degrees may be taken; viz. by the fore observation all under 120°, with a Sextant; and by the back observation, all between 90 and 180 degrees.

3d. Although Mr. Hadley at first directed that the line of sight should be parallel to the plane of the instrument; and for preserving it so, proposed, that in the telescope should be fixed two parallel wires, which in using should be parallel to the quadrant, and the contact of the objects should be observed in the middle between the two wires; yet these circumstances not being sufficiently attended to; therefore the two parallel wires are now placed in the focus of the eye-glass, and divide the diameter of the field of view into three equal parts; and when the telescope is adjusted parallel to the plane of the instrument, it will remain so during the observation.

4th. As every glass mirror has two reflexions, viz. one from the face, and one from the silvered back; these double reflexions not only cause some confusion among the reflected rays, but considerable errors might arise should the face and back of the glass be not parallel planes: None of these could happen from a plane mirror which has but one reflexion; and therefore, the upper part of the index speculum, has its back rough ground and blacked; whereby this part will reflect the rays only which fall on its face; and these sufficiently strong when the object observed is bright; but when otherwise, the object may be observed from the lower part of the speculum, which is silvered; the line limiting these two parts is parallel to the plane of the quadrant: This improvement was directed by the Rev. Mr. *Maskebyne*, Astronomer Royal.

The methods how these several adjustments are to be made, together with some other particulars relating to the use of the instrument, are fully treated of in the book which accompanies the sale of Mr. *Dollond's* quadrants.

51.

To make an artificial Horizon.

One great inconvenience that mariners have to struggle with at sea, is the frequent want of an horizon: For though the atmosphere may, at the height of 10 or 12 degrees and upwards, be clear enough to give a view of the Sun or other objects, yet all below that height is oft so hazy as to hinder a distinct sight of the horizon; and consequently an observation made at such a time cannot have the correctness wished for: And although water in a vessel will have its surface horizontal, and an observation may be made on the image of the Sun, seen therein, when that water is perfectly still; yet the tremulous motion of its surface, from the least wind, or motion of the ship, prevents this expedient from being generally useful at sea; and therefore many methods have been proposed; among which *Serjon's* whirling Speculum, or Top, was, for a while, thought a proper instrument, but it has been found imperfect: Some artists use the following method.

Into a wooden, or iron, circular box, of about 2½ or 3 inches diameter, and about ½ inch deep, pour about a pound or more of quicksilver; and on this lay a metal speculum, or piece of plain glass, whose diameter

diameter is about $\frac{1}{2}$ inch less than that of the box; this will float in the quicksilver, and shew the image of the Sun very steady. This apparatus being slung in jimbals, will preserve a tolerable good horizon.

The speculum, or glass, should be homogeneous, and have parallel sides: There are some workmen who can work the two planes of a piece of glass, so that they shall be demonstratively parallel.

Or the fine surface of the quicksilver will do of itself.

In all observations made with these artificial horizons, a piece of coloured glass should be fixed before the vane used, to preserve the eye; and the screens κ may be taken off.

Correction of the apparent, or observed, Altitude.

In observations of the Sun, it is usual to observe his upper or lower edge; to which, his semidiameter being applied, gives the apparent place of the center: For this purpose, among the tables page 303 is one shewing, for every ten days throughout the year, the Sun's semidiameter; as this is constantly altering with his distance from the Earth: But it is generally reckoned sufficiently accurate to allow 16 minutes for the Sun's semidiameter.

Every altitude observed at Sea, must be corrected: Two corrections are common to the center of every object; and three to the Moon.

52. I. BY THE DIP. OF THE HORIZON.

The observer's eye being raised above the level of the sea; he sees an horizon below the level of the true one; and consequently the instrument gives an altitude too great, when a fore observation is used; and too small, with a back observation.

As this error depends on the height of the eye above the water, according to the size of the ship; therefore a table of these corrections to different elevations has been computed and put in the following table for ready use. The principles upon which these computations depend, are given in the appendix to this IXth book.

53. II. BY THE REFRACTION.

The haziness, or vapours, in the atmosphere causes celestial objects to appear higher or with greater altitudes than what they really have: These refractions which are greatest at the horizon, and diminish in quantity as the altitudes increase, have been collected by eminent astronomers, and are here disposed of in the following table; And the apparent altitude, above the true horizon, is to be diminished by the quantity of the refraction on that altitude.

For the principles, see the Appendix.

54. III. BY PARALLAX.

From the explanation of parallax, at art. 66. B. V. it is evident that the Moon when seen at the same time from the surface and center of the Earth, will not appear in the same place in the heavens; and this arises because the ratio of the semidiameter of the earth, to the distance between it and the Moon (which on a mean is about $\frac{1}{2}$), is a quantity that considerably affects the observations on that planet; and therefore must be allowed for: But the distance of the Moon from the Earth constantly altering, between the Apogee and Perigee, the parallax also will be continually altering. The Moon's horizontal parallax, which lies between the limits of 53 minutes and 62 min. is given in the Nautical Almanack for every twelve hours throughout the year.

In the following table of parallaxes, is given the Moon's parallax to every two degrees of altitude and every minute of horizontal parallax; whereby her parallax to any altitude and horizontal parallax, may be found.

55. TABLES for correcting the observed ALTITUDES of CELESTIAL OBJECTS.

Sun's semi diam.			D.p. of Horizon.		Refraction on Altitude.							
Day of month.	Sun's diam.	Day of month.	Alt. of Eye.	Alt. of Hor.	Alt.	Ref.	Alt.	Ref.	Alt.	Ref.	Alt.	Ref.
"	"	"	feet.	min.	Deg.	"	Deg.	"	Deg.	"	Deg.	"
Jan. 1 16 21	31	31	5	13	4 11	51	21	2 27	40	68	59	34
10 16 21	21	21	8	3	4 10	48	22	2 20	41	65	60	33
20 16 20	11	11	10	33	5	54	23	2 14	42	63	61	32
Feb. 1 16 18	Dec. 1	12	4	4	5 1	9	8	2 7	43	61	62	30
10 16 17	21	15	15	4 1	8	28	25	2 2	44	59	63	29
20 16 15	11	18	4 1	4 1	7	7	20	26	1 56	45	57	64
Mar. 1 16 13	Nov. 1	20	5	5 1	6	29	27	1 51	46	55	65	26
10 16 10	21	25	5 1	5 1	9	5	48	28	1 47	47	53	66
20 16 7	11	30	6 1	6 1	10	5	15	29	1 42	48	51	67
Apr. 1 16 4	Oct. 1	35	7 1	7 1	11	4	47	30	1 38	49	49	68
10 16 2	21	40	7 1	7 1	12	4	4	31	1 35	50	48	69
20 15 59	11	45	7 1	7 1	13	4	3	32	1 31	51	46	70
May 1 15 56	Sept. 1	50	7 1	7 1	14	3	35	33	1 28	52	44	71
10 15 54	21	55	8 1	8 1	15	3	30	34	1 24	53	43	72
20 15 51	11	60	8 1	8 1	16	3	17	35	1 21	54	41	73
Jun. 1 15 51	Aug. 1	65	9 1	9 1	17	3	4	36	1 18	55	40	74
10 15 50	21	75	9 1	9 1	18	2	54	37	1 16	56	38	75
20 15 49	11	85	10 1	10 1	19	2	44	38	1 13	57	37	76
30 15 40	July 1	100	10 1	10 1	20	2	55	39	1 10	58	35	77

56. TABLES of the MOON'S PARALLAX in ALTITUDE.

Horizontal parallaxes			Horizontal parallaxes		
Deg.	'	"	Deg.	'	"
1	53	54	50	34	35
7	53	54	52	32	33
12	52	53	54	31	32
14	51	52	56	29	30
16	51	52	58	28	29
18	50	51	60	26	27
20	50	51	62	25	26
22	49	50	64	24	24
24	48	49	66	22	22
26	47	48	68	20	20
28	47	48	70	18	18
30	46	47	72	17	17
32	45	46	74	15	15
34	44	45	76	13	13
36	43	44	78	11	11
38	42	43	80	9	9
40	41	42	82	7	7
42	39	40	84	6	6
44	38	39	86	4	4
46	36	37	88	2	2
48	35	36	90	0	0

SECTION VIII.

Of Comparing and Correcting Time.

PROBLEM I.

57. Given the estimated account of the ship's longitude from London; to find the corresponding times at the ship and at London.

RULES.

Let the given time be always reckoned P.M. (afternoon) from the preceding noon; or from the noon 24 hours earlier, when necessary; and turn the degrees in the difference of longitude into time. (V. 133.)

I. Then east diff. long. subtracted from the ship's time gives Lond. time, P.M. from the noon used.

And east diff. longitude, added to London time, gives the ship's time, P.M. from the noon used.

Or. The excess above 24 hours is P.M. from the following noon.

II. Also west diff. longitude added to the ship's time, gives Lond. time, M. from the noon used.

Or. The excess above 24 hours, is P.M. from the following noon.

And west diff. long. subtracted from Lond. time, gives the ship's time, P.M. from the noon used.

EXAMPLES.

I. *What London time answers to a ship's noon, at a place 4 hours to the eastward of London?*

Here ship's time is 24^h from preceding noon.

Then $24^h - 4^h = 20^h$ P. M. London time.

Or. Ship's noon answers to 8 o'clock morning at London.

III. *What London time is it, when it is noon in a ship 5 h. to the west of London?*

0^h P.M. + 5^h W. = 5^h P.M. at London.

V. *What London time answers to 2 h. 38 m. P.M. in a ship 22 deg. to the E. of London?*

22° E. makes $1^h 28^m$. (V. 133.)

Then $2^h 38^m - 1^h 28^m$ E.

= $1^h 10^m$ P.M. at London.

VII. *What time is it at London, when a ship 40° W. of London reckons it 3 h. 20 m. P.M.*

40° W. = $2^h 40^m$ W. (V. 133.)

Then $3^h 20^m$ P.M. + $2^h 40^m$ W.

= $7^h 0^m$ P.M. London.

II. *When it is 8 o'clock in the morning at London, what is the corresponding time in a ship 4 h. to the eastward?*

Here 8 o'clock morning, is 20^h P.M. from noon of preceding day.

Then 20^h P.M. + $4^h = 24^h$ P.M. ship's time.

Or. 8 o'clock in the morning at London answers to noon in the ship.

IV. *When it is 5 h. P.M. at London; what hour is it in a ship 5 h. W. of London?*

5^h P.M. - 5^h W. = 0^h P.M. or noon in the ship.

VI. *When it is 1 h. 10 m. P.M. at London; what time is it in a ship 22° E. of London?*

22° E. makes $1^h 28^m$. (V. 133.)

Then $1^h 10^m$ P.M. + $1^h 28^m$ E.

= $2^h 38^m$ P.M. at the ship.

VIII. *When it is 7 h. P.M. at London; what ship time is it 40° W. of London?*

40° W. = $2^h 40^m$ W. (V. 133.)

Then 7^h P.M. - $2^h 40^m$ W. = 3^h

20^m P.M. ship.

IX. *When*

IX. *What London time answers to 5 h. 26 m. P.M. in a ship 120° E. of London?*

120° E. makes 8^h E. (V. 133).
Then 29^h 26^m P.M. — 8^h E. = 21^h 26^m P.M.
Or, 9^h 26^m in the morn. at Lond.

XL. *What London time answers to 5 h. 42 m. P.M. in a ship 110° W. of London?*

110° W. = 7^h 20^m W. (V. 133).
Then 5^h 42^m P.M. + 7^h 20^m W. = 13^h 2^m P.M. London.
Or, 1^h 2^m morning at London.

XIII. *A ship 28° 30' E. of London, reckons 7^h 42^m in the morning: What hour is it at London?*

28° 30' E. = 1^h 54^m E.
Then 19^h 42^m P.M. — 1^h 54^m E. = 17^h 48^m P.M.
Or, 48^m after 5 in the morning.

XV. *What London time answers to 6 h. 45 m. in the morning, in a ship 66° W. of London?*

66° W. = 4^h 24^m W.
Then 18^h 45^m P.M. + 4^h 24^m W. = 23^h 9^m P.M.
Or, 11^h 9^m in the morning at London.

XVII. *When it is 4 h. 26 m. morning, in a ship 98° 45' E. of London: What hour is it there?*

98° 45' E. = 6^h 35^m E.
Then 16^h 26^m P.M. — 6^h 35^m E. = 9 51^m P.M. at London.

XIX. *What London time answers to 10 h. 40 m morning, in a ship 160° W. of London?*

160° W. = 10^h 40^m W.
Then 22^h 40^m P.M. + 10^h 40^m W. = 33^h 20^m P.M. or 9^h 20^m P.M. at London.

Experience having shewn the necessity of variety of examples to most beginners, therefore were the above twenty introduced.

Vol. II.

X

Ex

X. *When it is 9 h 26^m morning at London; what ship time is it at 120° E. of London?*

120° E. makes 8^h E.
Then 21^h 26^m P.M. + 8^h E. = 29^h 26^m P.M.
Or 5^h 26^m P.M. in the ship.

XII. *What is it in a ship 110° W. of London, when it is 1 h. 2 m. in the morning at London?*

110° W. = 7^h 20^m W. (V. 133).
Then 13^h 2^m P.M. — 7^h 20^m W. = 5^h 42^m P.M. ship.

XIV. *When it is 5 h. 48 m. morning at London; what hour is it in a ship 28° 30' E. of London?*

28° 30' E. = 1^h 54^m E.
Then 17^h 48^m P.M. + 1^h 54^m E. = 19^h 42^m P.M. ship.
Or, 7^h 42^m in the morning.

XVI. *When it wants 51 m. of noon at London; what hour in a ship 66° W. of London?*

66° W. = 4^h 24^m W.
Then 23^h 9^m P.M. — 4^h 24^m W. = 18^h 45^m P.M.
Or, Time in the ship is 6^h 45^m morning.

XVIII. *What hour is it in a ship 98° 45' E. of London, when it is 9 h. 51 m. P.M. at London?*

98° 45' E. = 6^h 35^m E.
Then 9^h 51^m P.M. + 6^h 35^m E. = 16^h 26^m P.M.
Or, 4^h 26^m morning in the ship.

XX. *When it is 9 h. 20 m. P.M. as London; what hour in a ship 160° W. of London?*

160° W. = 10^h 40^m W.
Then 33^h 20^m P.M. — 10^h 40^m W. = 22^h 40^m ship.
Or, 10^h 40^m morning in the ship.

58. PROBLEM II.

To find the apparent time at sea, and thereby to regulate, or compare, the going of a watch.

Among the methods proposed for this purpose, that by equal altitudes (V. 215, 297) seems very fit to be practised at sea. At the times when the watch stands in need of being regulated for shewing the time of the observations intended, the ship lying by, let the Sun's altitude be taken at any time in the forenoon, 3, 4, or 5 hours distant from noon, set down that time and altitude: In the afternoon, wait for the Sun's having the same altitude exactly (the index of the quadrant being already set to the morning altitude), and note down that time; then the half sum of these two times, is the apparent time shewn by the clock or watch when the Sun was on the meridian of that place.

If it was wanted to reduce the apparent time to mean time,

Seek in the Nautical Almanack * for the given day, and take out the equation of time; which applied, as its title directs, to the apparent time, gives the mean time. Astronomical tables are usually fitted to mean time.

As it is possible that an altitude taken on the morning side of the meridian cannot have a corresponding one on the evening side, by the interposition of clouds; it is therefore best to take several in the forenoon, in order to have a greater probability of securing a corresponding altitude in the afternoon; and if several observations of equal altitudes can be made on both sides of the meridian, it will be best to find the noon to each pair, and take a mean of all the noons thus found for the true one.

If equal altitudes are taken within a month or six weeks of an equinox, it will be proper to make an allowance for the change in the Sun's declination between the forenoon and afternoon observations, as is shewn in Book V. articles 217, 297.

If there is reason to suspect that the watch gains or loses considerably in a day, other sets of observations should be taken on successive days, whereby the daily variation of the watch may be found, and allowed for.

EXAM. I. *May 20th, 1772, at 8^h 40^m forenoon, and at 3^h 15^m in latitude 49° N. at 8^h 10^m 58^s afternoon, by my watch, the Sun had forenoon, and at 3^h 58^m 34^s afternoon, the Sun had equal altitudes: Required the going of the watch?*

Now, add together

12 ^h 0 ^m	12 ^h 0 ^m 0 ^s
+ 8 40	+ 8 10 58
+ 3 16	+ 3 58 34
23 56	24 9 32

Take the half sum

Gives the appar. noon = 11 58

The watch noon

= 12 0

Watch too fast

2 min.

May 20th ap. noon

11 58

Equa. time subtr.

4

Mean noon, by watch.

11 54

Now add together

12 ^h 0 ^m 0 ^s	+ 8 10 58
+ 3 58 34	+ 3 58 34

Take half the sum

24 9 32	12 4 46
---------	---------

Gives the middle time

12 4 46	19
---------	----

Corr. on mid. time

12 4 27	12 0 0
---------	--------

Gives the appar. noon

12 0 0	4 27
--------	------

The watch noon

4 27	12 4 27
------	---------

Watch too slow

12 4 27	8 4
---------	-----

March 18th ap. noon

8 4	12 12 11
-----	----------

Equa. time add

12 12 11	12 12 11
----------	----------

Mean noon, by watch

* The Nautical Almanack is published annually, under the direction of the Commissioners of Longitude, which Almanack every skilful mariner should have with him at Sea.

*Other methods to find the apparent time.**I. By an observation of the Sun.*

1st. At a convenient time, take the altitude of the Sun's upper or lower limb, which correct for the femidiameter, dip, and refraction; whereby the correct zen. dist. of the Sun's center will be known.

2d. At the time of the observation, find the Ship's lat. and long. by account; (VIII. 60.) and correct the Sun's decl. for the day, by the diff. long. and time from noon; and thereby get the true polar distance.

3d. With the co-latit. polar dist. and zen. dist. find the hour angle. (V. 152)

Which turned into time, shews the app. time when the obser. was made.

II. By an observation of a Star.

1st. Correct the observed altitude by the dip. and refraction. (55)

2d. Find the ship latitude and longitude by account (VIII. 60.)

3d. Seek the stars decl. or polar dist. and its right ascension. (V. 312.)

4th. With the co-lat. polar dist. and zen. dist. find the hour angle (V. 155.)

5th. Apply the hour angle (in time) to the star's right ascension, and it gives the right ascension of the mid-heaven. (V. 133.)

6th. Find the Sun's right ascension to the place and time (V. 288.)

7th. The difference between the Sun's right ascension and that of the mid-heaven is the apparent time of the observation. Regulate the watch by the apparent time thus found.

The logarithmic work for the Sun in art. 3d; or for the star in art. 4th. may be performed by either of the following rules.

RULE I. Let the diff. between the co-lat. and polar dist. be called *D*.

Find the half sum, and half diff. of the zen. dist. and *D*.

Then, To the ar. co. of the log. fines of the co-lat. and polar dist.

Add the log. fines of the said half sum and half diff.

And Half the sum of these four logs. is the log. sine of $\frac{1}{2}$ the hour angle.

RULE II. Add together the co-alt. co-lat. and polar dist. call the half sum *A*.

Let the diff. between the half sum *A* and co-alt be called *D*.

Then, To ar. co. of the log fines of the co-lat. and polar dist.

Add the log. fines of *A* and *D*.

And Half the sum of these 4 logs. is the log. co-sine of $\frac{1}{2}$ the hour angle.

EXAMPLES.

I. January 26 1773, at 2^h 56^m 56^s P. M. by my watch, the true alt. of the sun's center was 9 44', the Ship's lat. 51° 42' N. long. 33° W. Required the error of the watch?

33° W = 2^h 12^m W. (V. 133)
Now 2^h 58^m 56^s P. M. + 2^h 12^m W.
= 5^h 10^m 56^s P. M. (57)

Sun's decl. at noon 18° 33' 3" S.
decr. of decl. in 5^h 11^m 32^s (V. 481)

Sun's decl. at Ship 18 29 43 S.
+ 90

Polar dist. 108 29 43

Co. lat. 38 18 ar. co. log. fine 0,20776

Po. dist. 108 30 ar. co. log. fine 0,02304

Co. alt. 80 16

P 70 12

150 28 | 75 14 log. fin. 9,98541

10 04 | 5 2 log. fin. 8,94317

Sum 19,15938

22° 19' 40" Log. fine 9,57969

2

44 39 20 = 2^h 58^m 37^s ap. time

2 58 50 watch time

Watch too fast 19

III. Sept. 12th 1773, in lat 46 16 N. long. 41½° W. at 8^h 3½^m forenoon, the true alt. of the Sun's center was 26° 17': Required the error of the Watch?

Ship's time 20 3 30 P. M.

Long. by ac. 2 46 0 W.

Greenw. time 22 49 30 P. M.

Sun's dec. Sep. 11 at noon 4° 22' 6" N.
decr. of decl. in 2¼ h. 21 15

Sun's decl. at Ship 4 0 51 N.

Polar dist. 85 59 0

Co. lat. 43 44

Pol. dist. 85 59

Co. alt. 63 43

D 42 15

105 58 | 52 59 L 9,90225

21 28 | 10 44 L 9,27007

Sum

19,33372

27 40 L 9,66686

Or 20 18 40 P. M. ap. time

20 330 watch time

Watch too slow 15 10

II. Being in lat. 43° 16' N. long 23° 30' E. at 7^h 16½^m forenoon, the true alt. of the Sun's center was found to be 29° 4': How did the watch go May 24th 1773?

Ship's st. 19^h 16^m 30^s P. M.

Long. 1 34 0 E.

Greenw. t. 17 42 30 P. M. or 5^h 42½^m morn

Sun's dec. May 23 noon 20 41 36

incr. of decl. in 18^h 8 12

Sun's decl. at Ship 20 49 48 N.

Polar dist. 69 10 12

Co. alt. 60 56

Co. lat. 46 44 ar. co. log. fine 0,13777

Po. dist. 69 10 ar. co. log. fine 0,02937

Sum 176 50

Sum A 88 25 log. fine 9,99983

Co. alt. 60 56

Diff. B 27 29 log. fine 9,66416

Sum 19,83113

34 35 Log. Co fine 9,91556

à noon 69 10 = 4^h 36^m 40^s app. time

Or 19 23 20 P. M. ap. time

19 16 30 Watch time

Watch too slow 6 50

IV. Dec. 17th 1773, in lat. 50° 34' S. long 55° 40' W. at 2^h 53½^m P. M. by a watch the true alt. of the Sun's center was 41° 32': What error was there in the watch?

Ship's time 2^h 53^m 30^s P. M.

Long. by acc. 3 42 40 W.

Greenw. time 6 46 10 P. M.

Sun's decl. Dec. 17 at noon 23° 24' 17" S
incr. of decl. in 6½ h. 23

Sun's decl. at Ship 23 24 50 S.

Polar distance 66 35 10

Co. alt. 48 28

Co. lat. 39 26 L's 0,19710

Pol. dist. 66 35 L's 0,03733

Sum 154 29

Sum A, 77 14½ L's 9,98914

48 28

Diff. B, 28 46½ L's 9,68248

Sum 26° 10' L's 9,95302

à noon 52 20 = 3^h 29^m 29^s app. time

2 53 30 watch time

Watch too slow 35 50

SECTION IX.

59. *To find the Latitude at Sea.*

Find the meridian altitude or zenith distance of the Sun or Star by article 37, or 44, or 45, or 48, 49.

Take also the corrections on the dip of the horizon, and on refraction in arts. 52, 53, 55.

Seek the Sun's or Star's declination in the tables at art. 309, 312, Book V; or in the nautical almanack, observing to correct the Sun's decl. for places more than 12° to the E. or W. of London, as at art. 311, Book V.

In latitudes near the poles, when the Sun's declination exceeds the co-latitude, the Sun then does not set; and therefore two altitudes may be taken on the meridian; the least of which is called the altitude below the pole: And the same may be understood of any other celestial object, which does not set.

Let the declination and the altitude, or zenith distance, be always named either north or south, according to the season of the year, and the situation of the object; and when it so happens that they are of no magnitude, they may then be named either north or south at pleasure.

These remarks being premised, all the varieties of working a meridian observation for the latitude, are contained in the solution to the following problem.

P R O B L E M I.

The declination of the Sun or a Star, and also its correct meridian altitude, or zenith distance, being known; to find the latitude of the place of observation.

WHEN THE OBJECT RISES AND SETS.

60. CASE I. *If the zen. dist. and decl. have the same name.*

RULE. Their difference shews the latitude required.

Like the declination, when it is greater than the zenith distance: But unlike, when it is less than the zenith distance.

61. CASE II. *If the zen. dist. and decl. have contrary names.*

RULE. Their sum shews the latitude required.

And is always like the declination.

When altitudes are given, take them from 90° for zenith distances.

62. WHEN THE OBJECT DOES NOT SET.

If the greatest meridian altitude or zenith distance can be taken, the preceding rules will serve: But when the least, or that below the pole, can only be taken. Then

RULE. To the meridian altitude below the pole, add the co-declination, the sum shews the latitude, of the same name with the declination.

The demonstrations of these rules will be shewn in the Appendix.

X 3

EXAM.

EXAM. I. *The Sun's meridional zenith distance was observed to be 48° $20'$ southerly, when he entered the sign Aries: Required the lat. of the place?*
 The zenith distance $48^{\circ} 20' \text{ S.}$

Declination $00^{\circ} 00' \text{ S.}$

Latitude sought $48^{\circ} 10' \text{ N. (60)}$

EXAM. III. *Being at sea, the Sun was observed in the zenith, when his declination was $22^{\circ} 10' \text{ N.}$: What is the latitude of that place?*

The zenith distance $00^{\circ} 00' \text{ N.}$

Declination $22^{\circ} 10' \text{ N.}$

Lat. sought $22^{\circ} 10' \text{ N. (60)}$

EXAM. V. *The Sun was observed to be $47^{\circ} 25'$ distant from the zenith in the south, when his declination was $14^{\circ} 18' \text{ S.}$: What is the lat. of the place of observation?*

The zenith distance $47^{\circ} 25' \text{ S.}$

Declination $14^{\circ} 18' \text{ S.}$

Lat. sought $33^{\circ} 07' \text{ N. (60)}$

EXAM. VII. *When the Sun's declination was $23^{\circ} 09' \text{ S.}$ his meridional zenith distance was observed to be $8^{\circ} 25' \text{ S.}$: Required the latitude of the place?*

The zenith distance $8^{\circ} 25' \text{ S.}$

Declination $23^{\circ} 09' \text{ S.}$

Lat. sought $14^{\circ} 44' \text{ S. (60)}$

EXAM. IX. *The Sun's meridional zenith distance was observed to be $32^{\circ} 57' \text{ S.}$ when his declination was $10^{\circ} 38' \text{ N.}$: What is the latitude of the place?*

The zenith distance $32^{\circ} 57' \text{ S.}$

Declination $10^{\circ} 38' \text{ N.}$

Lat. sought $43^{\circ} 35' \text{ N. (61)}$

EXAM. XI. *Observed the meridional latitude of the Star Arcturus to be $61^{\circ} 14' \text{ S.}$: Required the latitude?*

Mer. zen. dist. $28^{\circ} 46' \text{ S.}$

Arct. declin. $20^{\circ} 26' \text{ N. (V. 312)}$

Lat. sought $49^{\circ} 12' \text{ N.}$

EXAM. II. *When the Sun entered the sign Libra, his meridional altitude was observed to be $53^{\circ} 32'$ in the north: What is the latitude of that place?*

The co alt. or zen. dist. $36^{\circ} 28' \text{ N.}$

Declination $00^{\circ} 00' \text{ S.}$

Lat. sought $36^{\circ} 28' \text{ S. (61)}$

EXAM. IV. *The Sun's merid. zenith distance was observed $21^{\circ} 08' \text{ S.}$, when his decl. was the same: Required the latitude of the place of observation?*

The zenith distance $21^{\circ} 08' \text{ S.}$

Declination $21^{\circ} 08' \text{ S.}$

Lat. sought $00^{\circ} 00' \text{ S. (60)}$

EXAM. VI. *The Sun's meridional altitude was observed to be $54^{\circ} 27'$ north, when his declination was $19^{\circ} 57' \text{ north}$: Required the latitude of the place?*

The co-alt. or zen. dist. $35^{\circ} 33' \text{ N.}$

Declination $19^{\circ} 57' \text{ N.}$

Lat. sought $15^{\circ} 36' \text{ S. (60)}$

EXAM. VIII. *The Sun having $23^{\circ} 11' \text{ N.}$ declination, his meridional altitude was found to be $79^{\circ} 23' \text{ N.}$: What is the latitude of the place of observation?*

The co-alt. or zen. dist. $10^{\circ} 37' \text{ N.}$

Declination $23^{\circ} 11' \text{ N.}$

Lat. sought $12^{\circ} 34' \text{ N. (60)}$

EXAM. X. *Suppose the meridional altitude of the Sun was observed to be $48^{\circ} 14' \text{ N.}$, when his decl. was $17^{\circ} 38' \text{ S.}$: Required the latitude of that place?*

The co alt. or zen. dist. $41^{\circ} 46' \text{ N.}$

Declination $17^{\circ} 38' \text{ S.}$

Lat. sought $59^{\circ} 24' \text{ S. (61)}$

EXAM. XII. *The altitude of the Star Sirius, when on the meridian was $24^{\circ} 32' \text{ S.}$: Required the latitude?*

Mer. zen. dist. $65^{\circ} 28' \text{ S.}$

Syrius's declin. $16^{\circ} 24' \text{ S. (V. 312)}$

Lat. Required $48^{\circ} 54' \text{ N.}$

Exam.

63. Examples, wherein the foregoing Corrections are introduced.

I. *By a fore observation, the altitude of the Sun's lower edge was found, by a Hadley's quadrant, to be $40^{\circ} 20' S.$, when the declination was $9^{\circ} 56' N.$, the eye being 20 feet above the horizon: Required the latitude of that place?*

Apparent alt. $\odot$ lower edge	$40^{\circ} 20' S.$
Semidiameter. Add	$0 \ 16$
Apparent altitude $\odot$ center	$40 \ 36$
Dip. of the horiz. Subtract	$0 \ 5$
App. alt. cor. by the dip.	$40 \ 31$
Refraction. Subtract	$0 \ 1$
True altitude $\odot$ center	$40 \ 30$
True zenith distance	$49 \ 30 S.$
Declination	$9 \ 56 N.$
Latitude (61)	$59 \ 26 N.$

III. *Suppose the eye of an observer, at 35 feet above the water, should with Hadley's quadrant, by a fore observation, find the altitude of Sirius to be $53^{\circ} 35' S.$, when it passed the meridian; having before-hand set his watch and found the time of Sirius's passage (V. 133): Required the latitude of the place of observation?*

Apparent alt. of Sirius	$53^{\circ} 35' 0'' S.$
Dip. of the horiz. Subt.	$6 \ 30$
Appar. alt. above hor.	$53 \ 28 \ 30$
Refraction. Subtract	$0 \ 0 \ 40$
True altitude of Sirius	$53 \ 27 \ 50$
True zenith distance	$36 \ 32 \ 10 S.$
Sirius's decl. (V. 312)	$16 \ 23 \ 41 S.$
Latitude (60)	$20 \ 8 \ 29 N.$

II. *By a back observation, with Hadley's quadrant, the apparent altitude of the Sun's lower edge was $25^{\circ} 12' S.$, when the declination was $21^{\circ} 14' S.$, and the eye 30 feet above the horizon: In what latitude was that observation made?*

Apparent alt. $\odot$ lower edge	$25^{\circ} 12' S.$
Semidiameter. Subtract	$0 \ 16$
Apparent altitude $\odot$ center	$24 \ 56$
Dip of the horizon. Add	$0 \ 6$
App. alt. cor. by the dip	$25 \ 02$
Refraction. Subtract	$0 \ 2$
True altitude $\odot$ center	$25 \ 00$
True zenith distance	$65 \ 0 S.$
Declination	$21 \ 14 S.$
Latitude (60)	$43 \ 46 N.$

IV. *In a ship where the eye is 25 feet above the horizon, an observer having regulated his watch and found the time when Arcturus passes the meridian (V. 133), its altitude was then found to be $61^{\circ} 21' S.$ by a back observation with Hadley's quadrant: What latitude was that observer in?*

Appar. alt of Arcturus	$61^{\circ} 21' 0'' S.$
Apparent zenith dist.	$28 \ 39 \ 0$
Dip. of the horiz. Add	$5 \ 30$
Zen. dist clear of the dip.	$28 \ 44 \ 30$
Refraction. Add	$0 \ 0 \ 30$
True zenith distance	$28 \ 45 \ 0 S.$
Arcturus's decl. (V. 312)	$20 \ 29 \ 15 N.$
Latitude (61)	$40 \ 14 \ 15 N.$

V. May 26, 1772, being by account in longitude $67^{\circ} 30' E.$ of London, and the eye 18 feet high, the meridian altitude of the Sun's lower limb, by a fore observation, was $26^{\circ} 12' N.$: Required the ship's latitude?

Ship's time	$24^h 00^m P.M.$	May 25.
Lon. $67^{\circ} 30' E.$	$= 4^h 30^m E.$	
Long. time	$19^h 30^m P.M.$	$6^h 30^m \times 3.$
May 25. $\odot$ decl. $21^{\circ} 06' 19'' N.$	diff. $10' 18''$	
Incr. of decl.	$8' 12''$	(V. 285)
At the ship	$21^{\circ} 14' 31'' N.$	decl.
App. alt. $\odot$ lower limb	$26^{\circ} 12'$	
Semi-diameter	$+ 16$	
App. alt. $\odot$ center	$26^{\circ} 38'$	
Dip. on 18 feet	$- 4' 40''$	
Refraction	$- 1' 45''$	
True alt. $\odot$ center	$26^{\circ} 31' 35''$	
Zenith diff.	$63^{\circ} 28' 25'' N.$	
Declination	$21^{\circ} 14' 31'' N.$	
Latitude	$42^{\circ} 13' 54'' S.$	

The method in the foregoing problem for finding the latitude from the meridian altitude of a celestial object, is the most easy when it can be done: but many circumstances at sea frequently deny this opportunity: And on this account were the problems in Section VI. Book V. introduced; from whence it was judged proper to deduce some of the following practical methods for ready use.

PREPARATION.

Take one, two, or more altitudes of the Sun in the forenoon, noting by a watch the time of each: And, if the meridian altitude cannot be obtained, take one, two, or more, altitudes of him in the afternoon, noting also the times by the watch.

If it is possible, let the morning altitudes, if three, be taken at equal intervals of time; if not, let the afternoon observations, if they can, be taken at the same altitudes with the morning ones.

64. PROBLEM II.

If one morning and one afternoon observation are taken when the Sun has the same altitude; and the name of the elevated pole is known; to find the elevation, or latitude. See Book V. articles 182, 217.

RULE 1st. To 12 hours, add the morning and afternoon times, shewn by the watch; and half the sum gives the time by the watch when the Sun was on the meridian of that place, supposing no change of declination: Let the interval of time between that noon and the time of either observation be turned into degrees and minutes (V. 132) for the hour angle, which is here supposed to be less than 90° , by the observations being within six hours of noon.

2d. To the logarithmic co-sine of the hour angle, add the logarithmic tangent of the Sun's distance from the elevated pole, their sum is the logarithmic tangent of the first arc, of the same kind with the polar distance.

3d. Add

VI. June 12th, 1772, being in a high northern latitude, $83^{\circ} W.$ of London, and the eye 20 feet high, the meridian altitude, below the pole, of the Sun's lower limb was $8^{\circ} 15' N.$ by a fore observation. Required the latitude?

Ship's time	$12^h 0^m P.M.$	
Long. $83^{\circ} W.$	$= 5^h 40^m W.$	
Long. time	$17^h 40^m P.M.$	$= 4^h 25' \times 4.$
June 12. $\odot$ decl. $23^{\circ} 13' 17'' N.$	diff. $3' 16''$	
Incr. of decl.	$2' 28''$	
At the ship	$23^{\circ} 15' 45'' N.$	decl.
App. alt. $\odot$ lower limb	$8^{\circ} 15' N.$	
Semi-diameter	$+ 16$	
Ap. alt. $\odot$ center	$8^{\circ} 31'$	
Dip. on 20 feet	$- 5$	
Refraction	$- 8' 26''$	
True alt. $\odot$ center	$8^{\circ} 20' 8'' N.$	
$\odot$ co-declination	$66^{\circ} 44' 15'' N.$	
Latitude	$75^{\circ} 4' 23'' N.$	

3d. Add together the ar. co log. co-fine of the polar distance, the log. fine of the altitude, and the log co-fine of the foresaid first arc; their sum is log. co-fine of a second arc, always less than 90 degrees.

4th. The difference of the first and second arcs is the complement of the required latitude when the first arc exceeds the second, otherwise the difference is the latitude.

EXAM. I. Being in north latitude, at 8 h. 24 m. 30 s. A. M. the Sun's altitude was observed to be $30^{\circ} 22'$; and at 3 h. 39 m. 20 s. P. M. he was observed to have the same altitude; the declination on that day being $10^{\circ} 17\frac{1}{2}'$ N. Required the latitude of the place where those observations were made?

12 ^h 0 ^m of	L, co-fine hour $\angle$	$54^{\circ} 21\frac{1}{2}'$	9,76350
8 24 30	L, tangent polar distance	79 $42\frac{1}{2}'$	10,74093
3 39 20	L, tangent, first arc	72 37	10,50443
2) 24 3 50	Ar. co. L, co-fine polar distance	79 $42\frac{1}{2}'$	0,74797
12 1 55	L, fine altitude	30 22	9,70375
8 24 30	L, co fine first arc	72 37	9,47533
4) 3 37 25	L, co-fine second arc	32 17	9,92705
hour $\angle$ $54^{\circ} 21\frac{1}{2}'$	The co-latitude	40 20	
	Hence the latitude is	49 40 N.	

EXAM. II. In north latitude, on a day when the Sun's declination was $14^{\circ} 55'$ S., his altitude at 9 h. 7 m. 18 s. A. M. was found to be $15^{\circ} 45'$; and he was observed to have the same altitude at 2 h. 25 m. 44 s. P. M. What was the latitude of that place?

12 ^h 0 ^m of	L, co-fine hour angle	$39^{\circ} 48'$	9,88552
9 7 18	L, tangent polar distance	104 55	10,57448
2 25 44	L, tangent first arc	109 7	10,46000
2) 23 33 2	Ar. co. L, co-fine polar distance	104 55	0,58937
11 46 31	L, fine altitude	15 45	9,43367
— 9 7 18	L, co-fine the first arc	109 7	9,51520
4) 2 39 13	L, co-fine second arc	69 48	9,53824
hour $\angle$ $30^{\circ} 48'$	Complement of the latitude	39 19	
pol. dist. $104^{\circ} 55'$	The latitude	50 41 N.	

EXAM. III. Being in the torrid zone, at 8 h. 20 m. A. M. the Sun's altitude was observed to be $35^{\circ} 56'$, and at 3 h. 50 m. P. M. he had the same altitude, the declination being $19^{\circ} 48'$ N. Required the latitude of that place?

12 ^h 0 ^m	L, co-fine hour $\angle$	$56^{\circ} 15'$	9,74474
8 20	L, tangent polar distance	70 12	10,44367
3 50	L, tangent first arc	57 $3\frac{1}{2}'$	10,18841
2) 24 10	Ar. co. L, co-fine polar distance	70 12	0,47014
12 5	L, fine altitude	35 36	9,76852
— 8 20	L, co-fine first arc	57 $3\frac{1}{2}'$	9,73546
4) 3 45	L, co-fine second arc	70 25	9,97412
hour $\angle$ $56^{\circ} 15'$	Latitude sought	13 $21\frac{1}{2}'$ N.	
pol. dist. $70^{\circ} 12'$			

65.

PROBLEM III.

When three zenith distances between the limits of 30 and 70° are taken within an hour of noon, at equal intervals of time between 15 and 30 minutes.

CASE I. If the first and third zenith distances are equal.

RULE. The middle one is the meridian zenith distance.

EXAMPLE. *On a day when the Sun's declination was 18° 34' N. : some time near noon his zenith distance was found to be 32° 29' S. ; and 22 minutes after, the zenith distance was 32° 11' ; and 22 minutes afterwards the zenith distance was 32° 29' : Required the latitude of that place ?*

Here are equal zenith distances, at equal intervals of time from the middle observation ; which therefore is a meridian observation.

Then $32^{\circ} 11' S + 18^{\circ} 34' N. 50^{\circ} 45' N.$ the latitude. (61)

(66) CASE II. If the two first, or two last, zenith distances are equal. Then that on the meridian falls between those equal ones.

RULE. From one of the equal zenith distances, take one-eighth part of the difference between it and the greater zenith distance ; and the remainder is that on the meridian.

EXAMPLE. *Within an hour of noon, and at intervals of 22 minutes by a watch, the following zenith distances were observed, viz. 1st. $32^{\circ} 50\frac{1}{2}'$; 2d. $32^{\circ} 15\frac{1}{2}'$; 3d. $32^{\circ} 15\frac{1}{2}'$: Required the meridian zenith distance ?*

1st. $32^{\circ} 50,6' \}$ diff. $35,1'$; its $\frac{1}{8} = 4,8'$; then $32^{\circ} 15,5' - 4,8 = 32^{\circ} 10,7'$.
2d. $32^{\circ} 15,5'$

The meridian zenith distance is $32^{\circ} 11'$.

67. CASE III. When the three zenith distances are unequal.

If the two first are ascending, or the last descending ; then noon falls in that interval whose two zenith distances are nearest to equality.

RULE. Let the difference between the greatest and middle be called A. the difference between the greatest and least be called B. the difference between B and four times A be called C. the difference between B and C be called D.

Then C multiplied by itself, and the product divided by four times D, the quotient is a number of minutes, which subtracted from the greatest zenith distance, leaves the meridian zenith distance.

EXAM. I. *Being in north latitude, the Sun was thrice observed near noon when the zenith distances and times by a watch were as follow.*

1st. At 11h. 27m. the zenith distance = $32^{\circ} 51'$,

2d. 11h. 51m. the zenith distance = $32^{\circ} 14'$,

3d. 12h. 15m. the zenith distance = $32^{\circ} 19'$.

Required the meridian zenith distance ?

Zenith distance, 1st. $32^{\circ} 51'$ | A = 37
2d. $32^{\circ} 14'$ | B = 32
3d. $32^{\circ} 19'$ | C = 32
 $4A - B = 148 - 32 = 116 = C.$
 $C - B = 116 - 32 = 84 = D.$

And $\frac{116 \times 116}{84 \times 4} = 40'$, the correction.

Then $32^{\circ} 51' - 40' = 32^{\circ} 11'$, is the meridian zenith distance.

EXAM.

EXAM. II. Being in north latitude, three observations were made of the Sun near noon, at equal intervals of time, viz.

1st. At 11h. 2 m. the zenith distance was $34^{\circ} 11'$,
 11h. 20 m. 33 9,
 11h. 38 m. 32 29,

Required the meridian zenith distance?

Here $A=62$; $B=102$; $C=146$; $D=44$.

And $\frac{146 \times 146}{44 \times 4} = 121' = 2^{\circ} 1'$, the correction.

Then $34^{\circ} 11' - 2^{\circ} 1' = 32^{\circ} 10'$, is the meridian zenith distance.

EXAM. III. Being in the torrid zone, and the Sun having $19^{\circ} 48' S.$ decl., three observations were taken at equal intervals of time, near noon, viz.

At 10h. 56 m. the zen. dist. was $36^{\circ} 41 S.$ } Required the latitude.
 11 22 34 26
 11 48 33 16

Here $A=2^{\circ} 15' = 135$; $B=3^{\circ} 25' = 205$; $C=335$; $D=130$;

And $C^2 = 112225$; $4D = 520$. Then $\frac{112225}{520} = 216' = 3^{\circ} 36'$.

And $36^{\circ} 41' - 3^{\circ} 36' = 33^{\circ} 05'$, the meridian zenith distance.

Then $33^{\circ} 05' - 19^{\circ} 48' = 13^{\circ} 17' N.$ the latitude sought. (60)

EXAM. IV. Being in a low north latitude, and the Sun having $14^{\circ} 24' S.$ declination; when the watch showed 11h. 42 m., 12h. 20 m., and 12h. 58 m., the Sun's zenith distances were $43^{\circ} 19'$, $43^{\circ} 23'$, and $45^{\circ} 19'$, respectively, Required the latitude?

11h. 42 m. zen. dist.	$43^{\circ} 19'$	} $B = 2^{\circ} 0' = 120$.
12 20	$43 23$	
12 58	$45 19$	
L, C	344	} $A = 1 56 = 116$. $4A = 464$.
L, 4d subtr.	896	
L, correction	132	
	5,07312	} Greatest zenith distance $45^{\circ} 19'$
	2,95231	
	2,12081	
		} Take the correction 2 12
		} Meridian zenith distance $43 7 S.$
		} Declination
		} Latitude (60)

68. PROBLEM IV.

When three zenith distances, between the limits of 30° and 70° are taken within an hour and half of noon, at unequal intervals of time.

RULE 1st. Let the difference in minutes between the greatest and middle zenith distances be called A : and the interval of time called a . The difference between the first and last zenith distances be called B . And the corresponding interval of time called b .

2d. Let the difference between the products of A by b and B by a be taken.

From the logarithm of this difference take the logarithm of b , gives a logarithm, call it N .

3d. Add

3d. Add together; the logarithm of N .

twice the log. of the diff. between a and $\frac{1}{2}b$.

Ar. co. log. of the diff. between a and b .

Ar. co. logarithm of a .

Their sum gives a logarithm, call it M .

4th. Seek the numbers to N , M , $\frac{1}{2}b$; their sum gives E .

From four times E take B , gives C ; and B taken from C , gives D .

And the logarithm of four times D taken from twice the logarithm of C , give the logarithm of the correction in minutes

5th. Then the greatest zenith distance lessened by the correction, gives the meridian zenith distance.

EXAM. I. Three observations being made of the Sun, at a place in north

latitude, viz. at 10 h. 40 m. the zenith distance was $35^{\circ} 53'$,

11 9 was $33^{\circ} 44\frac{1}{2}'$,

11 51 was $32^{\circ} 14'$

How much was the Sun's meridian zenith distance?

10 h. 40	zen. dist. $35^{\circ} 53'$	
11 9	$a = 29$ m.	$A = 128,5'$
11 51	$b = 71$	$B = 219$
		$\frac{1}{2}b = 35,5$
		$\frac{1}{2}a = 42$

$128,5 \times 71 = (A \times b) = 9123,5; 219 \times 29 = (B \times a) = 6351$; the diff. = $2772,5$.

Logarithm diff. = $2772,5$ $\frac{1}{2}B = 109,5$

Logarithm $b = 71$ $N = 39,5$

Logarithm $N = 39,5$ $M = 1,3$

Twice log. $\frac{1}{2}b = 6,5$ $E = 150,3$ and $4E = 601,2$

Ar. co. log. $\frac{1}{2}a = 42$ $B = 219$

Ar. co. log. $a = 29$ $C = 382,2$

Logarithm $M = 1,3$ $D = 163,2$ and $4D = 652,8$

Twice log. $C = 382,2$ $\frac{1}{2}C = 191,1$ Great zenith distance $35^{\circ} 53'$

Logarithm $\frac{1}{2}C = 191,1$ $223 = 3^{\circ} 43'$

Log. correction = 223 Merid. zenith distance $32^{\circ} 10'$

EXAM. II. Being to the northward of the tropic of Cancer, when the Sun's

declination was $14^{\circ} 24' 8''$; at the times 11 b. 42 m., 12 h. 26 m. 12 b.

58 m., seen by a watch, the Sun's zenith distances were $43^{\circ} 19'$, $43^{\circ} 33'$,

$45^{\circ} 19'$, respectively: Required the latitude?

11 h. 42 m. 1st. zen. dist. $43^{\circ} 19'$

12 26 $b = 76$ m. 2d $43^{\circ} 33'$ $B = 120$ $\frac{1}{2}b = 6$

12 58 $a = 32$ 3d. $45^{\circ} 19'$ $A = 106$ $\frac{1}{2}a = 44$

$A \times b = 106 \times 76 = 8056$; $B \times a = 120 \times 32 = 3840$; diff. of prod. 4216 .

L 4216 $\frac{1}{2}B = 60$

L, b 76 $N = 55,5$

L, N $55,5$ $M = 1,4$

2L, $\frac{1}{2}b = 6$ $E = 116,9$, and $4E = 467,6$

L, $\frac{1}{2}a = 44$ $B = 120$

L, a 32 $C = 347,6$

L, M $1,4$ $D = 227,6$, and $4D = 910,4$

2L, C $347,6$ Greatest zenith distance $45^{\circ} 19'$

L, 4D $910,4$ Correction 133 $2^{\circ} 13'$

K, correction $132,7$ Merid. zenith distance $43^{\circ} 6' S$.

Declination $28^{\circ} 42' N$.

PROBLEM V.

69. *When three altitudes are taken at equal intervals of time, at any distance from noon.* See Book V. article 202.

RULE 1st. Seek the natural sines of the three given altitudes. (V. 185) Call the difference between the first and third altitudes, D; between the second and third, F.

Put E for the sum of D and twice F, when the degrees in the third altitude falls between those of the first and second; otherwise put E for the difference between D and twice F.

2d. Add together the log. co-tan. of half one interval of time (in deg.) the log. of E, and the ar. co. log. of D (each in numbs.)

Their sum is the logarithmic tangent of an angle A; or of its supplement, if the first altitude exceeds the third, convert the angle A into time. (V. 132)

To A add one interval; then the difference between this sum and six hours will shew the time from noon when the last observation was made.

Hence, by the intervals, the other times are known relative to the noon of the place of observation.

3d. Add together the ar. co. log. co. sine of A.

the ar. co. log. sine of one interval observed,

and the log. of E, their sum is the log of a number B, and the logarithm of B, add twice the logarithmic sine of half the least time from noon; seek the number, and add it to the natural sine of the greatest altitude, gives the natural sine of the meridian altitude.

5th. The number B lessened by the natural sine of the meridian altitude, gives the natural sine of the midnight depression.

Then the half sum, and the half difference, being taken, of the degrees and minutes answering to the sines of the meridian altitude and midnight depression, gives the co-latitude and declination.

If the zenith falls between the equator and the Sun; the half sum is the co-declination, and the half difference is the latitude.

EXAM. I. *Being between the tropics, the Sun's altitude was observed some time in the forenoon, to be $73^{\circ} 22' 1''$; after waiting 26 minutes, its altitude was $78^{\circ} 50'$, and 26 minutes after, its altitude was $82^{\circ} 55'$: Required the latitude of that place?* (See Book V. note to art. 185.)

78 50	,98107	,03421 = D and ,01161 = E;	
82 55	,99237	also 26 m. = $6^{\circ} 30'$,	
		its half = $3 15'$.	
L, i $3^{\circ} 15'$	11,24577	L, S, A $80^{\circ} 30'$	0,78239
L, S = 0,01161	8,06483	L, S, $6 30$	0,94614
L, D = 0,03421	1,46584	L, D $0,03421$	8,53416
L, S, A = $80^{\circ} 30'$	10,77644	L, S $1,8310$	10,26269
		2L, $1^{\circ} 30' (= \frac{12 m.}{2})$	6,83584
		0,001255	7,09853
		,99237	
Or A = 5 hours 22 minutes.			
Now 5 h. 22 m. + 26 m. = 5 h. 48 m.			
Then 6 h. — 5 h. 48 m. = 0 h. 12 m.			
So the greater alt. was at 11 h. 48 m.			
the middle at 11 22			
the least at 10 56			
Mer. alt. 0,99362 = nat. sine $83^{\circ} 32'$.			
Mid. dep. 0,8374 = nat. sine $56 52'$,			
half sum = $70^{\circ} 12'$ the co-declinat.			
half diff. = $13 20$ the latitude.			

EXAM.

EXAM. II. In the north temperate zone, the Sun having south declination, his altitude was observed to be $8^{\circ} 37\frac{1}{2}'$; and 66 minutes afterwards, his altitude was $13^{\circ} 51\frac{1}{2}'$; and 66 minutes after that, his altitude was $16^{\circ} 34'$: Required the latitude?

At some time, alt. $8^{\circ} 37\frac{1}{2}'$ nat. sine, 14997
 66 m. 13 $51\frac{1}{2}'$ } ,13517 = D.
 66 m. 16 34 } ,23955
 ,28514 } ,04559 = F.
 2F = ,09118 & E = ,04398.

66 m. = 16 30'. And 6 h. = 1 h. 6 m = 4 h. 54 m.
 L, A $8^{\circ} 15'$ 10,83865 L, A 65° 59' 0,39040
 L, E = 0,04398 8,64325 L, 16 30 0,54666
 L, D = 0,13517 0,86913 L, D 0,13517 9,13087
 L, A = 65° 59' 10,35103 L, B 1,1693 10,06793
 2L, 3° 45½ 7,63312

Or A = 4 h. 23 m. 56 f.

Now 4 h. 23 m. 56 f. + 1 h. 6 m. = 5.29.56.

And 6 h. = 5 h. 29 m. 56 f. = 0 h. 30 m. 4 f.

11 29 56 at last alt.

10 23 56 mid alt.

9 17 56 least alt.

Greater alt.

s, merid. alt. 0,29016 = 16° 52

s, dep. 0,8792 = 61 33

the $\frac{1}{2}$ sum = $39^{\circ} 12\frac{1}{2}'$ the co-latitude

the $\frac{1}{2}$ diff. = 22 20½ the declination

EXAM. III. Being in a high north latitude, and the Sun to the eastward of the meridian, his altitude was observed to be $41^{\circ} 53'$; and 2 h. 16 m. after, his altitude was $50^{\circ} 23\frac{1}{2}'$; and 2 h. 16 m. after, his altitude was again observed to be $44^{\circ} 54'$: Required the latitude?

In forenoon,

1st interval 2 h. 16 m. = $34^{\circ} 0'$ alt. $41^{\circ} 53'$ sine, 66.62

50 23½ ,77042 } D = ,03° 25

2d interval 2 h. 16 m.

44 54

,70587

F = ,06455

2F = 12910

Here as the 3d altit. falls between 1st and 2d; D + 2F = E = 16735

Now L, A 17° 0' 10,51466 L, A 86° 0' 1,15642

L, E 0,16735 9,22362 L, 34 0 0,25244

L, D 0,03825 1,41737 L, D 0,03825 8,58263

L, A 86° 0' 11,15565 L, B 0,98060 9,99149

And $86^{\circ} = 5$ h. 44 m.

5 h. 44 m. + 2 h. 16 m. = 8 h. 0 m.

8 h. = 6 h. = 2 h. à noon at 3d alt.

1st altitude at 11 44

2d altitude at 11 44

3d altitude at 2 0

2L, 15° (= $\frac{2 \text{ h.}}{2}$)

0,06569

,70587, sine 3d altitude.

50° 30' ,77156, sine merid. altitude.

,98060, number B.

12 4 ,20904, sine of mid. deprec.

Hence the latitude is $58^{\circ} 43' \text{ N.}$; and declination $19^{\circ} 13' \text{ N.}$

EXAM.

EXAM. IV. Being in a low northern latitude, and the Sun bearing south declination, his altitude was observed in the forenoon to be $61^{\circ} 47'$, and 2 h. 30 m. after, his altitude was $64^{\circ} 29\frac{1}{2}'$, and 2 h. 30 m. after, his altitude was $36^{\circ} 54'$: Required the latitude?

Forenoon

1st interval 2 h. 30 m. = $37^{\circ} 30'$ } alt. $61^{\circ} 47'$ fine, 88117 *
 2d interval 2 h. 30 m. } $64^{\circ} 29\frac{1}{2}'$, 90249 } $D = .29215$
 $36^{\circ} 54'$ } $36^{\circ} 54'$, 58902 } $F = .31347$
 $2F = .62694$

Now $L, l = 18^{\circ} 45'$	10,46922	$L, l, 73^{\circ} 30'$	$2F - D = E = .33479$
$L, E = 0,33479$	9,52477	$L, l, 37^{\circ} 30'$	0,54666
$L, D = 0,29215$	0,53439	$L, D, 0,29215$	0,21555
		$L, B, 1,6897$	9,46561
$L, A, 106^{\circ} 30'$	10,52838	$2L, 27^{\circ}$	10,22782
And $A = 7$ h. 6 m.		$0,34826$	9,31409
7 h. 6 m. + 2 h. 30 m. = 9 h. 36 m.		58902	9,54191
And 9 h. 36 m. - 6 h. = 3 h. 36 m.		$69^{\circ} 36'$, 93728, fine merid. altitude.	
the 3d alt. at 3 h. 36 m. P. M.		$48^{\circ} 48'$, 75224, fine mid. depression.	
2d 1 6 P. M.		Hence latitude $30^{\circ} 48' N$.	
1st 10 36 A. M.		Declination $10^{\circ} 24' N$.	

70.

PROBLEM VI.

When three altitudes are taken at unequal intervals of time †.

RULE 1st. Let the interval between the 1st and 3d times be called M , between the 1st and 2d m ,
 between the 2d and 3d n ;

(V. 132)

Seek the natural sines of the three converted into degrees.

(IV. 256)

And let the difference between the 1st and 3d be called D , between the 2d and 3d F .

2d. Add together the log. sines of $\frac{1}{2} m$ and $\frac{1}{2} n$, gives the log. of $\frac{1}{2} A$.

3d. To the log. co-sine of $\frac{1}{2} m$ add the log. sine of $\frac{1}{2} n$, gives the log. of $\frac{1}{2} B$.

4th. Add together the ar. co. log. of D , the log of F , and the log. sine of $\frac{1}{2} M$, the sum is log. of $\frac{1}{2} C$.

Take the difference between the numbers $\frac{1}{2} B$ and $\frac{1}{2} C$, call it $\frac{1}{2} E$. Or let $\frac{1}{2} E$ be the sum of $\frac{1}{2} B$ and $\frac{1}{2} C$, when the degrees in the 3d altitude falls between those of the 1st and 2d altitudes.

5th. From the logarithm of $\frac{1}{2} E$ take the logarithm of $\frac{1}{2} A$, the remainder (10 being added to the index) is the logarithmic tangent of an arc G ; or of its supplement, when the first altitude exceeds the second.

* See Book IV. 256.

† See Book V. article 202.

Take

Take the difference between G and the complement of $\frac{1}{2}M$, the remainder is an arc H , shewing the time from noon when the last altitude was taken: Hence the other times are known.

6th. Add together the ar. co. log. co. sine of G ,

the ar. co. log. sine of $\frac{1}{2}M$,

and, the log. of D , their sum is the log. of a number I .

7th. To log. of I add twice the log. sine of $\frac{1}{2}H$, gives the log. of K .
8th. The number K added to the natural sine of the third altitude, gives the natural sine of the meridian altitude.

And the number I lessened by the natural sine of the meridian altitude, gives the natural sine of the midnight depression.

The half sum and half difference of the degrees shewing the meridian altitude and midnight depression, will give the co-latitude and declination; or co-declination and latitude, when the zenith falls between the Sun and the equator.

EXAM I. Being in north latitude, and the Sun having south declination; at the times 8 h. 40 m., 10 h. 00 m., and 11 h. 48 m., shewn by a watch, the altitudes of the Sun were $30^{\circ} 41'$, $45^{\circ} 37'$, and $56^{\circ} 44'$, respectively: Required the latitude?

$$M = 3h. 8m. = 47^{\circ} \text{ 1st alt.} = 30^{\circ} 41' \text{ nat. sine } 0,51037$$

$$m = 1 \text{ } 20 = 20 \text{ 2d } = 45 \text{ } 27 \frac{1}{2} \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} D = 0,32578$$

$$n = 1 \text{ } 48 = 27 \text{ } 3d = 56 \text{ } 44 \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} F = 0,12148$$

$$L, \frac{1}{2}M. \quad 10^{\circ} 0' \\ L, \frac{1}{2}n. \quad 13 \text{ } 30$$

$$L, \frac{1}{2}A. \quad 0,04054$$

$$L, \frac{1}{2}m. \quad 10^{\circ} 0' \\ L, \frac{1}{2}n. \quad 13 \text{ } 30$$

$$L, \frac{1}{2}B. \quad 0,22990$$

$$L, D. \quad 0,32578$$

$$L, F. \quad 0,12141$$

$$L, \frac{1}{2}M. \quad 23^{\circ} 30'$$

$$L, \frac{1}{2}C. \quad 0,14860$$

$$L, \frac{1}{2}E. \quad 0,08130$$

$$L, \angle G. \quad 63^{\circ} 30'$$

$$66 \text{ } 30 = \text{compl. of } \frac{1}{2}M.$$

$$u = 3 \quad 0 = \text{oh. 12m. \& noon.}$$

$$9,23967$$

$$9,36818$$

$$8,60785$$

$$9,99335$$

$$9,36818$$

$$9,36153$$

$$0,48707$$

$$9,08425$$

$$9,60070$$

$$9,17202$$

$$8,91009$$

$$10,30224$$

$$\text{Here } \frac{1}{2}B - \frac{1}{2}C = \frac{1}{2}E.$$

$$\text{And log. } \frac{1}{2}B - \log. \frac{1}{2}A = L, G.$$

$$\text{Also } 12 \text{ h.} - 12 \text{ m.} = 11 \text{ h. } 48 \text{ m.}$$

the time of the last altitude.

So the watch shewed true time.

$$L, \frac{1}{2}O. \quad 63^{\circ} 30' \quad 0,35047$$

$$L, \frac{1}{2}M. \quad 23 \text{ } 30 \quad 0,39930$$

$$L, D. \quad 0,32578 \quad 9,51293$$

$$L, I. \quad 1,83110 \quad 10,26270$$

$$2L, \frac{1}{2}H. \quad 1^{\circ} 30' \quad 6,83584$$

$$L, K. \quad 0,00125 \quad 7,09854$$

$$0,83615 = \text{fine } 3d \text{ alt.}$$

$$56^{\circ} 52' \quad 0,83740 = \text{fine mer. alt.}$$

$$83 \text{ } 34 \quad 0,99370 = \text{fine mid. dep.}$$

$$\text{Hence latit.} = 13^{\circ} 21' N.$$

$$\text{Declination} = 19 \text{ } 47 \text{ } S.$$

EXAM. II. Observed the Sun's altitude to be $16^{\circ} 34'$ and 50 minutes after, the altitude was $14^{\circ} 16'$; and 82 minutes after, the altitude was $8^{\circ} 37 \frac{1}{2}'$: Required the latitude?

$M =$

EXAM. IV. Suppose three successive altitudes of the Sun were, $61^{\circ} 47'$, $66^{\circ} 39\frac{1}{2}'$, $36^{\circ} 5\frac{1}{2}'$, taken at intervals of 2 h. 14 m. and 2 h. 46 m.: Required the times of the day, and the latitude?

$m = 5$ h. om. $= 75^{\circ} 0'$ 1st alt. $= 61^{\circ} 47'$ nat. sine 0,88117 } $D = 0,29215$
 $m = 2$ 14 $= 33$ 30 2d $= 66$ 39 $\frac{1}{2}$ } $E = 0,32824$
 $m = 2$ 46 $= 41$ 30 3d $= 36$ 5 $\frac{1}{2}$ }

$L, \frac{1}{2} m$ $16^{\circ} 45'$ 9,45969 Here $\frac{1}{2} C - \frac{1}{2} B = \frac{1}{2} E$.

$L, \frac{1}{2} n$ 20 45 9,54936 And a is a supplement.

$L, \frac{1}{2} A$ 0,10211 Also 3 h. 36 m. = time of 3d alt.

$L, \frac{1}{2} m$ $16^{\circ} 45'$ 9,00905 12 50 = time at 2d alt.

$L, \frac{1}{2} n$ 20 45 9,98117 10 36 = time at 1st alt.

$L, \frac{1}{2} m$ 0,33926 $L, \frac{1}{2} A$ 1,68969 0,54666

$L, \frac{1}{2} n$ 0,29215 $L, \frac{1}{2} m$ 37 30 0,21555

$L, \frac{1}{2} B$ 0,34825 L, D 0,29215 9,46561

$L, \frac{1}{2} C$ 0,58902 = fine of 3d alt. L, I 1,68969 10,22782

$L, \frac{1}{2} E$ 0,93727 = fine merid. alt. $L, \frac{1}{2} H$ 27 $^{\circ} 0'$ 9,31409

$L, \angle O$ $106^{\circ} 30'$ 10,52839 L, K 0,34825 9,54191

52 30 = compl. of $\frac{1}{2} m$. Hence latitude $= 30^{\circ} 48' N$.

$n = 54$ = 3 h. 36 m. à noon. Declination $= 10$ 24 N.

71.

PROBLEM VII.

Given two altitudes of the Sun, the interval of time between the two observations, the Sun's declination, and the latitude by dead reckoning; to find the true latitude.

SOLUTION.

1st. Convert the interval of time into degrees, and take its half.

Also take the half sum and half difference of the two altitudes.

First Operation.

2d, Add together the five following logarithms.

log. cof. of the half sum of the two altitudes.

log. fine of the half diff. of the two altitudes.

Ar. co. log. fine of the half interval.

Ar. co. log. cof. of the declination.

Ar. co. log. cof. of the lat. by account; their sum* is the log. fine of an arc.

3d. Take the diff. between this arc and the half interval; and the half of this diff. is half a time from noon.

4th. Add together the three following logarithms.

log. co-fine of the declination.

log. co-fine of the latitude by account.

Twice log. fine of half a time from noon, in degrees.

* Rejecting the 10's in the sum of the indices, in this and the following logarithmic work.

Seek the number to the logarithm of the sum, then that number doubled is a number, called the correction; which being added to the natural sine of the greatest observed altitude, gives the natural sine of the mer. alt., which is thereby known: and hence the lat. is known by Prob. I.

72.

REMARKS.

But to know if the latitude thus found is accurate, make a *second operation*. Thus,

To the sum of the first four logs, used in the 2d article of the rule, add the ar. co. log. co-sine of the latitude found by article 4th. The sum is the log. sine of an arc.

Work by the 3d and 4th articles, for a corrected latitude; And if it comes out the same, to a minute or two, as that found by the first operation, it may be esteemed as the true latitude.

Or, if the difference between the results of the two operations do not exceed about $\frac{1}{4}$ th of the difference between the latitude assumed, and that first found, the last result may be accounted sufficiently correct.

But if the difference between the results of both operations, is much greater than the said $\frac{1}{4}$ th part, then a third operation with the last found latitude will generally give the latitude exact enough.

When the latitudes found by the first and second operations, have one greater and one less than the latitude by account; or if these latitudes found, differ by a degree or more: then make a third operation with the half sum, or mean, of these two latitudes; and if the latitude found differs from the said mean; then half their sum will generally give the latitude very near the truth.

When both observations are made in the forenoon, or both in the afternoon;

The half interval, is half the difference of the times from noon:

And the first found arc, is half the sum of the times from noon.

But, when the observations are, one before, and the other, after, noon.

The $\frac{1}{2}$ interval, is the $\frac{1}{2}$ sum; and the 1st found arc, is the $\frac{1}{2}$ diff. of the times from noon.

The $\frac{1}{2}$ diff. added to the $\frac{1}{2}$ sum, gives the time of the observation farthest from noon:

And the $\frac{1}{2}$ diff. taken from the $\frac{1}{2}$ sum, gives the time of the observ. nearest to noon.

73. The times of observation, noted down, are best when taken to seconds, or at least to quarters, or thirds of a minute of time: They are also best when taken between the times of 9 h. A. M. and 3 h. P. M. and the interval of time between the observations, should not exceed five hours, nor be much less than the time from noon at the taking of the greatest altitude: Or, in general, the interval should not be much less than about $\frac{1}{4}$ of an hour.

This problem is best suited to situations where the Sun's meridian zenith distance is not much less than half the latitude: For in latitudes where the Sun approaches near to the zenith, the observations are to be taken much nearer to noon; and the rule instead of approximating, will in some cases give the results of successive operations wider and wider from the truth; and in such cases it is best to apply Prob. III.

Y 2

74. About

74. About the year 1740, Mr. *John Douwes*, a mathematician at Amsterdam, and examiner of the marine cadets there, first communicated, to the sea officers of his nation, a solution of this problem, by the help of a set of solar tables which he had fitted for that purpose: Manuscript copies thereof coming into the hands of several English officers, who held the method in high esteem, it was in the autumn of the year 1759 published, but without any demonstration, which has since been found by several persons: And Dr. Pemberton communicated to the Royal Society the whole composition, and shewed the limits, or where the method diverges, together with other useful particulars: This curious paper of the Doctor's was published in the Philosophical Transactions for the year 1760.

Mr. Douwes's tables, consisting of four columns, have in the first column the time to every half minute, extended to six hours: The second column, called half the elapsed time, contains the arithmetic complement of the logarithmic fines of the degrees corresponding to the times they stand against, in the first column: The third column, called middle times, are the logarithms of twice the natural fines of the times, in degrees, which they stand against; and in the fourth column, are the logarithmic versed fines of the times, in degrees, they stand against. But the preceding solution requires no other tables than are in the common books of navigation; the solar tables may indeed save about a minute of time in the computation; and those who are desirous of doing so, may work by them*.

EXAM. I. At 9 h. 18 m., and at 11 h. 30 m. by a watch, the Sun's altitudes respectively were $8^{\circ} 37\frac{1}{2}'$, and $16^{\circ} 34'$; the Sun's declination was $22^{\circ} 23' S$: δ and the latitude by account $50^{\circ} 40' N$: Required the true latitude?

$9^h. 18^m. \left. \begin{array}{l} 2h. 12m. = 33^{\circ} \\ altitudes \left\{ \begin{array}{l} 8^{\circ} 37\frac{1}{2}' \\ 16^{\circ} 34' \end{array} \right. \right\} \text{nat. sine } 0,28514.$

$\frac{1}{2}$ interval or $\frac{1}{2}$ diff. times	$\left. \begin{array}{l} 25 \\ 11 \end{array} \right\} = 16^{\circ} 30'$	$25 \ 11\frac{1}{2}$	$12^{\circ} 35\frac{1}{2} = \frac{1}{2}$ sum alts.	
$Ls, \frac{1}{2}$ sum alts. = $15^{\circ} 35\frac{1}{2}'$	$9,98942$	$7 \ 56\frac{1}{2}$	$3 \ 58\frac{1}{2} = \frac{1}{2}$ diff. alts.	
$Ls, \frac{1}{2}$ diff. alts. = $3 \ 58\frac{1}{2}$	$8,84041$	$Ls, \text{ declinat.} = 22^{\circ} 23'$	$9,96598$	
$Ls, \frac{1}{2}$ diff. times = $16 \ 30$	$0,54666$	$Ls, \text{ latitude} = 50 \ 40$	$9,80197$	
$Ls, \text{ declinat.} = 22 \ 23$	$0,03402$	$2Ls, \frac{1}{2} \text{ time} = 3 \ 43\frac{1}{2}$	$\left\{ \begin{array}{l} 8,82317 \\ 8,82317 \end{array} \right\}$	
$Ls, \text{ latitude} = 50 \ 40$	$0,19803$	$L, \frac{1}{2} \text{ correct.} = ,00247$	$7,39429$	
$L, \text{ an arc} = 23 \ 57\frac{1}{2}$	$9,60854$	Correction = ,00494		
$16 \ 30$		nat. s. of $16^{\circ} 34', 28514$ great alt.		
		$s, \text{ merid. alt.} = 29008$	$16^{\circ} 52'$	
		Meridional zenith distance $73 \ 8 S$.		
		Declination	$22 \ 23 S$.	
ts half	$7 \ 27\frac{1}{2}$	True latitude	$50 \ 45 N$.	
	$= 3 \ 43\frac{1}{2} \text{ a t. from noon}$			

* Among the several publications of these Solar tables, the largest and most useful, is in the Nautical Almanack for the year 1771.

EXAM.

EXAM. II. At 12 h. 12 m. and at 3 h. 20 m. by a watch, the Sun's altitudes were $56^{\circ} 44'$, and $30^{\circ} 41'$ respectively; the Sun's declination was $19^{\circ} 48' S.$, and the latitude, by dead reckoning, $13^{\circ} 14' N.$: Required the true latitude?

$$\begin{array}{r} 12^h 12^m \\ 3 \ 20 \end{array} \left. \begin{array}{l} \} 3 \text{ h. } 8 \text{ m.} = 47^{\circ}; \text{ altitudes} \\ \} 56^{\circ} 44' \text{ nat. sine } 0,83615. \\ \} 30 \ 41' \end{array} \right\} \begin{array}{r} 87 \ 25 \frac{3}{4} \\ 2) \quad \quad \quad \end{array} \begin{array}{r} 43 \ 43 \\ 26 \ 3 \end{array} \begin{array}{l} = \frac{1}{2} \text{ sum altitudes.} \\ 13 \ 1\frac{1}{2} = \frac{1}{2} \text{ diff. altitudes.} \end{array}$$

$$\begin{array}{l} L', \frac{1}{2} \text{ sum alts.} = 43^{\circ} 43' \quad 9,85900 \quad L', \text{ declinat.} = 19^{\circ} 48' \quad 9,97353 \\ L', \frac{1}{2} \text{ diff. alts.} = 13 \ 1\frac{1}{2} \quad 9,3529 \quad L', \text{ latitude} = 13 \ 14 \quad 9,98831 \\ L', \frac{1}{2} \text{ diff. times} = 23 \ 30 \quad 0,39930 \quad 2L', \frac{1}{2} \text{ time} = 1 \ 29\frac{3}{4} \quad \left. \begin{array}{l} 8,41630 \\ 8,41630 \end{array} \right\} \\ L', \text{ declination} = 19 \ 48 \quad 0,02647 \quad L, \frac{1}{2} \text{ correct.} = ,000623 \quad 6,79444 \\ L', \text{ latitude} = 13 \ 14 \quad 0,01169 \quad \text{Correction} = ,00125 \\ L, \text{ an arc} = 26 \ 29\frac{1}{2} \quad 9,64936 \quad s, \text{ great alt.} = ,83615 \\ \quad \quad \quad 23 \ 30 \quad \quad \quad \quad \quad \quad \quad \quad s, \text{ merid. alt.} = ,83740 = 56^{\circ} 52' \\ \quad \quad \quad 2 \ 59\frac{1}{2} \quad \quad \quad \quad \quad \quad \quad \quad \text{Merid. zenith distance} = 33 \ 8 \ S. \\ \text{Its half} = 1 \ 29\frac{3}{4} \text{ tim. fr. n.} \quad \text{Declination} = 19 \ 48 \ S. \\ \text{True latitude} = 13 \ 20 \ N. \end{array}$$

EXAM. III. Required the latitude of a place from the following observations made with a regulated watch and a Hadley's quadrant?

Altitude $41^{\circ} 53'$ at 9 h. 28 m. A.M. } interval 4 h. 32 m. = 68° .
Altitude $44^{\circ} 54'$ at 2 h. om. P.M. }

$$\begin{array}{r} 86 \ 47 \\ 3 \ 1 \end{array} \begin{array}{r} 43 \ 23\frac{1}{2} \\ 2) \quad \quad \quad \end{array} \begin{array}{r} 23\frac{1}{2} \\ 3 \ 1 \end{array} \begin{array}{l} = \frac{1}{2} \text{ sum of the altitudes: declination } 19^{\circ} 12' N. \\ = \frac{1}{2} \text{ diff. altitudes: lat. by account } 59 \ 0 \ N. \end{array}$$

$$\begin{array}{l} L', \frac{1}{2} \text{ sum alts.} = 43^{\circ} 23\frac{1}{2}' \quad 9,86134 \quad L', \text{ declinat.} = 19^{\circ} 12' \quad 9,97514 \\ L', \frac{1}{2} \text{ diff. alts.} = 1 \ 30\frac{1}{2} \quad 8,42032 \quad L', \text{ latitude} = 59 \ 0 \quad 9,71184 \\ L', \frac{1}{2} \text{ sum times} = 34 \ 0 \quad 0,25244 \quad 2L', \frac{1}{2} \text{ time} = 14 \ 59 \quad \left\{ \begin{array}{l} 9,41252 \\ 9,41252 \end{array} \right. \\ L', \text{ declinat.} = 19 \ 12 \quad 0,02485 \quad L, \frac{1}{2} \text{ correction} = ,03251 \quad 8,51202 \\ L', \text{ latitude} = 59 \ 0 \quad 0,28816 \quad \text{Correction} = ,06502 \text{ to be added.} \\ L', \frac{1}{2} \text{ diff. times} = 4 \ 2 \quad 8,84711 \quad s, \text{ altitude } 44 \ 54,70589 \\ \text{à noon at 1st alt. } 38 \ 2 = 2 \text{ h. } 32 \ \frac{1}{2} \text{ m.} \\ \text{à noon at 2d alt. } 29 \ 58 = 1 \text{ h. } 59 \ 52'' \quad s, \text{ merid. altitude, } 77^{\circ} 09' = 50^{\circ} 26' \\ \text{Hence the latitude} = 58 \ 46 \ N. \end{array}$$

EXAM. IV. Being by my dead reckoning in lat. $30^{\circ} 30' N$. the declination being $10^{\circ} 24' N$.; the Sun's altitude in the forenoon was $61^{\circ} 47'$, and 5 hours after, in the afternoon, his altitude was $36^{\circ} 51'$: Required the true latitude?

The interval, or diff. of time = 5 h. | Altitudes. $\left\{ \begin{array}{l} 61^{\circ} 47' \text{ nat. line, } 88117. \\ 37^{\circ} 30' \end{array} \right.$

$$\begin{array}{r} 97 \ 52\frac{1}{2} \ 48 \ 56\frac{1}{2} = \frac{1}{2} \text{sum alt.} \\ 2) \hline 25 \ 41\frac{1}{2} \ 12 \ 51 = \frac{1}{2} \text{diff. alts} \end{array}$$

$$\begin{array}{l} Ls, \frac{1}{2} \text{ sum alts.} = 48^{\circ} 56\frac{1}{2}' \quad 9,81750 \quad Ls', \text{ declinat.} = 10^{\circ} 24' \quad 9,99280 \\ Ls, \frac{1}{2} \text{ diff. alts.} = 12 \ 51 \quad 9,34713 \quad Ls', \text{ latitude} = 30 \ 30 \quad 9,93532. \\ Ls, \frac{1}{2} \text{ diff. times} = 37 \ 30 \quad 0,21555 \quad 2Ls, \frac{1}{2} \text{ times} = 10 \ 31 \quad \left\{ \begin{array}{l} 9,26165 \\ 9,26165 \end{array} \right. \\ Ls', \text{ declinat.} = 10 \ 24 \quad 0,00720 \quad L, \frac{1}{2} \text{ correct.} = ,02828 \quad 8,45142 \\ Ls', \text{ latitude} = 30 \ 30 \quad 0,06468 \quad \text{Correction, } ,05656 \text{ to be added.} \\ Ls, \frac{1}{2} \text{ sum times} = 16 \ 27 \quad 9,45206 \quad \text{, greatest alt. } ,88117 \end{array}$$

$$\text{à noon at 1st alt.} = 21 \ 3 = 1 \text{ h. } 24\frac{3}{4} \text{ m.}$$

$$\text{à noon at 2d alt. } 53 \ 57 = 3 \text{ h. } 35\frac{1}{4} \text{ m.}$$

$$\text{1st altitude, at } 10 \text{ h. } 35\frac{1}{4} \text{ min.}$$

$$\text{2d altitude, at } 3 \text{ h. } 35\frac{1}{4} \text{ min.}$$

$$\begin{array}{l} \text{, merid. altitude, } 93773 = 69^{\circ} 40' \\ \text{Merid. zenith distance} = 20 \ 20 \text{ S.} \\ \text{Sun's declination} = 10 \ 24 \text{ N.} \end{array}$$

$$\text{Latitude} = 30 \ 44 \text{ N.}$$

Second operation.

Sum of 1st four logs.

$$Ls', \text{ new lat. is} = 30 \ 44$$

$$Ls, \frac{1}{2} \text{ sum times} = 16 \ 30$$

$$\text{à noon at 1st alt.} = 21 \ 0 = 1 \text{ h. } 24 \text{ m.}$$

$$\text{à noon at 2d alt.} = 54 \ 0 = 3 \text{ h. } 36 \text{ m.}$$

$$\text{1st altitude at } 10 \text{ h. } 36 \text{ m. A.M.}$$

$$\text{2d altitude at } 3 \text{ h. } 36 \text{ m. P.M.}$$

$$\begin{array}{l} 9,38738 \quad Ls', \text{ declinat.} = 10 \ 24 \quad 9,99280 \\ 0,06573 \quad Ls', \text{ new lat.} = 30 \ 44 \quad 9,93427 \\ \hline 2Ls, \frac{1}{2} \text{ time} = 10 \ 30 \quad \left\{ \begin{array}{l} 9,26063 \\ 9,26063 \end{array} \right. \\ L, \frac{1}{2} \text{ correct.} = ,02808 \quad 8,44833 \\ \text{Correction} = ,05616 \text{ to be added.} \\ \text{, greatest alt.} = ,88117 \\ \text{, merid. alt.} = ,93733 = 69^{\circ} 36'. \end{array}$$

Hence the true latitude will be $30^{\circ} 48' N$.

EXAM.

EXAM. V. Being by account in latitude $16^{\circ} 20' S.$, and the Sun's declination $22^{\circ} 48' S.$; at $8^h 31^m 44^s$ forenoon, the altitude of the Sun's center was $40^{\circ} 38'$; and at $1^h 17^m 44^s$ P.M. its altitude was $70^{\circ} 26'$: Required the true latitude?

Times $13^h 17^m 44^s$
 $8^h 31^m 44^s$
 $4^h 46^m = 71^{\circ} 30'$; its $\frac{1}{2} = 35^{\circ} 45'$.

Alts. $70^{\circ} 26'$ Nat. sine $= 0,94225$

$40^{\circ} 38'$

$111 \quad 4; \frac{1}{2} \text{ sum} = 55^{\circ} 32'$
 $29 \quad 48; \frac{1}{2} \text{ diff.} = 14 \quad 54$

$L, \frac{1}{2} \text{ sum alts.} = 55 \quad 32$
 $L, \frac{1}{2} \text{ diff. alts.} = 14 \quad 54$
 $L, \frac{1}{2} \text{ sum times} = 35 \quad 45$
 $L, \frac{1}{2} \text{ declination} = 22 \quad 48$
 $L, \frac{1}{2} \text{ latitude} = 16 \quad 20$
 $9,75276 \quad L, \frac{1}{2} \text{ declination} = 22^{\circ} 48'$
 $9,41016 \quad L, \frac{1}{2} \text{ latitude} = 16 \quad 20$
 $0,23340 \quad L, \frac{1}{2} \text{ least time} = 9 \quad 43$
 $0,03534 \quad L, \frac{1}{2} \text{ least time} = 9 \quad 43$
 $0,01789 \quad L, N^{\circ} \frac{1}{2} \text{ correct.} = 02520$
 $9,44955 \quad \text{Correction} = 05040$
 $N, \text{ gr. alt.} = 94225$

Least time $= 19 \quad 26$; its $\frac{1}{2} = 9^{\circ} 43'$
 $N, \frac{1}{2} \text{ mer. alt.} = 99265 = 83^{\circ} 03'$
 Mer. zenith distance $6 \quad 57 \quad S.$
 Sun's declination $22 \quad 48 \quad S.$
 Latitude $15 \quad 51 \quad S.$

Second Operation.

Sum of four first logs $9,43166 \quad L, \frac{1}{2} \text{ decl.} = 22^{\circ} 48'$
 $L, \frac{1}{2} \text{ latitude} = 15 \quad 51$
 $0,01683 \quad L, \frac{1}{2} \text{ latitude} = 15 \quad 51$
 $9,44849 \quad L, \frac{1}{2} \text{ least time} = 9 \quad 43\frac{1}{2}$
 $35 \quad 45 \quad L, \frac{1}{2} \text{ least time} = 9 \quad 43\frac{1}{2}$
 $9,44849 \quad L, N^{\circ} \frac{1}{2} \text{ correct.} = 02528$
 $9,44849 \quad \text{Correction} = 05056$
 $N, \text{ gr. alt.} = 94225$
 $N, \frac{1}{2} \text{ mer. alt.} = 99281 = 83^{\circ} 07'$

Hence the true latitude may be taken at $15^{\circ} 55' S.$

Suppose the latitude, declination, and altitudes the same as above; but the times had been at $8^h 31^m 44^s$, and $10^h 42^m 16^s$ in the forenoon; or at $1^h 17^m 44^s$ and $3^h 28^m 16^s$ in the afternoon; to correct the latitude by account?

Times $10^h 42^m 16^s$ Or $3^h 28^m 16^s$
 $8^h 31^m 44^s \quad 1 \quad 17 \quad 44$
 $2 \quad 10 \quad 32 \quad 2 \quad 10 \quad 32 = 32^{\circ} 38'$; its $\frac{1}{2} = 16^{\circ} 19' = \frac{1}{2} \text{ diff. times.}$

Consequently, in the first part of the operation, if the $16^{\circ} 19'$ be used instead of $35^{\circ} 45'$; the sum of the five logs. will give the log. of $35^{\circ} 45'$, for the half sum of the times; and the half the least time will be $9^{\circ} 43'$: Therefore the same result will be obtained, and the final latitude will be $15^{\circ} 55' S.$

If three zenith distances, circumstanced as in Problem III. or IV. could be taken, the latitude would be found with less trouble.

Y 4

EXAM.

EXAM. VI. *Wanting to go through the 8° N. channel among the Maldivas, and by account being in lat. 7° 20' N. with 22° 48' N. declination, at 7^h 25^m forenoon, the true altitude of the Sun's center was 22° $\frac{1}{2}$, and at 10^h 31^m its altitude was 63° 40': Required the ship's true latitude?*

Time 10^h 31^m 48'
7 25 40

Alts. 63 49 N., 89 610

Interval 3 6 8 = 46° 32'; its $\frac{1}{2}$ = 23° 16'
22 30

$L, \frac{1}{2}$ sum alt. = 43° 05'
 $L, \frac{1}{2}$ diff. alt. = 20 35
 $L, \frac{1}{2}$ diff. times = 23 16
 $L, \frac{1}{2}$ declination = 22 48
 $L, \frac{1}{2}$ declination = 7 20

$L, \frac{1}{2}$ latitude = 7 20

Least tim. à noon = 22 3; its $\frac{1}{2}$ = 11° 1 $\frac{1}{2}$

$L, \frac{1}{2}$ sum alt. = 9,86354
 $L, \frac{1}{2}$ latitude = 9,54601
 $L, \frac{1}{2}$ diff. times = 0,40339
 $L, \frac{1}{2}$ declination = 0,03533
 $L, \frac{1}{2}$ declination = 0,00357
Correction = 0,03344
 $N, \frac{1}{2}$ gr. alt. = 89 610
 $N, \frac{1}{2}$ merid. alt. = 96 298 = 74° 22'.
Mer. zen. dist. = 15 38 N.
Declination = 22 48 N.
Latitude = 7 10 N.

Second Operation.

Sum of first four logs.

$L, \frac{1}{2}$ latitude = 7° 10'

$L, \frac{1}{2}$ sum times 45 17 $\frac{1}{2}$
 $\frac{1}{2}$ diff. times 23 16

Least time à noon 22 1 $\frac{1}{2}$; its $\frac{1}{2}$ = 11° 0 $\frac{1}{2}$

$L, \frac{1}{2}$ decl. = 22° 48'
 $L, \frac{1}{2}$ latitude = 7 10
 $L, \frac{1}{2}$ $\frac{1}{2}$ l. time = 11 0 $\frac{1}{2}$
 $L, \frac{1}{2}$ $\frac{1}{2}$ l. time = 11 0 $\frac{1}{2}$
 $L, \frac{1}{2}$ $\frac{1}{2}$ l. time = 0,93337
Correction = 0,0674
89 610

$N, \frac{1}{2}$ mer. alt. 99 284 = 74° 20'.
Hence the latitude is 7° 8' N. time.

If the altitude had been at 1^h 28^m 12^s, and at 4^h 34^m 20^s, both after-noon, the latitude would have been the same.

But had the times been at 7^h 25^m 40^s forenoon, and at 1^h 28^m 12^s afternoon. Or, at 10^h 31^m 48^s forenoon, and at 4^h 34^m 20^s afternoon, with the same altitudes, declination, and latitude, by account, to correct the latitude.

Times 13 28 12

7 25 40

6 2 32

3 1 16 = 45° 19'.

Or 16 34 20

10 31 48

6 2 32

6 2 32

Then the log. of 45° 19' being used for the third log. instead of that of 23° 16': the sum of these five logs. will be the log. of 23° 16', the half difference of the times; and, consequently, the least time will be the same as in the former operation; and the final result for the latitude will be the same.

EXAM.

EXAM. VII. Wanting to make the English Channel, I observed the true altitude of the Sun's center $24^{\circ} 30'$, and $3^h 4^m$ after, its altitude, was $52^{\circ} 16'$; our latitude by account was $49^{\circ} 12'$, and the Sun's declination 23° : Required the Ship's true latitude?

$3^h 4^m = 46^{\circ}$ its $\frac{1}{2} = 23^{\circ}$ the $\frac{1}{2}$ interval. $2d$ alt. $= 52^{\circ} 16'$ Nat. sine 79087
 $1st$ alt. $= 24^{\circ} 30'$

$$\begin{array}{r} 76 \\ 27 \end{array} \begin{array}{r} 46 \\ 46 \end{array} \begin{array}{r} 38 \\ 13 \end{array} \begin{array}{r} 23 \\ 53 \end{array}$$

$$\begin{array}{r} L, s, \frac{1}{2} \text{ sum alts.} = 38 \ 23 \\ L, s, \frac{1}{2} \text{ diff. alts.} = 13 \ 53 \\ L, s, \frac{1}{2} \text{ interval} = 23 \ 0 \\ L, s, \frac{1}{2} \text{ declination} = 23 \ 0 \\ L, s, \text{ latitude} = 49 \ 12 \end{array} \quad \begin{array}{r} L, s, \text{ declination} = 23 \ 0 \\ L, s, \text{ latitude} = 49 \ 12 \\ L, s, \frac{1}{2} \text{ least time} = 15 \ 4\frac{1}{2} \\ L, s, \frac{1}{2} \text{ least time} = 15 \ 4\frac{1}{2} \\ L, N^{\circ} \frac{1}{2} \text{ correct.} = 94069 \end{array}$$

Correction 08137

$$= 79087$$

$$L, s, \frac{1}{2} \text{ sum times} = 53 \ 09$$

$$N, s, \text{ mer. alt.} = 87224 = 60 \ 43$$

$$\text{Zen. dist.} = 29 \ 17$$

$$\text{Decl.} = 23$$

$$\text{Latitude } 52 \ 17$$

Second Operation.

$$\begin{array}{r} \text{Sum of four first logs} \\ L, s, \text{ latitude} = 52 \ 17 \\ L, s, \frac{1}{2} \text{ sum times} = 58 \ 44 \\ \text{Least time à noon } 35 \ 44; \text{ its } \frac{1}{2} = 17 \ 52 \end{array} \quad \begin{array}{r} L, s, \text{ decl.} = 23 \ 0 \\ L, s, \text{ latitude} = 52 \ 17 \\ L, s, \frac{1}{2} \text{ l. time} = 17 \ 52 \\ L, s, \frac{1}{2} \text{ l. time} = 17 \ 52 \\ L, N^{\circ} \frac{1}{2} \text{ correct.} = 95301 \end{array}$$

$$10602$$

$$\text{Latitude } 52 \ 17 \text{ first found.}$$

$$\text{Latitude } 49 \ 15 \text{ second found.}$$

$$101 \ 32$$

$$\text{Mean latitude } 50 \ 46$$

$$\text{Zen. dist.} = 63^{\circ} 45'$$

$$\text{Decl.} = 23$$

$$\text{Latitude } 49 \ 15$$

Third Operation.

$$\begin{array}{r} \text{Sum of first four logs.} \\ L, s, \text{ latitude} = 50 \ 46 \\ L, s, \frac{1}{2} \text{ sum times} = 55 \ 46 \\ \text{Least t. à noon } = 32 \ 46; \text{ its } \frac{1}{2} = 16 \ 23. \end{array} \quad \begin{array}{r} L, s, \text{ decl.} = 23 \ 0 \\ L, s, \text{ latitude} = 50 \ 46 \\ L, s, \frac{1}{2} \text{ least time} = 16 \ 23 \\ L, s, \frac{1}{2} \text{ least time} = 16 \ 23 \\ L, N^{\circ} \frac{1}{2} \text{ correct.} = 94632 \end{array}$$

$$09264$$

$$79087$$

$$N, s, \text{ mer. alt.} = 88351 = 62 \ 04$$

$$\text{Zen. dist.} = 37 \ 56$$

$$\text{Decl.} = 23$$

$$\text{Latitude } 50 \ 56$$

$$\text{Former mean latitude } 50 \ 46$$

$$101 \ 42$$

$$\text{True latitude } 50 \ 51$$

PRO.

75. PROBLEM VIII.

Given the latitude by account, the Sun's declination, two observed altitudes, the interval of time, and the course and distance run, between the observations; to find the latitude corresponding with the noon of that day.

SOLUTION.

1st. Find the distance run in the interval of time; and with this distance and the course, corrected, find the difference of latitude, whereby the latitude at the time of the observation nearest to noon will be known, if the latitude at the least altitude was given. If the latitude at the greater is first given, there is no need of using this article.

2d. Let the latitude by account be used at the time of taking the greatest altitude; and at the time of observing the least altitude, take also the Sun's bearing, or the rhumb he is seen on; and take the difference between the Sun's rhumb and the ship's rhumb, or course she has steered, or the point opposite.

3d. With this angle as a course, and the distance run between the observations, find (by the traverse table) a difference of latitude.

Which difference of latitude, applied to the least altitude, gives an altitude to be used with the greater alt. as if the ship had been stationary at both observations.

The diff. lat. is to be added, when the course is within 90 degrees from the Sun.

But is to be subtracted, when above 90 degrees from the Sun.

4th. With these altitudes (reduced to the same zenith), the observed interval of time, together with the declination and the latitude by account, at the greatest altitude, find the meridian altitude by the preceding problem; and hence the latitude will be obtained for the noon of that day.

REMARK. The introducing of the Sun's bearing at either observation, if it could be taken with sufficient accuracy, would render the other observation useless; for, with the azimuth, the polar distance and the zenith distance, taken at any time, the latitude and hour from noon may be found.

But instead of observing the azimuth, it may be computed from the latitude by account*, with sufficient exactness for obtaining the correction to be applied to one of the zenith distances to fit it for the other, as if the ship had been there stationary: For the small error incident to the correction of the zenith distance, by being computed from an azimuth somewhat erroneous, will bear to the error in that azimuth, a ratio but little different from that of the sine of the distance run by the ship, to the co-secant of the angle which the ship's course makes with the azimuth circle of the Sun†.

Note. In the second operation of the examples to the VIIth and VIIIth Problems, the arith. compl. of the co-sine of the latitude is added to the sum of the four first logarithms, according to the precept in Problem VII, whereby the conformity of the second operations with the first are more expressly shewn; yet the same logarithm would be produced by deducting the co-sine of the latitude from the said sum.

* As proposed by Dr. Pemberton.

† *Cotes. Assum. Err. in Mixt Math. Theorem. 22.*

EXAM. I. Sailing S. E. 11 knots an hour, at 9^h 36^m forenoon, the true altitude of the sun's center was 52° 2', and his bearing was S. 60° 30' E. and at 11^h 24^m, its altitude was 63° 45', the latitude by account at this time being 48° 46' N. and the Sun's declination 23° 20' N.: Required the mid-day latitude?

Now 11h. 24m.—9h. 36m. = 1h. 48m. is the interval of time.
 And 1h : 1h. 48m :: 11k : 20 kn. nearly the dist run. in 1h. 48m.
 Also 60° 30' the Sun's rhumb, lessened by 45° the ship's rhumb, leaves 15° 30' for the diff. rhumbs.
 Then with the course 15° 30', and dist. 20 m. the diff. lat. is 19 m.
 And 52° 2' + 19' = 52° 21' for the lesser altitude corrected; or fitted to the zenith used at the taking of the greatest altitude.

Now 1h. 48m. = 27° 0'	Alts. 63° 45'	N. S. 89687
half interval = 13 30	52 21	N. S. 79176
	half sum 58 3	half diff. 5 42.
$L_s \frac{1}{2}$ sum alts. = 58° 03'	9,72360	L_s declin. = 23 20
$L_s \frac{1}{2}$ diff. alts. = 5 42	8,99704	L_s Lat. = 48 46
$L_s \frac{1}{2}$ diff. times. = 13 30	0,63182	$L_s \frac{1}{2}$ gr. time = 17 40
L_s declination = 23 20	0,03706	$L_s \frac{1}{2}$ gr. time = 17 40
L_s Lat. = 48 46	0,18103	L_s No $\frac{1}{2}$ Corr. = 05561
$L_s \frac{1}{2}$ sum times = 21 50	9,57055	Correction = 11122
Gr. time à noon = 35 20 its $\frac{1}{2}$ = 17 40		N. s. least alt. = 79176
Lr. time à noon = 8 20 its $\frac{1}{2}$ = 4 10		N. s. mer. alt. = 90298 =
		mer. zen. dist.
		Declin.
		Lat.
		L_s declin. = 23 20
		L_s Lat. = 48 46
		$L_s \frac{1}{2}$ l. time = 4 10
		$L_s \frac{1}{2}$ l. time = 4 10
		L No $\frac{1}{2}$ Corr. = 003195
		Correction = 00639
		N. s. Gr. alt. = 89687
		N. s. mer. alt. = 90326 = 64 35

9,96294
 9,81897
 9,48213
 9,48213
 8,74617

64 33
 25 27 S.
 23 20 N.
 48 47 N.

9,96294
 9,81897
 8,86128
 8,86128
 7,50447

Here the second part of the work is done with $\frac{1}{2}$ the greatest time from noon, and with half the least time from noon; the correction by the first is applied to the nat. sine of the least alt. and the correction by the second, to that of the greatest alt. and the meridian alts. differ only 2 min. which arises from the imperfection of the tables.

Therefore it is quite indifferent whether the nat. sine of the greater or lesser alt. is used.

Here the lat. 48° 47' computed, differing by 1 min. only from the latitude by account, may be taken for the true latitude.

EXAM.

75. PROBLEM VIII.

Given the latitude by account, the Sun's declination, two observed altitudes, the interval of time, and the course and distance run, between the observations; to find the latitude corresponding with the noon of that day.

SOLUTION.

1st. Find the distance run in the interval of time; and with this distance and the course, corrected, find the difference of latitude, whereby the latitude at the time of the observation nearest to noon will be known, if the latitude at the least altitude was given. If the latitude at the greater is first given, there is no need of using this article.

2d. Let the latitude by account be used at the time of taking the greatest altitude; and at the time of observing the least altitude, take also the Sun's bearing, or the rhumb he is seen on; and take the difference between the Sun's rhumb and the ship's rhumb, or course she has steered, or the point opposite.

3d. With this angle as a course, and the distance run between the observations, find (by the traverse table) a difference of latitude.

Which difference of latitude, applied to the least altitude, gives an altitude to be used with the greater alt. as if the ship had been stationary at both observations.

The diff. lat. is to be added, when the course is within 90 degrees from the Sun.

But is to be subtracted, when above 90 degrees from the Sun.

4th. With these altitudes (reduced to the same zenith), the observed interval of time, together with the declination and the latitude by account, at the greatest altitude, find the meridian altitude by the preceding problem; and hence the latitude will be obtained for the noon of that day.

REMARK. The introducing of the Sun's bearing at either observation, if it could be taken with sufficient accuracy, would render the other observation useless; for, with the azimuth, the polar distance and the zenith distance, taken at any time, the latitude and hour from noon may be found.

But instead of observing the azimuth, it may be computed from the latitude by account*, with sufficient exactness for obtaining the correction to be applied to one of the zenith distances to fit it for the other, as if the ship had been there stationary: For the small error incident to the correction of the zenith distance, by being computed from an azimuth somewhat erroneous, will bear to the error in that azimuth, a ratio but little different from that of the sine of the distance run by the ship, to the co-secant of the angle which the ship's course makes with the azimuth circle of the Sun†.

Note. In the second operation of the examples to the VIIth and VIIIth Problems, the arith. compl. of the co-sine of the latitude is added to the sum of the four first logarithms, according to the precept in Problem VII, whereby the conformity of the second operations with the first are more expressly shewn; yet the same logarithm would be produced by deducting the co-sine of the latitude from the said sum.

* As proposed by Dr. Pemberton.

† *Cotef. Aestim. Err. in Mixt Math. Theorem. 22.*

EXAM. I. Sailing S. E. 11 knots an hour, at 9^h 36^m forenoon, the true altitude of the sun's center was 52° 2', and his bearing was S. 60° 30' E. and at 11^h 24^m, its altitude was 63° 45', the latitude by account at that time being 48° 46' N. and the Sun's declination 23° 20' N.: Required the mid-day latitude?

Now 1 h. 24 m.—9 h. 36 m. = 1 h. 48 m. is the interval of time.

And 1 h : 1 h. 48 m :: 11 k : 20 kn. nearly the dist run. in 1 h. 48 m.

Also 60° 30' the Sun's rhumb, lessened by 45° the ship's rhumb, leaves 15° 30' for the diff. rhumbs.

Then with the course 15° 30', and dist. 20 m. the diff. lat. is 19 m.

And 52° 2' + 19' = 52° 21' for the lesser altitude corrected; or fitted to the zenith used at the taking of the greatest altitude.

Now 1 h. 48 m. = 27° 0'
half interval = 13 30

Alts. 63° 45' N. S. 89687
52 21 N. S. 79176
half sum 58 3 half diff. 5 42.

L^s $\frac{1}{2}$ sum alts. = 58° 03'
L^s $\frac{1}{2}$ diff. alts. = 5 42
L^s $\frac{1}{2}$ diff. times. = 13 30
L^s declination = 23 20
L^s Lat. = 48 46

9,72360
8,99704
0,63182
0,03706
0,18103
Corr. =,05561

L^s $\frac{1}{2}$ sum times = 21 50
Gr. time à noon = 35 20 its $\frac{1}{2}$ = 17 40
Lr. time à noon = 8 20 its $\frac{1}{2}$ = 4 10

9,96294
9,81897
9,48213
9,48213
8,74617

Correction =,11122

N^s. least alt. =,79176

N^s. mer. alt. =,90298 =

mer. zen. dist.

Declin.

Lat.

Here the second part of the work is done with $\frac{1}{2}$ the greatest time from noon, and with half the least time from noon; the correction by the first is applied to the nat. sine of the least alt. and the correction by the second, to that of the greatest alt. and the meridian alts. differ only 2 min. which arises from the imperfection of the tables.

64 33
25 27 S.
23 20 N.
48 47 N.

9,96294
9,81897
8,86128
8,86128
7,59447

L^s, declin. = 23 20

L^s Lat. = 48 46

L^s, $\frac{1}{2}$ l. time = 4 10

L^s, $\frac{1}{2}$ l. time = 4 10

L^s N^o $\frac{1}{2}$ Corr. =,003195

Correction =,00639

N^s. Gr. alt. =,89687

N^s, mer. alt. =,90326 = 64 35

Therefore it is quite indifferent whether the nat. sine of the greater or lesser alt. is used.

Here the lat. 48° 47' computed, differing by 1 min. only from the latitude by account, may be taken for the true latitude.

EXAM.

EXAM. II. *Being by account in latitude 51° N., when the Sun's declination was 22° 23' S.; at 1h. 20m. P. M., the Sun's altitude was 14° 46' when he bore S. 36° 45' W.; and having run NE. b. E. eight knots an hour, at 2h. 42m. the Sun's altitude was 8° 37½': Required the mid-day latitude?*

Here the observed difference of time is 1h. 22m.

Now 1h. : 1h. 22m. : 8kn. : 11kn. nearly, the dist. run betw. the obs. With Co. NE. b. E. and dist. 11 gives 6m., the diff. lat. made in that interval.

Then $51^{\circ} 6' + 0^{\circ} 6' = 51^{\circ} 6'$ the latitude at the 2d observation.

Here the Sun's rhumb is S. 36° 45' W. or N. 36° 45' E.

And the Ship's rhumb is N. 56 15 E.

Their diff. is $19^{\circ} 30'$, which as a course, with the dist. 11, gives $10\frac{1}{2}'$ for a diff. lat.

And the Ship running N. from the Sun lessens the altitude.

Then $8^{\circ} 37\frac{1}{2}' - 10\frac{1}{2}' = 8^{\circ} 27'$ is the alt. to be used at the 2d observ.

To find the latitude by Problem VII.

1h. 20' } interval = 1 22m. = 20° 30'. Alts. { $14^{\circ} 46'$ $\frac{1}{2}$ sum = $11^{\circ} 36\frac{1}{2}'$.	
2 42' } half diff. times = 10 15	$8 27$ $\frac{1}{2}$ diff. = $3 8\frac{1}{2}'$.
$L, \frac{1}{2}$ sum alts. = $11^{\circ} 36\frac{1}{2}'$ 0,00898	$L, \frac{1}{2}$ declination $22^{\circ} 23'$ 9,96598
$L, \frac{1}{2}$ diff. alts. = $3 8\frac{1}{2}'$ 8,74111	$L, \frac{1}{2}$ latitude $51 6$ 9,79793
$L, \frac{1}{2}$ diff. times = 10 15 0,74972	$2L, \frac{1}{2}$ time $21 37$ 9,13262
$L, \frac{1}{2}$ declination = 22 23 0,03402	$L, \frac{1}{2}$ correction 0,07880 8,89653
$L, \frac{1}{2}$ latitude = 51 6 0,20207	Correction 0,15760
$L, \frac{1}{2}$ sum times = 32 59 9,73590	Nat. fine 0,14694 of $8^{\circ} 27'$.
Greatest time 43 14 from noon.	L , of $17^{\circ} 44'$ 0,30454 merid. alt.
	And $72 16 =$ merid. zen. distance.
	(60.)

Then $72^{\circ} 16' - 22^{\circ} 23' = 49^{\circ} 53' N.$ is the latitude

Second Operation.

Sum of the first four logs. 9,53383	$L, \frac{1}{2}$ declination = $22^{\circ} 23'$ 9,96598
$L, \frac{1}{2}$ latitude 49° 53' 0,19088	$L, \frac{1}{2}$ latitude = 49 53 9,80912
$L, \frac{1}{2}$ sum times 32 2½ 9,72471	$2L, \frac{1}{2}$ greatest time 21 8½ 9,11438
Greatest time 42 17½ fr. noon.	$L, \frac{1}{2}$ correction 0,07753 8,88948

Then the correction 0,15506 + 0,14694 the nat. fine of $8^{\circ} 27'$, gives 0,30200 the nat. fine of $17^{\circ} 35'$ the merid. alt. and the merid. zen. dist. = $72^{\circ} 25'$.

Therefore the latitude will be $50^{\circ} 2' N.$

And by making a third operation, the latitude sought will be found to be $50^{\circ} 1' N.$

The work in this example is done with the lat. $51^{\circ} 6'$, at the least alt.

But if done with 51° at the greatest alt. the results will be :

The 1st operation gives a lat. $49^{\circ} 54'$; the 2d $50^{\circ} 2'$; and the 3d $50^{\circ} 1'$.

EXAM.

EXAM. III. Running with a brisk gale ten knots an hour, on an E.N.E. course; at $10^h 36^m$ forenoon, the altitude of the Sun's center was observed to be $61^{\circ} 47'$; and at $3^h 36^m$ P.M. his altitude was again found to be $36^{\circ} 51'$, and his bearing S. $80^{\circ} 30' W.$ by our account, the latitude at that time was $30^{\circ} N.$, and the Sun's declination $10^{\circ} 24' N.$: Required the latitude at noon?

Here the observed difference of time is five hours.

And $1h : 5h :: 10 kn : 50 kn.$ the distance run between the observations.

The dist. 50, and course E.N.E. gives 19 for a diff. lat. made in 5 h.

Then $(30^{\circ} 0' - 0^{\circ} 19') = 29^{\circ} 41' N.$ = lat. by account at first observation.

Now the Sun's rhumb is S. $80^{\circ} 30' W.$ or N. $80^{\circ} 30' E$

And the ship's rhumb, or course, is N. $67^{\circ} 30' E.$

Their diff. is $13^{\circ} 08'$, which as a course, with the dist. 50, gives a diff. lat. 49^m .

Then $36^{\circ} 5' - 49' = 35^{\circ} 16'$ is the corrected alt. for the 2d observ.

To find the latitude by Problem VII.

Interval $gh.$ = $75^{\circ} 0'$.	Alts. $61^{\circ} 47'$	$\frac{1}{2}$ sum alts. = $48^{\circ} 31\frac{1}{2}'$.
$\frac{1}{2}$ Interval = $37^{\circ} 30'$	$35^{\circ} 16'$	$\frac{1}{2}$ diff. alts. = $13^{\circ} 15\frac{1}{2}'$.
$L_s, \frac{1}{2}$ sum alts. = $48^{\circ} 31\frac{1}{2}'$	$9, 82147$	$L_s, \frac{1}{2}$ decl. = $10^{\circ} 24'$
$L_s, \frac{1}{2}$ diff. alts. = $13^{\circ} 15\frac{1}{2}'$	$9, 36048$	$L_s, \frac{1}{2}$ latitude = $30^{\circ} 0'$
$L_s, \frac{1}{2}$ sum times = $37^{\circ} 30'$	$0, 21555$	$L_s, \frac{1}{2}$ time = $27^{\circ} 16\frac{1}{2}'$
$L_s, \frac{1}{2}$ decl. = $10^{\circ} 24'$	$0, 00719$	$L_s, \frac{1}{2}$ time = $27^{\circ} 16\frac{1}{2}'$
$L_s, \frac{1}{2}$ latitude = $30^{\circ} 0'$	$0, 06247$	$L_s, N^{\circ} \frac{1}{2}$ corr. = $0, 17888$
$L_s, \frac{1}{2}$ diff. times = $17^{\circ} 3'$	$9, 46716$	Correction = 35776
Gr. time $\hat{a}$ noon = $54^{\circ} 35'$	$its \frac{1}{2} = 27^{\circ} 16\frac{1}{2}'$	N_s , of $35^{\circ} 16' = 57740$
		N_s , mer. alt. = $93516 = 69^{\circ} 15'$.
		And the latitude at noon $31^{\circ} 26' N.$ (61)

By a 2d operation the latitude will be found to be $31^{\circ} 26' N.$

By a 3d

By a 4th

the true latitude $31^{\circ} 30'$

By working with the latitude $29^{\circ} 41'$ at the 2d observation, the final result will be the same; which is shewn in the following form of a practical method of performing the work of these examples: Where $9, 40469$ is used as the sum of the four first logarithms.

First Operation.		Second Operation.		Third Operation.	
$9, 40469$	$9, 99281$	$9, 40469$	$9, 99281$	$9, 40469$	$9, 99281$
$0, 06109$	$9, 93891$	$0, 06724$	$9, 93276$	$0, 06869$	$9, 93131$
$9, 46578$	$9, 66068$	$9, 47193$	$9, 66258$	$9, 47338$	$9, 66295$
$16^{\circ} 59\frac{1}{2}'$	$9, 66068$	$17^{\circ} 15'$	$9, 66258$	$17^{\circ} 18'$	$9, 66295$
$37^{\circ} 30'$	$0, 17910$	$37^{\circ} 30'$	$0, 17813$	$37^{\circ} 30'$	$0, 17778$
$54^{\circ} 29\frac{1}{2}'$	$0, 35820$	$54^{\circ} 45'$	$0, 35626$	$54^{\circ} 48'$	$0, 35556$
$27^{\circ} 14\frac{1}{2}'$	$0, 57740$	$27^{\circ} 22\frac{1}{2}'$	$0, 57740$	$27^{\circ} 24'$	$0, 57740$
	93560		93366		93296
Mer. alt.	$69^{\circ} 20'$	Mer. alt.	$69^{\circ} 01'$	Mer. alt.	$68^{\circ} 54'$
Zen. dist.	$20^{\circ} 40'$	Zen. dist.	$20^{\circ} 59'$	Zen. dist.	$21^{\circ} 6'$
Decl.	$10^{\circ} 24'$	Decl.	$10^{\circ} 24'$	Decl.	$10^{\circ} 24'$
Latitude	$31^{\circ} 4'$	Latitude	$31^{\circ} 23'$	Latitude	$31^{\circ} 30'$

EXAM.

EXAM. IV. January 20, 1772, being by account in latitude $56^{\circ} 30' S.$, longitude $94^{\circ} W.$ at $7^h 32^m$, and at $10^h 28$ forenoon, by fore observations, the altitudes of the Sun's lower limb, were $29^{\circ} 48'$, and $49.53\frac{1}{2}$; and his bearings $N. 80^{\circ} E.$ and $N. 30^{\circ} E.$ the eye 20 feet above the water, and between the observations the ship had run 8 knots an hour, on a course $S. 50^{\circ} E.$: Required the mid-day latitude?

Now ship time $19^h 32^m P. M.$ + $6^h 16^m W. = 25^h 58^m$. Or, $1^h 58^m P. M.$ London.

Or. $22^h 28^m P. M.$ + $6^h 16^m W. = 28^h 44^m$. Or, $4^h 44^m P. M.$ Lond. time. (57.)

The Sun's declination fitted to these times, are $20^{\circ} 8'$, and $20^{\circ} 7'$. (V. 281.) (57.)

App. alt. $\odot$ lower edge $29^{\circ} 48' 15''$ And, $49^{\circ} 53' 30''$
 $\odot \frac{1}{2}$ diameter + $16' 20''$

App. alt. $\odot$ center $30^{\circ} 4' 35''$ $50^{\circ} 9' 50''$
 Dip. on 20 feet $5'' 0''$ $5'' 0''$

Refraction $29^{\circ} 59' 35''$ $50^{\circ} 4' 50''$
 $1' 38''$ $48''$

True alt. $\odot$ center $29^{\circ} 57' 57''$ $50^{\circ} 4' 2''$
 Or, $29^{\circ} 58''$ $50^{\circ} 4''$

Now the interval is $2^h 56^m$. And $1^h : 2^h 56^m :: 8kn : 23kn$: the distance run.

With the course $S. 50^{\circ} E.$ and distance 23^m the diff. lat. is 15^m .

Then $56^{\circ} 30' + 15' = 56^{\circ} 45'$ the latitude in at second observation.

Again. The difference between the Sun's bearing $N. 80^{\circ} E.$ and the ship's course $N. 30^{\circ} E.$ gives 50° .

And with course 50° and dist. 23^m the diff. lat. is 15^m .

So $29^{\circ} 58' + 15' = 30^{\circ} 13'$ the corrected alt. at 1st observation.

The interval = $2^h 56^m = 44^{\circ}$ Alt. $50^{\circ} 4'$ | $\frac{1}{2}$ sum alts. = $40^{\circ} 08'\frac{1}{2}$
 Its half = 22° $30' 13''$ | $\frac{1}{2}$ diff. alts. = $9^{\circ} 55\frac{1}{2}'$

First Operation.

9,88335	9,97262
9,23643	9,74189
0,42647	9,72873
0,02738	" 9,72873
0,25811	9,17197
9,83174	,14860
42,945	,29720
22	,50327
64 45	,80047
32 28 $\frac{1}{2}$	53° 11'
Zen. dist. 36 49 N.	
Decl. 20 8 S.	
Latitude 56 57	

Second Operation.

9,57363	9,97262
0,26331	9,73669
9,83694	9,73249
43° 23'	9,73249
22	9,17429
65 23	,14940
32 41 $\frac{1}{2}$	
Correction, 29880	
Least alt. ,50327	
Mer. alt. ,80207	
Mer. alt. 53° 20	
Zen. dist. 36 40	
Declination 20 8	
Latitude 56 48	

Third Operation.

9,57363	9,97262
0,26157	9,73843
9,83520	9,73126
43° 10 $\frac{1}{2}$	9,73126
22	3,17357
65 10 $\frac{1}{2}$	,14910
32 35 $\frac{1}{2}$	,29820
	,50327
	,80147
Mer. alt 53 16	
Mer.zen.dist. 36 44	
Declination 20 8	
Latitude 56 52	

But

But working with the latitude $56^{\circ} 45'$, and the declination $20^{\circ} 7'$, at the second observation.

First Operation.

9.88330
9.23643
0.42647
0.02734
0.26099
9.83453
43° 06'
22 00
65 06
32 33

Cor.

Mer. alt.

Zen. dist.

Declination

Latitude

9.97266 9.57354
9.73901 0.26215
9.73081 9.83569
9.73081 43° 14'
9.17329 22
.14904
.29808
.50327
.80135

N°

Corr.

N, least alt.

N, mer. alt.

Merid. alt.

Zen. dist.

Declination

Latitude

53° 16'

36 44 N.

20 07 S.

56 51 S.

9.97266
9.73785
9.73160
9.73160
9.17371
.14920
Corr. 29840
N, least alt. 50327
N, mer. alt. 80167
Merid. alt. 53° 17'
Zen. dist. 36 43 N.
Declination 20 07 S.
Latitude 56 50 S.

Second Operation.

Hence it appears, that in the work of such examples, as the preceding, it is quite indifferent whether the latitude by account and declination relate to the observation nearest to noon, or farthest from noon: However, for the sake of uniformity, it may be recommended to use the latitude, the declination and the natural sine, which do all relate to the same altitude.

In the appendix will be shewn how this Problem was considered by *Facio*; but his solution requiring too many trigonometrical operations to be of ready use, the preceding method, by approximation, was substituted instead thereof.

76. Mr. *Richard Graham's* method of solving this problem, published in the Philosophical Transactions for the year 1734, N° 435, by an instrumental manner of Solution, may indeed give the latitude within some minutes of the truth, by a very easy and quick operation; which is in substance as follows.

Description of the Instrument.

To the meridian of a globe (properly divided) let a piece of a like meridian and called the beam compass, of about 110 degrees long (and alike divided) be fitted, sliding on the upper side of the meridian*; and let two sliding Vernier's scales, fitted to those circles, and to five minutes of their degrees, be so applied, that a wire fixed at 0 degrees at the center, or end, of the beam-compass, shall pass through a hole at the beginning of the divisions of one Vernier, which is to slide on the

* In Mr. Graham's instrument, made on purpose for this problem, the beam-compass slides under the meridian.

meridian

meridian, that wire serving as the center of motion of the beam compass, when the Vernier is skewed to the meridian: The other Vernier is to slide on the beam compass, and carry with it a pointer, or index, at the beginning of its divisions.

With this Instrument the problem is thus solved.

At the time of the first observation let the Sun's bearing be also observed, and take the difference between the rhumbs on which the Sun was observed, and the rhumb on which the ship steered between the observations: Then say,

As radius is to the co sine of that difference of rhumbs; so is the distance run between the observations, to the minutes of correction to be applied to the first zenith distance, and is to be added when the ship sails from the Sun, otherwise it is to be subtracted.

Then slide the Vernier's scale on the meridian to the given declination, and the Vernier on the beam-compass to the first zenith distance as corrected, and let these scales be fixed by their screws: Rectify the globe for noon (VI. 59), turn it till the hour-index points at the time of the first observation, there hold the globe fast, and move the beam-compass on its center, nearly into the azimuth of the Sun at that observation, and describe an arc on the globe with the pointer of the Vernier,

Slide the Vernier on the beam-compass to the second observed zenith distance; turn the globe (the proper way) till the hour-index points at the time of the second observation, there hold the globe fast, and with the beam-compass cut the former arc, their intersection will shew the place of the ship at the time of the second observation: Bring the meridian over that point, and the latitude is found; also the hour-index will point at the true time of the day when the last observation was made.

SECTION X.

77. *To find the Longitude at Sea.*

If an accurate account could be kept, of the true courses which a ship has steered, as directed by the compass, and of the distance she has run, as measured by the log. or by any other contrivance; then both the latitude and longitude of the place where the ship is at any time, would be known on settling the ship's account to that time.

For the course and distance being known, the difference of latitude and departure are readily found by the traverse table; and the difference of longitude being also found (VIII. 60), the true lat. and long. are known.

But as the ship's motion is liable to be disturbed from a variety of causes, such as, a continual deflection from the course set, by the ship's playing to the right and left round her center of gravity, and the unequal care of the *helmsmen*: the distance sailed being faulty, on account of tumbling seas rolling with or against the ship; the unsteadiness of the wind; unknown currents; sudden storms, and many other impediments, which seem impossible to be surmounted; consequently her place, according to the reckoning kept, may be justly suspected to be erroneous: Mariners judge of this by finding the ship's latitude every day, if they can, by celestial observations on the Sun or Stars; and if the latitude found by their reckoning agrees with that found by observation, it is presumed that the ship's place is well determined: But if they disagree, the account of longitude stands in need of being corrected, for the latitude found by observation is always to be depended upon.

78. The error in the ship's reckoning is frequently attributed to unknown currents: For by various causes, yet indetermined, there are many counter-motions of the water in the open seas, as well as those observed near the shores, where the motions may be tolerably well accounted for. Some of the observed currents in the great seas, may perhaps be owing to the tide's following the moon, and to the libratory motion the waters may have thereby; and the unsettled setting and drift of these currents may possibly depend on the change in the moon's declination: However, it is well known from observations, that the trade-winds occasion a considerable current, within their limits, particularly within the Torrid Zone, where the motion is perpetually towards the west, at the rate of 8 or 10 miles a day: But at the extremities of the trade-winds, or near the latitudes of 30 degrees north or south, it is likely that the currents are compounded of the said western motion, and of one towards the equator (by observation 2. Art. 84. B. VI.): Therefore all ship's sailing within these limits, should allow a course each day for this current.

Although some of the methods recommended for correcting a sea, reckoning, are little better than guess-work; as it cannot with certainty be known whether the ship is to the eastward or westward of the point wherein the reckoning places her; yet it was judged proper to introduce them here; as they may sometimes be useful.

Methods of Correcting the Reckoning of Longitude.

79. METHOD I. When the error is supposed to happen from a current.

Consider whether the difference may have not been occasioned by a current, and, if it is possible, try it (by article 47, Book VII.) ; or make such an estimate of its setting and drift, as may be judged reasonable : Then with the setting and drift, as a course and distance, find the difference of latitude and departure, with which increase or diminish the dead reckoning ; and if the latitude, thus corrected, agrees with the latitude by observation, then the departure thus corrected may be safely taken as true, and so the ship's place determined, with regard to longitude.

EXAM. I. Suppose a ship in 24 hours finds by her dead reckoning she has made 96 miles of diff. lat. N., and 38 miles of departure west ; but by observation, finds her difference of latitude is 112, and on trial finds a current, which in 24 hours makes 16 miles of diff. lat. N., and 10 miles of depart. E. : Required the ship's departure ?

Diff. lat. by account	96 m. N.	Departure by account	38 m. W.
Diff. lat. by current	16 m. N.	Departure by current	10 m. E.
True diff. lat.	112 m. N.	True departure	28 m. W.

Here the dead reckoning, corrected by the current, gives the difference of latitude 112 miles, which is the same as found by observation ; therefore the departure 28 is taken as the true one.

EXAM. II. Yesterday noon we were in latitude $36^{\circ}15' N.$, and have sailed these 24 hours, 1st. $SE. \frac{1}{2} E.$ 55 m. 2d. $NE.$ by $N. 20 m.$ 3d. $WSW.$ 70 m. 4th. $S.$ by $W. \frac{1}{2} W.$ 20 m., and all the time in a current setting $SSW. \frac{1}{4} W. \frac{3}{4} m.$ an hour : By an observation this day at noon, we were in latitude $34^{\circ} 58' N.$: Required the true departure the ship has made ?

Courses.	Diff.	N.	S.	E.	W.
$SE. \frac{1}{2} E.$	55				
$NE. b. N.$	20	16,6	34,9	42,5	
$WSW.$	70		26,8	11,1	64,7
$S. b. W. W.$	20		19,1		5,8
$SSW. \frac{1}{4} W.$	15		12,8		7,7
		16,6	93,6	53,6	78,2
		diff. lat.	77,0	dep	24,6

Departed lat. $36^{\circ}15' N.$
Present lat, $34^{\circ} 58' N.$

Diff. latitude $1^{\circ} 17' = 77 m.$

Here the current being taken into the account, produces a diff. lat. by dead reckoning equal to that found by observation ; and therefore the departure by the dead reckoning is taken as the true departure.

80. METHOD II. *When the error is thought to arise from the courses and distances.*

If the difference of latitude is much more than the departure, or the direct course has been within three points of the meridian; then it is most likely the error is in the distance.

And if the departure is much greater than the difference of latitude, or the direct course is within three points of the parallel, or more than five points from the meridian, the error may be ascribed to the course.

But if the courses are in general near the middle of a quadrant, the error may be either in the course, or in the distance, or in both.

For, to cause an alteration in the difference of latitude, the first of these cases requires a greater error in the course than can well be supposed to have been committed; in the second case, the distances must be so faulty as would scarce escape observation; and in the last, it is often doubtful whether to attribute the error to the course or distance; but it is usually corrected in both.

81.

CASE I.

When, by the dead reckoning, the diff. lat. is more than once and a half the departure: Or, which is the same, when the course is less than three points.

RULE. To the diff. lat. and the departure by account, find the course.

With this course, and the diff. lat. by observ. find a new departure. With the new departure, and the co-mid. lat., find the diff. long.

EXAM. Yesterday noon we were in lat. $39^{\circ} 18' N.$, and by an observation, are this day noon in lat. $37^{\circ} 48' N.$; our dead reckoning gives 116 miles of southing, and 64 miles of easting: Required the ship's true difference of long.?

By the TRAVERSE TABLE.

The diff. lat. 106, and the departure 64, gives the course $2\frac{1}{2}$ points.

The course $2\frac{1}{2}$ points, and the diff. lat. by observ. 90, gives a dep. 54.

The co-mid. lat. $51^{\circ} 27'$, and true dep. 54, gives the diff. long. 69.

82.

CASE II.

When by the dead reckoning, the departure is more than once and a half the difference of latitude: Or, the course is more than five points.

RULE. With the diff. latitude and departure by account, find the distance.

With this distance, and the diff. lat. by observ. find the true departure.

With the co mid. latitude and true departure, find the diff. longitude.

EXAM. Yesterday noon we were in latitude $48^{\circ} 52' N.$, and were to day at noon in latitude $50^{\circ} 18' N.$; our dead reckoning shews we have made 68 miles of northing and 112 miles of westing: Required the true difference of longitude the ship has made?

By the TRAVERSE TABLE.

The diff. lat. 68 and departure 112, gives 132 for the distance.

The dist. 132, and diff. lat. by observ. 86, gives the true depart. 100.

The co-mid latitude $40^{\circ} 25'$, and true departure 100, gives 155 miles for difference of longitude.

Z 2

84. CASE

83. CASE III.

When the difference of latitude and departure by account are nearly equal, or the direct course has been between three and five points of the meridian.

RULE. To the diff. latitude and departure by account, find the distance. With this dist. and diff. lat. by observation, find another departure. Take the half sum of the two departures for the true one.

With the co-mid. latitude and true departure find the diff. longitude.

EXAM. *These last 24 hours we have made 84 miles of northing, and 76 of easting; we were yesterday noon in latitude $52^{\circ}40' N.$, and are this noon in latitude $54^{\circ}22' N.$: What difference of longitude has the ship made?*

By the TRAVERSE TABLE.

To the diff. latitude 84 and departure 76, the distance is 113 miles.

With the dist. 113 and diff. lat. by observ. 102, find a departure 47,7.

The sum of the two departures 76 and 47,7 is 123,7, its half is 61,8.

To the co-mid. lat. $36^{\circ}29'$ and true depart. 61,8 the diff. long. is 103.

84. Dr. Halley having collected a great multitude of observations made on the variation of the needle in many parts of the world, he by the help of the latitude and longitude of the places of those observations, was enabled to draw on a Mercator's chart certain lines, shewing the variation of the compass in all those places over which they passed, in the year 1700, the time he published the first chart of this kind, and called the Variation Chart. Consequently, the longitude of any of those places could be found by the chart, having its latitude and the variation of the needle in that place given.

85. METHOD III. To find the longitude at sea by the variation chart.

Find the variation of the compass. For which see article 28.

Draw a parallel of lat. on the chart through the lat. found by observ.

And the point where it cuts the curved line, whose variation is the same with that observed, will be the ship's place.

EXAM. *A ship finds by a good observation that she is in the latitude of $18^{\circ}20' N.$; and that the variation of the compass is 4° west: Required the ship's place?*

Take from the graduated meridian the distance between the equator and the latitude of $18^{\circ}20' N.$: Lay a ruler along the equator, and slide one point of the compasses along its edge till the other point cuts the curve of 4° west variation, and the intersection gives the ship's place, whose longitude will be found to be about $27^{\circ}10'$ west from London.

86. This method is attended with two inconveniencies.

First, That wherever the variation lines run east or west, or nearly so, this way of finding the longitude becomes imperfect: But among all the trading parts of the world, this imperfection is at present found chiefly on the west coasts of Europe, between the latitudes of 45° and 53° ; on the eastern shores of North America, and in some parts of the western ocean and Hudson's bay, lying between the said shores; therefore,

fore, for the other parts of the world, a variation chart may be esteemed as of the greatest use.

87. But the variation curves, even where they run east and west, may be sometimes applied to good use in correcting the latitude, when meridian observations cannot be had, as it frequently happens on the northern coasts of America, in the Western ocean, and about Newfoundland: For if the variation can be obtained exactly, then the east and west curve answering to that variation on the chart, will shew the latitude.

Secondly. As the deflection of the magnetical meridian from the true one, is subject to continual alteration; therefore a chart, to which the variation lines are fitted for any year, must in time become useless, unless new lines, shewing the state of the variation at that time, be drawn on that chart: But as the change in the variation is very slow; therefore new variation charts being published every seven or eight years, will answer the purpose wanted.

88. Notwithstanding the great use which Dr. Halley's Variation Chart afforded to most mariners, yet it has been useless, for want of public encouragement to renew it from time to time. However, in the year 1746, Mr. William Mountaine, F. R. S. and Mr. James Dodson, published a new Variation Chart, fitted for that year, which was well received; and several instances of its great utility having been communicated to them, they fitted the Variation lines anew for the year 1756, and the year following published the third Variation Chart, and also presented to the Royal Society a curious paper concerning the variation of the magnetic needle, with a set of tables annexed, containing the result of upwards of fifty thousand observations, in six periodical reviews, from the year 1700 to 1756, inclusive, and adapted to every five degrees of latitude and longitude, in the more frequented oceans: which paper and tables were printed in the Society's Transactions for the year 1757.

The engraving of new plates on every publication of such charts, is a charge too great for private persons to incur; and should, like the nautical almanack, be done at the expence of the public; especially as its use is so easy, and so well fitted to the knowledge and time of mariners in general.

89. METHOD IV. To find the longitude, by the help of a perfect time-keeper.

This method is, a mechanical solution of the following problem.

The time in any unknown place, relative to the meridian of that place, being given, to find what o'clock it is, at that instant, under the meridian of some known place; suppose London, Greenwich, or any other place.

If a clock or watch was so contrived, as in all seasons and in all places to go uniformly; such a watch being regulated to London or Greenwich time, would shew always the time of the day at London or Greenwich: then the time of the day under any other meridian being found, the difference between that time and the London or Greenwich time corresponding, would give the difference of longitude.

90. That very excellent artist, Mr. John Harrison, after more than

thirty years close attention to this subject, has discovered many curious inventions to remove the irregularities to which all watch and clock-work were liable, when constructed by the common methods: these his inventions are so extraordinary, and seem so well adapted to remove the imperfections of clocks, that the commissioners for the longitude have granted him some considerable gratuities to enable him to bring his notions to the desired perfection: Experiments of his watch were made in the latter end of the year 1761, and the beginning of the year 1762, between Portsmouth and Jamaica; and although the success did not fully come up to the wishes of the public. yet proved sufficient to induce the commissioners to enable Mr. Harrison to make a second trial, between Portsmouth and Barbadoes, which succeeded so well, that the commissioners of longitude ordered him the sum of 10,000 pounds, upon his disclosing the mechanism of his WATCH, to a number of proper persons appointed to receive his communications; which have since been published, and *watches* of a similar construction have been made by other artists. Mr. Harrison has even improved his former works; and it is probable, that such watches may hereafter become more common, and be afforded for less than 100l. or a fourth part of their present value.

But to whatever degree of perfection such a movement may be brought, yet as every mechanic instrument must be liable by various accidents to be injured, it is certainly to be wished, that astronomical methods could be also so far improved, that the marine artist might be enabled to find his longitude from time to time with sufficient exactness by celestial observations.

91. Various astronomical methods have been proposed for finding the longitude at sea, some of which will hereafter be mentioned.

These methods chiefly depend on having an Ephemeris or Almanac fitted to the meridian of some place, suppose Greenwich, for which the Nautical Almanac is fitted, which Ephemeris shall contain for every day of the year the Sun's place, right ascension, declination, the times of the planets passing the meridian, as also their rising and setting, the times of the eclipses of the Sun and Moon, together with those of Jupiter's satellites; the distances of the Moon from the Sun and some zodiacal stars at certain times; and in general, the times when any other remarkable celestial appearances may be seen at the place for which the Ephemeris is calculated: Then, as these celestial appearances are seen at, nearly, the same instant of absolute time from all parts of the Earth where they are visible; therefore by knowing the relative times of the day when such appearances are seen in two distant places, the difference of those times are known, and consequently the difference of longitude between those places.

92.

METHOD V. *By the Sun's declination.*

By some of the means shewn in Book V., between the articles 185 and 204, or by any other means, let observations of the Sun be taken, from whence his declination may be computed at those times.

Take the difference between this computed declination, and that for the noon of the same day at London, as shewn by the Ephemeris; from whence take also the daily difference of declination at that time, and say,

At

As the daily difference of declination, is to the above-found difference; so is 360 degrees, to the difference of longitude.

EXAM. Suppose from certain observations made on the 18th of April 1764, the Sun's declination should be found to be $11^{\circ} 10' 4''$: Required the longitude of the place of observation?

On April 18th the decl. at London is $11^{\circ} 5' 26''$ } daily diff. is $20' 39''$.
 10th it is $11^{\circ} 26' 5''$

10th

it is 11 26 5

The difference between $11^{\circ} 10' 4''$ and $11^{\circ} 5' 26''$ is $4' 38''$.

Then $20' 39'' : 4' 38' :: 360^\circ : 80^\circ 46\frac{1}{2}'$, the diff. of long east of London.

Because the observed declination was greater than that at London.

Had it been less, the difference of longitude would have been westward.

Here a small error in the declination computed, will make a very considerable error in the difference of longitude; and therefore the method is inserted as a caution to beginners.

93. METHOD VI. *By the Moon's culminating.*

Seek in the Ephemeris for the time of her coming to the meridian on the given day, and on the day following, and take their difference; also take the difference between the times of culminating, on the same day, as found in the Ephemeris, and as observed; and say,

As the daily difference in the Ephemeris, is to the difference between the Ephemeris and observation; so is 360 degrees, to the difference of longitude.

For as the whole difference arises in a day, or by the running through 360° ; consequently any part of that difference will require a proportional part of 360 degrees; upon the supposition of the moon's motion being uniform during 24 hours, which it is not.

EXAM. Suppose at sea, the Moon's center was observed to pass the meridian on a certain known day at 57 minutes after one in the morning: Required the longitude of that place?

In the Ephemeris I find the Moon's center passes the meridian of London on that day at 1 h. 47 m., and on the next day at 2 h. 31 m.; the diff. is 44 m., and the time observed is 10 m. later than at London.

Then, as 44 m : 10 m. :: 360° : $81^{\circ} 49'$. Which shews that the place is $81^{\circ} 49'$ to the west of the meridian of London.

Here also, a small error in the time of the Moon's culminating will produce a great error in the difference of longitude; and is mentioned on the same account as the former method was.

94. **METHOD VII.** *By the eclipses of Jupiter's satellites.*

On the day preceding the evening on which it is proposed to observe the eclipse, seek the time it will happen at London, in the Nautical Almanack.

Let the difference of longitude between London and the place of observation (known by the dead reckoning at sea, or by the printed maps) be turned into time; and if eastward, be added to the London time of the eclipse; but subtracted, if westward; and it will give the time nearly when the eclipse is to be looked for in that place; but it will be proper to be in to observe 20 or 30 minutes sooner; and let the instant when the eclipse begins or ends be noted, as shewn by a watch previously fitted to the place of observation (58): then the difference between this

Z 4

time

time and the London time, converted into degrees, will shew the difference of longitude; and if the London time precedes the observed time, the diff. of longitude is east; otherwise it is west.

95. The beginning of these eclipses, called *immersions*, happens always on the west side of Jupiter; and the endings, called *emersions*, are always seen to the eastward of the planet; and both cases happen at a very small distance from the planet, seldom exceeding its half diameter.

The observations, best for finding the difference of longitude, should be made on the first satellite, or that which moves nearest to Jupiter; but it should be observed, that its emersions are not visible from the time of Jupiter's conjunction with the Sun, to the time of his opposition; and the immersions are not visible from the time of the planet's opposition to the Sun, to the time of its conjunction.

EXAM. Suppose by the *Almanac* for 1773, an immersion of Jupiter's first satellite is visible at London on October 23d., at 13h. 23m. 15f.; and being by my dead reckoning in longitude 72 deg. to the west of London, I observed on the same day the immersion at 8h. 40m. 45f.; Required the difference of longitude?

The difference between 13h. 23m. 15f. and 8h. 40m. 45f., is 4 h. 42 m. 30 f.

And 4 h. 42 m. 30 f. is equal to $70^{\circ} 32'$ for the diff. of long. westward.

96. The eclipses of Jupiter's satellites may be well observed by one of Dollond's new achromatic telescopes of about three feet length, or by a reflecting telescope of 12 or 15 inches long; and as these eclipses happen almost daily, they prove the most ready means of determining the longitudes of places at land, and thereby the longitudes of sea-coasts may be better ascertained than they are at present: and whenever Jupiter is to be seen, might be applied at sea, even oftener than they would be wanted, could they be observed with sufficient accuracy in a ship under sail.

97. METHOD VIII. By Eclipses of the Moon.

This is performed much in the same manner as by the last method.

For if in two or more distant places, where an eclipse of the Moon is visible, the times of the beginning or ending are carefully observed, as also the times when any number of digits was eclipsed; or when the Earth's shadow begins to touch, or to leave, any remarkable spot on the Moon's face; then will the difference of the times, when the observations of like kind were made, give the difference of longitude between the places of observation. But these eclipses too seldom happen to derive from them any general use.

98. METHOD IX. By an Occultation of a fixed star by the Moon.

An occultation of a fixed star by the Moon, is when a star, lying in the line of the Moon's motion, is eclipsed by that planet.

The method of computing the difference of longitude between London, and a place where the time of the occultation is observed, is shewn in Halley's Astronomical Tables: and the method of delineating such an occultation, is shewn in De la Caille's Elements of Astronomy.

But the method of finding the difference of longitude by an occultation, is rather too intricate and tedious to become of general use at sea.

99. The great Dr. Halley, and after him the Abbé de la Caille, and others, have reckoned the best astronomical method for finding the longitude at sea to be that wherein the distance of the Moon from the Sun, or from a star is used : for the Moon's daily mean motion being about 13 degrees, her hourly mean motion is about half a degree, or 1 min. of a degree in two minutes of time ; and so an error of one minute of a degree in position will produce an error of two minutes in time, or half a degree in longitude : And if by observation it is determined what part of her daily motion the Moon has run through during the interval between a certain point of time under a known meridian, and the instant of time when the observations are made on her, under an unknown meridian, then her daily motion at that time, will have, to the part thereof determined by observation, the same ratio, which 24 hours has to the interval of time taken to describe that arc.

100. M. de la Caille, in the prosecution of his method, directs a number of observations to be made one after another, in which the interval of time is to be diligently noted : and observes ; that many errors will be subject here to arise from the instrument, the observations and the calculations, which must jointly affect the longitude deduced ; so that the most experienced artist ought not, in general, to expect on a single set of observations, to find his longitude nearer than to between two or three degrees of the truth : And therefore proposes that each set of observations should be taken by two or more observers at the same noted time ; whereby, from a mean taken between them, the longitude may probably be found within a degree and a half ; although it may happen, that by an extraordinary degree of diligence and skill, and by a lucky compensation of errors, a result within a degree of the truth may be obtained : But the Rev. Mr. Maskelyne, the Astronomer Royal, who has also had experience in this subject, recommends the set of observations to be divided among different persons, each observing, at the same time, his respective part, and the computations made either from a good Ephemeris, or from the best Lunar tables, the result may in general determine the longitude at sea to within a degree of the truth.

101. Since Sir Isaac Newton's discovery of the true cause of the inequalities in the Moon's motion, which had from the earliest times perplexed astronomers, attempts have been made to form juster tables of her motion than before was possible to be effected ; and herein the late professor of astronomy at Gottingen, *Mayer*, has succeeded to such a degree, that his tables come so near the truth in every part of the Moon's orbit, that they may be confided in for determining the longitude of a ship at sea, whenever the Moon is to be seen ; which has been proved by a comparison of the Tables with the observations made at the Greenwich observatory by the late Dr. Bradley and by the present Astronomer Royal.

Preparatives for finding the longitude by an observation of distance.

102. *First. To observe the angular distance between the Moon and any celestial object.*

I. *Between the Moon and Sun.*

Turn down one of the screens, and hold the Hadley's quadrant so, for a fore observation (44), that its plane shall pass through the Sun and Moon; look at the Moon through the transparent part of the horizon glass; and keeping her there, gently move the index till the Sun's image is brought into the silvered part of that glass; bring the nearest limbs of both objects into contact, and let the quadrant librate a little on the lunar ray, whereby the Sun will appear to rise and fall by the side of the Moon; and in this motion the nearest limbs are to touch one another exactly, by moving the index; when this is effected, the observation is made, and the degrees cut by the Vernier scale will shew their angular distance.

103.

II. *Between the Moon and a star.*

If the Moon is very bright, turn down the lightest screen, and direct the plane of the quadrant to both objects; look at the star, as in a fore observation, through the clear part of the horizon glass; keep it there, and by moving the index, bring the Moon's image into the silvered part of the same glass; let the quadrant gently librate about the star's ray, and the Moon will appear to rise and fall by the star; between the librations move the index till the Moon's enlightened limb is touched by the middle of the star, as by a tangent, and the observation is made.

R E M A R K S.

104. The enlightened limb of the Moon is always to be brought into contact with the Sun or star, even though the Moon's image is made to pass beyond the Sun or star before the desired contact can be obtained.

Between the new and full moons, the enlightened limb is turned towards the west: and during the time from the full to the new moon, the enlightened limb is turned to the east.

The observed distance of the nearest limbs of the objects, is to be increased by the sum of their semi-diameters: But when the contact is observed of their farthest limbs, by bringing one over the other, then the observed distance is to be lessened by the sum of their semi-diameters.

The stars have no apparent semi-diameter; but those of the Sun and Moon will be found in the Nautical Almanack for any given month and day.

105. When the face of the quadrant is held upwards, the motion of the index forwards brings the right-hand object, by reflection, into contact with the left-hand one seen directly: and when its face is held downwards, the like motion of the index brings the left-hand object, by reflection, into contact with the right-hand one seen directly.

Therefore in north lat. the quadrant is to be used with its face upwards, when the object to be seen by reflection is to the right-hand,

hand, or westward of that seen directly. But when the object to be seen directly lies to the westward of that to be seen by reflection, the quadrant is to be used with its face downwards.

106 A readiness in the use of the quadrant both ways, is only to be attained by practice; which the learner may make familiar to himself, by observing the position of land objects; and to attain an accuracy in observations, let him select 6, 8, 10, or more objects lying around his view; and beginning with any one of them, take the angular distance of the 1st and 2d, the 2d and 3d, the 3d and 4th, and so on all round, ending with the last and first; and the sum of all these angles should make 360 degrees.

In a Hadley's quadrant of 18 inches radius, if the limb and the Vernier's scale is well divided, the measure of an arc may be read to thirds or quarters of a minute, if the eye is assisted by a magnifying glass.

107. Secondly. *To correct the apparent angular distance observed between the Moon and any other celestial object: or to reduce the distance, observed at the surface, to what it would appear if observed from the center of the earth.*

It was shewn in book V. art. 66, that the apparent place in which the Moon is seen in the heavens, from the surface of the earth, is, on the account of parallax, in general, not her true place; so that her apparent place should be corrected, as well for the error of parallax as for that of refraction: but the parallax of the Sun is too small, and that of the stars yet smaller, to affect their places sensibly, and therefore the refraction only is to be applied to them: now because that the apparent places of celestial objects want correction, therefore their apparent distance does also stand in need of being corrected, especially where the Moon is concerned; how to do this is shewn in the following precepts.

1st. In the triangle formed by the observed distance of the objects, and their zenith distances; find the angles at each object comprehended between the observed distance and respective vertical, or azimuth circle. (IV. 154)

2d. By the Almanac find the Moon's horizontal parallax at the time of observation reduced to the meridian of the Almanac.

3d. Say, as radius is to the cosine of the Moon's altitude;

So is the horizontal parallax, to the parallax in altitude:

Diminish the parallax in alt. by the refraction on the Moon's altitude.

4th. Say, as radius is to the cosine of the said angle at the Moon;

So is the diminished parallax, to one error in distance.

Which is subtractive if the angle at the Moon is acute; otherwise it is additive.

5th. Also say, as rad. is to the cos. of the said angle at the Sun, or star,

So is the refraction on its zenith distance, to an error in dist.

Which is additive when the angle at the Sun or star is acute, otherwise this error is subtractive.

EXAM.

EXAM. I. Suppose on the 6th of August 1772, in a place 96 degrees to the west of London, at 5 h. 36 m. P. M. by a watch, the distance of the nearest limbs of the Sun and Moon should be observed to be $68^{\circ} 10' 24''$, at the time the altitude of the Sun's lower limb is $31^{\circ} 44' 12''$, and the altitude of the Moon's lower limb is $23^{\circ} 44' 12''$, the height of the eye above the water being allowed for: Required what would be the true distance of the centers of the Sun and Moon?

By the Almanac, the semidiameters of the Sun and Moon at that time will be found to be, each of $15' 48''$. Hence the apparent altitude of the Sun's center is 32° , of the Moon's center is 24° , and the apparent distance of the centers is $68^{\circ} 42'$.

Also $96^{\circ} W$. is 6 h. 24 m. to which 5 h. 36 m. being added, gives 12 h. P. M. at London; and the Moon's horizontal parallax at the given London time, is $58' 39'' = 58.65'$.

Given the Sun's zenith distance 58°
 the Moon's zenith distance 66
 the distance of their centers $68.42'$.

Required $\angle$ of position at the $\odot$. Required the $\angle$ of position at the $\odot$.

$68^{\circ} 42'$	Working by art. 154.	B. IV.	$68^{\circ} 42'$	Working by art. 154.	B. IV.
$66 \quad 0$	$L', 68^{\circ} 42'$	$0, 03073$	$58 \quad 0$	$L', 68^{\circ} 42'$	$0, 03073$
$58 \quad 0$	$L', 66 \quad 0$	$0, 03927$	$60 \quad 0$	$L', 58 \quad 0$	$0, 07158$
$2 \quad 42$	$L', 30 \quad 21$	$9, 70353$	$10 \quad 42$	$L', 38 \quad 21$	$9, 79772$
$60 \quad 42$	$30^{\circ} 21' \quad L', 27 \quad 39$	$9, 66658$	$76 \quad 42$	$38^{\circ} 21' \quad L', 27 \quad 39$	$9, 66658$
$55 \quad 18$	$27 \quad 39$	$19, 44011$	$55 \quad 18$	$27 \quad 39$	$19, 56161$
	$\frac{5, 31 \quad 39\frac{1}{2}}{L \text{ at the Moon} = 63 \quad 19}$	$\frac{9, 72005}{L \text{ at the Sun} = 74 \quad 16}$		$\frac{5, 37 \quad 8}{L \text{ at the Sun} = 74 \quad 16}$	$\frac{9, 78080}{L \text{ at the Sun} = 74 \quad 16}$

As Rad.

To $\odot$, $\odot$'s alt. 24°

So horizon. paral. $58 \quad 65'$

To paral. in alt. $53, 58$

Deduct. refraction $2, 3$

Error in alt. $51, 28$ minutes.

As Rad.

To $\odot$, $\angle$ at $\odot$ $63^{\circ} 19'$

So error in alt. $51, 28$

So error in dist. $23, 03$

Which is subtractive.

$10, 00000$ As Rad.

To $\odot$, $\angle$ at $\odot$ $74^{\circ} 16'$

So error in refr. $1, 8$

To error in dist. $0, 488'$

Which is additive.

Now $23, 03 - 0, 488 = 22, 54' = 22^{\circ} 32''$ Subtractive.

Then $68^{\circ} 42' - 0^{\circ} 22' 32'' = 68^{\circ} 19' 28''$ Which is the true distance of the centers of the Sun and Moon at the time of observation, as would be seen at the Earth's center.

EXAM.

EXAM. II. Suppose in the year 1773, being in latitude $32^{\circ} 12' N$.; and longitude $38^{\circ} 30' W$. of Greenwich by account, on July 8th, at 7 h. 38 m. P. M., the apparent distance of the star Regulus from the Moon's center was observed to be $62^{\circ} 41'$; and at the same time, the apparent altitude of Regulus was $71^{\circ} 4'$, and that of the Moon's center was $55^{\circ} 39'$: Required the true distance of the star, from the Moon's center?

THE WORK.

62,41	0,05135	62,41	0,05135
18,56	0,4883	34,21	0,24853
<u>34,21</u>		18,56	
43,45		28,20	
78,0	39 03	47,16	23 38
9,24	4 42	9,24	4 42
		19,25301	
25 02		9,62650	
		14 50	

R	1'50.04	9.80746					
	19"	1.27875					
	+12"	<u>1.08621</u>					
R	55.39	9.75147					
	56.8	1.75435					
	32.0	<u>1.50582</u>					
		,39					
		<u>31.21</u>					
R	29.40	1.49624					
	31.35	<u>1.43522</u>					
		-27.15					
		+12					
		<u>-27.3</u>					
		9.93898					

The precepts by which these examples are worked, are derived from principles shewn in the Appendix: But the operations may be much shortened thus.

In a triangle, with the three sides, viz. the two apparent zenith distances, and the apparent distance of the centers, find the azimuth angle.

With this azimuth angle, and the corrected zenith distances, find the true distances of the centers.

This work may be done very compendiously by the following precepts, from Book IV. Art. 253, 255.

108. *A compendium for finding the true distance between the Moon and Sun, or Star, from their observed altitudes and distance.*

PRECEPTS.

- 1st. To the Arith. Comp. of the log. sine of half the apparent distance, add the log. sine of half the diff. of the apparent altitudes; gives log. sine of a first arc.
- 2d. To the log. sine of half the apparent distance, add the log. cosine of the first arc, and reject radius gives the log. sine of a second arc.
- 3d. Add twice the log. sine of the second arc, and the Arith. Complements of the log. cosines of the two apparent altitudes, gives the log. of a third arc.
- 4th. To the log. of the third arc, add the log. co-sines of the correct altitudes, gives the log. of a fourth arc.

5th.

5th. From half the log. of the 4th arc, take the log. sine of the half the diff. of the corrected altitudes, gives the log. tangent of a 5th arc.

6th. From half the log. of the 4th arc, take the log. sine of the 5th arc; leaves the log. sine of half the true distance.

These precepts expressed in the following short way, may contribute to the memory, and the ease of operation.

1st. $L, s, \frac{1}{2}$ ap. dist. + $L, s, \frac{1}{2}$ diff. ap. alts
 2d. $L, s, \frac{1}{2}$ ap. dist. + $L, s, \frac{1}{2}$ 1st arc
 3d. $2L, s, 2d$ arc + $L, s, \frac{1}{2}$ one ap. alt. + $L, s, \frac{1}{2}$ oth. ap. alt. = $L, s, 2d$ arc.
 4th. $L, 3d$ arc + $L, s, \frac{1}{2}$ one cor. alt. + $L, s, \frac{1}{2}$ oth. cor. alt. = $L, 4th$ arc.
 5th. $\frac{1}{2}L, 4th$ arc - $L, s, \frac{1}{2}$ diff. cor. alts.
 6th. $\frac{1}{2}L, 4th$ arc - $L, s, \frac{1}{2}$ 5th arc
 = $L, s, \frac{1}{2}$ cor. dist.

These precepts applied to the preceding examples, the work will stand in the following order.

EXAM. I. (See page 348) Here are given the apparent altitudes of the centers, viz. of the Sun 32° ; of the moon 24° ; and their apparent distance $68^{\circ} 42'$; and the moon's horizontal parallax $58' 39''$: Required the true distance?

Diff. = $68^{\circ} 42'$; its $\frac{1}{2}$ = $34^{\circ} 21'$.	Alt. $\odot$ = $32^{\circ} 0'$ Alt. D = $24^{\circ} 0'$
Ap. alt. $\odot$ = 32°	Refr. — = $1 31$ Refr. — = $2 7$ *
Ap. alt. D = 24	Cor. alt. $\odot$ = $31 58 29$ Alt. cor. = $23 57 53$
Diff. = $8; \frac{1}{2}$ diff. = 4° .	Cor. alt. D = $24 51 32$ Paral. + = $53 39 \frac{1}{2}$
	Diff. cor. alts. = $7 6 57$ Cor. alt. D = $24 51 32$
L, s , dist. = $34 21$ $0, 24853$	$\frac{1}{2}$ Diff. cor. alts. = $3 33 28$
$L, s, \frac{1}{2}$ diff. ap. alts. = $4 0$ 8.84358	$9, 75147$ $L, s, \frac{1}{2}$ diff.
L, s , 1st. arc	$9, 09211$
	$9, 096' 6$ $L, s, \frac{1}{2}$ 1st. arc
	$9, 74813$ $L, s, 2d$ arc
	$9, 74813$ $L, s, 2d$ arc
	$0, 07158$ $L, s, \frac{1}{2}$ $\odot$ alt. 32°
	$0, 03927$ $L, s, \frac{1}{2}$ alt. 24°
	$19, 60711$ $L, 3d$ arc
	$9, 92854$ $L, s, \frac{1}{2}$ $\odot$ cor. alt. $31^{\circ} 58 \frac{1}{2}'$
	$9, 95778$ $L, s, \frac{1}{2}$ cor. alt. $24^{\circ} 51 \frac{1}{2}'$
	$39, 49343$ $L, 4th$ arc
$\frac{1}{2} L, s, 4th$ arc.	$19, 74671$
$L, s, 5th$ arc	$9, 90733$
$L, s, \frac{1}{2}$ cor. dist. = $34^{\circ} 9' 44''$ $9, 74938$	
True dist. = $68 10 28$	

* The refractions are taken from table art. 55.

+ The parallax are taken from table art. 56.

But the parallax on alt. is most readily found by the Gunter's scale.

The extent, on the fines, from rad. to the cosine of the alt.

Will, on the numbers, reach from the hor. par. to the paral. in alt.

EXAM.

EXAM. II. (See page 349) Here are given the apparent alt. of the moon $55^{\circ} 39'$ that of *Regulus* $71^{\circ} 04'$; their dist. $62^{\circ} 41'$; and the moon's hor. parallax $56' 48''$.

$$62^{\circ} 41'; \frac{1}{2} 31^{\circ} 20\frac{1}{2}'$$

Ap. alt.	71 04	0	55 59	0
Ref.	19			39
	71 03	41	55 38	21
	56 10	21	32	0
	14 53	20	56 10	21

Ap. alts. $71^{\circ} 04'$

$$\frac{55 39}{15 25}$$

$$\frac{7 42\frac{1}{2}}{7 42\frac{1}{2}}$$

$$\frac{7 42\frac{1}{2}}{7 42\frac{1}{2}}$$

$L',$

$$31^{\circ} 20\frac{1}{2}'$$

$L',$

$$7 42\frac{1}{2}$$

$L',$ 1st. arc

$$9, 71612 s, \frac{1}{2} \text{ dist.}$$

$$9, 98506 s, \text{ 1st. arc}$$

$$9, 70118 s, \text{ 2d. arc}$$

$$9, 70117 s, \text{ 2d. arc}$$

$$0, 48883 L', s, \text{ alt. } 71^{\circ} 4'$$

$$0, 24853 L', s, \text{ alt. } 55 39$$

$$20, 13971 3d \text{ arc}$$

$$9, 51129 s, \text{ alt. } 71^{\circ} 3\frac{3}{4}'$$

$$9, 74562 s, \text{ alt. } 56 10\frac{1}{2}$$

$$39, 39662 4th. \text{ arc}$$

$$19, 69831 \frac{1}{2} L, \text{ 4th. arc}$$

$$9, 11248 s, 7 26\frac{2}{3}$$

$$10, 58583 t, 5th. \text{ arc}$$

$\frac{1}{2} L',$ 4th. arc

$$31^{\circ} 03'$$

$s,$ 5th. arc

$$9, 98584$$

$$9, 71247$$

62 06 the cor. dist.

EXAM. III. Suppose the apparent dist. of the moon and a star was $51^{\circ} 28' 35''$; the stars altitude $24^{\circ} 48'$; the moon's altitude $12^{\circ} 30'$ and her horizontal parallax $56' 15''$: Required the true distance?

$$\text{Dist. } 51^{\circ} 28' 35''; \text{ its } \frac{1}{2} = 25^{\circ} 44' 17\frac{1}{2}''$$

$$\text{Ap. alt. } * 24^{\circ} 48'$$

$$D 12 30$$

$$\text{Diff.} = 12 18$$

$$\frac{1}{2} \text{ diff.} = 6 09 \text{ ap. alts.}$$

$$* 24^{\circ} 48' 0''$$

$$24 45 57$$

$$13 20 48$$

$$11 25 9$$

$$5 42 34\frac{1}{2} = \frac{1}{2} \text{ diff. true. alts.}$$

$$9, 63774 L, s, 25^{\circ} 44' 17\frac{1}{2}''$$

$$9, 98636 L', s, \text{ 1st. arc}$$

$$9, 62410 L', s, \text{ 2d. arc}$$

$$9, 62410 L', s, \text{ 2d. arc}$$

$$0, 04202 L', s, 24^{\circ} 48'$$

$$0, 01042 L', s, 12 30$$

$$19, 30064 L', s, 3d. \text{ arc}$$

$$9, 95810 L', s, 24^{\circ} 46'$$

$$9, 98840 L', s, 13 21$$

$$39, 24714 L, \text{ 4th. arc}$$

$$19, 62357 \frac{1}{2} L, \text{ 4th. arc}$$

$$8, 99775 L', s, 5 42 34$$

$$10, 62582 L', s, 5th. \text{ arc}$$

$L',$

$$25^{\circ} 44' 17\frac{1}{2}''$$

$$L',$$

$$6 9$$

$L',$ 1st. arc

$\frac{1}{2} L',$ 4th. arc

$L',$ 5th. arc

$$\frac{1}{2} \text{ Dist. } 25 35 23$$

$$\text{Dist. } 51 10 46$$

EXAM. IV. Let the apparent distance of the moon and a star be $102^{\circ} 30'$; the star's alt. $15^{\circ} 25'$; the moon's alt. $27^{\circ} 30'$, and her hor. par. $51' 3''$. Required the true distance.

Dist. $102^{\circ} 30'$; its $\frac{1}{2} = 51^{\circ} 15'$.

Ap. alt. $\ast 15^{\circ} 25'$

D $27^{\circ} 30'$

Diff. $12^{\circ} 5'$

$\frac{1}{2}$ diff. $6^{\circ} 2\frac{1}{2}$ ap. alts.

$L, s, 51^{\circ} 15'$

$Ls, 6^{\circ} 2\frac{1}{2}$

$L, s, 1st. arc$

0.10797

9.02223

9.13020

$\ast 15^{\circ} 25' 0''$ D $27^{\circ} 30' 0''$

— 3 25 — 1 49

15 20 35 27 28 11

28 18 56 + 50 45

12 58 21 28 18 56

6 29 10 = $\frac{1}{2}$ diff. true alts.

9.89208 $L, s, 51^{\circ} 15'$

9.99601 $L, s, 1st. arc$

9.88804 $L, s, 2d. arc$

9.88804 $L, s, 2d. arc$

0.01592 $L, s, 15^{\circ} 25'$

0.05207 $L, s, 27^{\circ} 30'$

19.84407 $L, 3d. arc$

9.94465 $L, s, 28^{\circ} 19'$

9.98424 $L, s, 15^{\circ} 20\frac{1}{2}$

39.77296 $L, 4th. arc$

19.88648 $\frac{1}{2} L, 4th. arc$

9.05293 $L, s, 6^{\circ} 29\frac{1}{2}$

10.83355 $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

19.88648

$L, s, 5th. arc$

9.99537

$\frac{1}{2}$ Diff. $51^{\circ} 6'$

9.89111

Diff. $102^{\circ} 12'$

EXAM. V. Given the apparent distance of the Moon from a Star, $43^{\circ} 35' 42''$; Star's alt. $11^{\circ} 17'$; Moon's alt. $9^{\circ} 38'$; and her horiz. paralax $54' 42''$. Required the true distance?

Now Rad : $1^{\circ} 5' 38'' : 54\frac{1}{2} : 54' = par. in alt. by Gunter's Scale.$

Dist. = $43^{\circ} 35' 42''$; its $\frac{1}{2} = 21^{\circ} 47' 51''$.

$\ast salt. = 11^{\circ} 17'$

D $salt. = 9^{\circ} 38'$

$\ast 11^{\circ} 17' 0''$ D $9^{\circ} 38' 0''$

— 4 40 — 5 30

11 12 20 9 32 30

10 26 30 + 54 0

0 45 50 10 26 30

Diff. = $1^{\circ} 39'$

$\frac{1}{2}$ diff. = $0^{\circ} 49\frac{1}{2}$ ap. alts.

$L, s, 21^{\circ} 47' 51''$

$L, s, 0^{\circ} 49\frac{1}{2}$

$L, s, 1st. arc$

0.43024

8.15829

8.58853

9.09967 $L, s, 1st. arc$

9.56943 $L, 2d. arc$

9.56943 $L, 2d. arc$

0.00617 $L, s, 11^{\circ} 17'$

0.00848 $L, s, 9^{\circ} 38'$

19.15351 $L, 3d. arc$

9.99275 $L, s, 10^{\circ} 26\frac{1}{2}'$

9.99164 $L, s, 11^{\circ} 12\frac{1}{2}'$

39.13700 $L, 4th. arc$

19.56895 $\frac{1}{2} L, 4th. arc$

7.82384 $L, s, 22^{\circ} 55'$

11.74511 $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

19.56895

$L, s, 5th. arc$

9.99993

$\frac{1}{2}$ Diff. $21^{\circ} 45\frac{1}{2}'$

9.56902

43 31 the true distance.

EXAM. VI. Given the observed distance of the Moon from the Sun 85° ; the Sun's alt. $5^{\circ} 0'$; the Moon's alt. $30^{\circ} 0'$; and her horizontal parallax $60^{\circ} 0'$: Required the true distance?

Now Rad : $1, 30^{\circ}$:: $60' : 51' 45''$ = par. in alt. by Gunter's Scale.

Dist. = $85^{\circ} 0'$; its $\frac{1}{2}$ = $42^{\circ} 30'$.

D's alt. = $30^{\circ} 00'$

☉'s alt. = $5^{\circ} 00'$

Diff. = $25^{\circ} 00'$

$\frac{1}{2}$ diff. = $12^{\circ} 30 \frac{1}{2}$ ap. alts.

L's, $42^{\circ} 30'$

L's, $12^{\circ} 30'$

L's, 1st. arc

6,17032

9,33534

9,50566

13 00 = $\frac{1}{2}$ diff. true alts.

9,82968 L's, $42^{\circ} 30'$

9,97619 L's, 1st. arc

9,80617 L, 2d. arc.

9,80617 L, 2d. arc

0,06247 L's, $30^{\circ} 0'$

0,00166 L's, $5^{\circ} 0'$

19,67647 L, 3d. arc

9,99845 L's, $4^{\circ} 50'$

9,93381 L's, $30^{\circ} 50'$

39,60873 L, 4th arc

19,80436 L, 4th arc

9,35209 L's, $13^{\circ} 0'$

10,45227 L's, 5th. arc.

$\frac{1}{2}$ L, 4th. arc

19,80436

L's, 5th arc

9,97450

Diff. $42^{\circ} 31'$

9,82986

$\frac{85}{2}$ the true distance.

EXAM. IV. Let the apparent distance of the moon and a star be $102^{\circ} 30'$; the star's alt. $15^{\circ} 25'$; the moon's alt. $27^{\circ} 30'$, and her hor. par. $51' 3''$. Required the true distance.

Dist. $102^{\circ} 30'$; its $\frac{1}{2} = 51^{\circ} 15'$.

Ap. alt. * $15^{\circ} 25'$

D $27^{\circ} 30'$

Diff. $12^{\circ} 5'$

$\frac{1}{2}$ diff. 6 $2\frac{1}{2}$ ap. alts.

$L, s, 51^{\circ} 15'$

$L, s, 6^{\circ} 2\frac{1}{2}$

$L, s, 1st. arc$

0.10797

9.02223

9.13020

* $15^{\circ} 25'$	0"	D $27^{\circ} 30'$	0"
—	—	—	—
15 20 35	3 25	—	1 49
28 18 56		+	27 28 11
12 58 21		+	50 45
6 29 10			28 18 56
9,89208 $L, s, 51^{\circ} 15'$			

9.09601 $L, s, 1st. arc$

9.88804 $L, s, 2d. arc$

9.88804 $L, s, 2d. arc$

0.01592 $L, s, 15^{\circ} 25'$

0.05207 $L, s, 27^{\circ} 30'$

19,84407 $L, 3d. arc$

9,94465 $L, s, 28^{\circ} 19'$

9,98424 $L, s, 15^{\circ} 20\frac{1}{2}$

39,77296 $L, 4th. arc$

19,88648 $\frac{1}{2} L, 4th. arc$

9,05293 $L, s, 6^{\circ} 29\frac{1}{2}$

10,83355 $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

19,88648

$L, s, 5th. arc$

$\frac{1}{2}$ Diff. $51^{\circ} 6'$

Diff. 102 12

9.99537

9.89111

EXAM. V. Given the apparent distance of the Moon from a Star, $43^{\circ} 35' 42''$; Star's alt. $11^{\circ} 17'$; Moon's alt. $9^{\circ} 38'$, and her horiz. parallax $54' 42''$. Required the true distance?

Now Rad : $s, 9^{\circ} 38' :: 54\frac{1}{2} : 54' = par. in alt. by Gunter's Scale.$

Diff. = $43^{\circ} 35' 42''$; its $\frac{1}{2} = 21^{\circ} 47' 51''$.

*'s alt. = $11^{\circ} 17'$

D's alt. = $9^{\circ} 38'$

Diff. = 1 39

* $11^{\circ} 17'$	0"	D $9^{\circ} 38'$	0"
—	—	—	—
11 12 20	4 40	—	5 30
10 26 30		+	9 32 30
0 45 50		+	54 0
0 22 55			10 26 30

$\frac{1}{2}$ diff. = $0 49\frac{1}{2}$ ap. alts

$L, s, 21^{\circ} 47' 51''$

$L, s, 0 49\frac{1}{2}$

$L, s, 1st. arc$

0.43024

8.15829

8.58853

9.09967 $L, s, 1st. arc$

9.56943 $L, 2d. arc$

9.56943 $L, 2d. arc$

0.00617 $L, s, 11^{\circ} 17'$

0.00848 $L, s, 9^{\circ} 38'$

19,15351 $L, 3d. arc$

9,99275 $L, s, 10^{\circ} 20\frac{1}{2}$

9,99164 $L, s, 11^{\circ} 12\frac{1}{2}$

39,13700 $L, 4th. arc$

19,56895 $\frac{1}{2} L, 4th. arc$

7,82384 $L, s, 22^{\circ} 55''$

11,74511 $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

19,56895

$L, s, 5th. arc$

$\frac{1}{2}$ Diff. $21^{\circ} 45\frac{1}{2}$

43 31 the true distance.

9.99993

9.56902

EXAM. VI. Given the observed distance of the Moon from the Sun 85° ; the Sun's alt. $5^{\circ} 0'$; the Moon's alt. $30^{\circ} 0'$; and her horizontal parallax $60^{\circ} 0'$; Required the true distance?

Now Rad $1^{\circ} 30^{\circ} :: 60' : 51' 45'' =$ par. in alt. by Gunter's Scale.

Dist. $= 85^{\circ} 0'$; its $\frac{1}{2} = 42^{\circ} 30'$.

D's alt. $= 30^{\circ} 00'$

☉'s alt. $= 5^{\circ} 00'$

Diff. $= 25^{\circ} 00'$

$\frac{1}{2}$ diff. $= 12^{\circ} 30' \frac{1}{2}$ ap. alts.

L's, 42 30

L's, 12 30

L's, 1st. arc

6, 17032

9, 33534

9, 50566

9, 82968 L's, 42 30

9, 97619 L's, 1st. arc

9, 80617 L, 2d. arc.

9, 80617 L, 2d. arc

0, 06247 L's, 30° 0'

0, 00166 L's, 5 0

19, 67647 L, 3d. arc

9, 99845 L's, 4 50

9, 93381 L's, 30 50

39, 60873 L, 4th arc

19, 80436 $\frac{1}{2}$ L, 4th arc

9, 35209 L's, 13° 0'

10, 45227 L's, 5th. arc.

$\frac{1}{2}$ L, 4th. arc

19, 80436

L's, 5th arc

9, 97450

Diff. $42^{\circ} 31'$

9, 82986

85 = the true distance.

SECONDLY.

109. To find the time at which two proposed celestial objects may be seen at a given distance; if observed from the center of the Earth under the meridian of a given Ephemeris.

In the Nautical Almanac, there is, ready computed, the distance of the enlightened limb of the Moon from the Sun, and also her distance from certain fixed stars, at the hours 3, 6, 9, 12, 15, 18, 21, and noon on the days when such observations are proper to be made.

Therefore seek in that Almanac for the computed distance of the proposed objects on the day given; and among those computed distances seek for the given distance, which if found there, the time is there also:

But if the given distance falls between two computed ones, A, B, (as it generally will) the corresponding time is to be found by proportional parts.

Saying, As the difference of the computed distances A, B,

Is to the difference between A and the given distance;

So is three hours of time,

To the proportional time to be added to that of the distance A.

EXAM. At what time on the 21st of July, 1773, will the distance between the Virgin's spike and the Moon's enlightened limb, be equal to $54^{\circ} 56'$, as seen from the center of the Earth under the meridian of Greenwich

On the 21st of July the given distance falls between the computed distances at 9 h. and at midnight, viz. between $A = 55^{\circ} 31' 30''$, and $B = 53^{\circ} 51' 53''$ so that the time is between 9 h. and 12 h. P. M.

The difference between A and B is $1^{\circ} 39' 37'' = 5977''$ and the difference between A and $54^{\circ} 56\frac{1}{4}'$ is $0^{\circ} 35' 15'' = 2115''$.

Then $5977'' : 2115'' :: 3^h : 1^h 3^m 59^s$.

And $9 h + 1 h. 3 m. 59 s. = 10 h. 3 m. 59 s.$ the time required.

110. METHOD X. To find the longitude from observations made on the altitudes of the Moon and Sun, or Star, and of their distance.

PRECEPTS.

1st. Let the watch be well adjusted to time, a few hours preceding the time of observation.

2^d. Let three persons, each having a good Hadley's quadrant, make observations; one taking the apparent angular distance of the Moon from the Sun or Star, another taking the altitude of the Moon; and the third, taking the altitude of the Sun or Star: These observations to be made at one and the same time, which is to be noted; the watch hanging in a convenient place for each observer to see.

3^d. Let the observations of altitudes be corrected for the centers of the objects, and that of the distance be corrected by parallax, to have the true distance of the centers.

4th. Find the time at Greenwich when this true distance happened.

5th. The difference between the Greenwich time and the ship time, is the difference of Longitude.

The

- The manner of adjusting the *watch* is shewn art. 58.
 Of taking the altitudes is shewn at arts. 44, 48. 63.
 Of taking the angular distance is shewn art. 125.
 Of correcting the distance by parallax, &c. at art. 108.
 Of finding the Greenwich time answering to this dist. at art. 109.

III.

R E M A R K S.

The best times of the day for taking the distance between the moon and Sun, is from about half an hour after Sun rise, to about ten o'clock in the forenoon; and in the afternoon from about two o'clock to within about half an hour of Sun-set. These observations may be taken during the four days preceding the Moon's first quarter, and the four days following the last quarter; which make eight days in each lunation, or about a fourth part of the time between new moon and new moon.

The best time for making observation between the Moon and Star, is during the evening or morning twilight; or when the Sea horizon can be seen distinct enough to take an altitude sufficiently accurate; and when the Moon has between 5° and 15° of altitude, or not exceeding 20° ; although any altitude may be used with a distinct horizon.

Choose a known star to the eastward or westward of the Moon, lying nearly in the line of the Moon's path; that is, in a right line conceived to pass through the moon's center perpendicularly to the right line joining her culps, or horns: but between the full Moon and her quadratures, the direction of her shortest diameter points out her path.

The distance of the star from the meridian should be two hours at least, but a greater distance is better; and the star's altitude should not be less than 5 degrees.

If two, three, or more sets of observations be made at the distance from one another of about seven or eight minutes; then the means of the watch times, of the distances, and of these respective altitudes, being taken, these means may be esteemed as a set of observations more to be depended on than any single set: Or the longitudes may be computed to each set, and the mean of the results be taken for the true longitude.

EXAM. I. Being by account in latitude 40° N. and longitude 28° W. of Greenwich, on the 24th of July, 1772; the watch being well regulated to apparent time, and the Hadley's quadrants well adjusted, the distance between the nearest limbs of the Sun and Moon, and the altitudes of their lower limbs were observed as follows, the height of the eye being allowed for: Required the Ship's true longitude?

Time 21 h. 20 m. Sun's alt. $46^{\circ} 10'$. Moon's alt. $6^{\circ} 30'$ Diff. $60^{\circ} 34'$

Means $21^{\circ} 25'$ $48^{\circ} 50'$ $16^{\circ} 20'$ $60^{\circ} 54'$

Now Sh. time 21 h. 24 m. P. M. ± 1 h. 52 W. = 23 h. 17 m. P. M. at Greenwich. (57)

At this time by almanac, the Sun's $\frac{1}{2}$ diam. = $15' 48''$ } Sum $31' 02''$
 Moons $\frac{1}{2}$ diam. = $15' 14''$

Her hor. parallax. = $55' 54''$

Then $60^{\circ} 44' + 31' 02'' = 61^{\circ} 15' 02''$ the apparent distance.

And $48' 50' + 15' 48'' = 49' 5' 48''$ the alt. of Sun's center.

And $8' 25' + 15' 14'' = 8' 40' 14''$ the alt. of Moon's center.

Also Rad: $5' 8'' \frac{2}{3} : 55' 54'' : 55' 10''$ the par. in alt. by the Gunter.

Now diff. $61^{\circ} 15' 2''$; its $\frac{1}{2} = 30^{\circ} 37' 31''$ $\odot$ alt. $49^{\circ} 5' 48''$ Moon alt. $8^{\circ} 40' 14''$

$\odot$ ap. alt. $49' 5' 48''$ Moon ap. alt. $8' 40' 14''$

Diff. $40' 25' 34''$ $\odot$ $49' 4' 59''$ Moon $8' 34' 13''$

$\frac{1}{2}$ Diff. $20' 12' 47''$ of ap. alts. $\text{Par.} \pm$ $55' 10''$

$19, 47' 48'' = \frac{1}{2}$ diff. true alts.

$9, 707' 08''$ $L, s, 30^{\circ} 37' 31''$

$9, 866' 19''$ $L, s, 1$ ft. arc

$9, 573' 27''$ $L, 2d.$ arc

$9, 573' 27''$ $L, 2d.$ arc

$0, 183' 89''$ $L, s, 49^{\circ} 5' 48''$

$0, 004' 99''$ $L, s, 8' 40' 14''$

$19, 335' 42''$ $L, 3d.$ arc

$9, 816' 21''$ $L, s, 49^{\circ} 4' 59''$

$9, 994' 01''$ $L, s, 9' 29' 23''$

$39, 145' 64''$ $L, 4th.$ arc

$19, 572' 82''$ $\frac{1}{2}$ $L, 4th.$ arc

$9, 529' 79''$ $L, s, 19^{\circ} 47' 48''$

$10, 043' 03''$ $L, s, 5th.$ arc

$\frac{1}{2}$ $L, s, 4th.$ arc $19, 572' 82''$

$L, s, 5th.$ arc $9, 869' 93''$

$\frac{1}{2}$ Diff. $30' 18''$ $9, 702' 89''$

$60' 36''$ the true distance.

Again, by the Nautical Almanac 1772.

The dist. at 21 h. is $61^{\circ} 37' 35'' = A.$ And $61^{\circ} 37' 35'' = A.$ (109)

24 h. is $60' 11' 37'' = B.$ $60' 36''$ at ship.

$1' 25' 58'' = 85' 58''$ $1' 1' 35'' = 61' 35''$

Now (I. 40) $85' 58'' = 85-966$; and $61' 35'' = 61,5833$.

Then (I. 46) As $85-96 : 61,583 : : 3h. : 2h. 8m. 56s.$ time above A.

21 time A.

Greenwich time $23^{\circ} 08' 56''$

Ship time $21^{\circ} 25' 0''$

True diff. longitude $1^{\circ} 43' 56''$

By account diff. long. $1^{\circ} 52''$

$8' 4'' = 2^{\circ} 1'$

EXAM. II. Suppose in 1773 Nov. 6th. at 14h. 15m. 14s. apparent time; being by account in lat. $48^{\circ} 19' S$. long. $116^{\circ} 15' E$. the observed distance between Aldebaran and the Moon's farthest limb should be $58^{\circ} 30' \frac{1}{2}$; the Star's alt. $25^{\circ} 10'$; the alt of the Moon's lower limb $8^{\circ} 2' 50''$. Required the ship's true longitude?

Now Sh. time 14h. 15m. 14s. P. M. $-7h. 45m = 6h. 30m. 14s$ at Greenw. (57)
At this time Δ 's semidiam. $= 15' 10''$; and hor. parallax $55' 40''$ by Alman.

And $58^{\circ} 30' 15'' - 15' 10'' = 58^{\circ} 15' 5''$ the apparent distance.

And $8. 250 + 15 10 = 8 18$ the ap. alt. of Δ 's center.

Then dist. $58^{\circ} 15' 5''$; its $\frac{1}{2} = 29^{\circ} 7' 32''$ * $25^{\circ} 10' 0'' \Delta 8^{\circ} 18' 00''$

* salt. $= 25 10$ — 2 16 — 6 17
 Δ 's alt. $= 8 18$ 25 7 59 8 11 43

Diff. $= 16 52$ 9 6 43 + 55 0

$\frac{1}{2}$ Diff. $= 8 26$ 16 1 10 9 6 43

8 0 38 $= \frac{1}{2}$ diff. true alts.

$L, s, 29^{\circ} 7' 32''$ 0, 31 271

$L, s, 8 26 0$ 9, 166 31

$L, s, \text{first arc}$ 9, 479 03

9, 687 28 $L, s, 29^{\circ} 7' 32''$

9, 979 34 $L, s, 1st. arc$

9, 606 62 $L, 2d. arc$

9, 666 62 $L, 2d. arc$

0, 043 32 $L, s, 25^{\circ} 10'$

0, 004 57 $L, s, 8 18$

19, 384 13 $L, 3d. arc$

9, 956 80 $L, s, 25^{\circ} 08'$

9, 994 48 $L, s, 9 07$

39, 332 41 $L, 4th. arc$

19, 606 20 $\frac{1}{2} L 4th. arc$

9, 144 12 $L, s, 8^{\circ} 0' 38''$

10, 522 08 $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

19, 666 20

$L, s, 5th. arc$

9, 981 23

$\frac{1}{2}$ Diff. $28^{\circ} 57' 22''$

9, 604 97

57 54 44 the true distance.

Now here is given the ship time 14h. 15m. 14s. and the Corrected distance between the Moon's center and Aldebaran $57^{\circ} 54' 44''$. Required the ship's longitude.

By the Nautical Almanack: The dist. at 6h is $57^{\circ} 38' 33'' A$ $57^{\circ} 38' 35'' A$
The dist. at 9h. 59 11 43 B 57 54 44 Ship.

Now (l. 40) $9' 8'' = 93.133$; and $16' 9 = 16.11$.

Then (l. 46) As $93.133 : 16.11 :: 3h. 43m. 13s.$ time above A.

Greenwich Time 04 31 13

Ship Time 144 15 14

True diff. longest 74 44

Diff. long by account 74 45

Error

59s. $= 14 \frac{1}{2}$ East.

Exam.

EXAM. III. Suppose on the 13th of April 1772, on board a Ship in lat. 50° N. and long. 50° W. of Greenwich by account, with a watch and instruments well adjusted, the following observations were taken of the Star Pollux and the Moon's nearest limb, then to the east of the Star, the height of the eye being allowed for: Required the Ship's true longitude?

Time 9h. 0m. P. M.; Alt. D's lower pt. $17^{\circ} 15' 0''$; Alt. * $55^{\circ} 0'$; dist. $44^{\circ} 48'$.
 9 7 16 15 30 54 30 45 10
 9 14 15 15 0 53 30 45 44

Sum 48 45 30 Sum 163 30 Sum 135 42
 Mean 9 7 P. M. mean alt. D 16 15 10; * 54 20 45 14

Now Sh. tim. 9h. 7m. P. M. + 3h. 20m. W. = 12h. 27m. P. M. Gr. tim. (57)
 At this time by Almanac the D's semidiam. = $16' 26''$
 and horiz. parallax = $60 18$.

Then $45^{\circ} 14' + 16' 26'' = 45^{\circ} 30' 26''$ the apparent distance.

And $16' 15' 10'' + 16' 26'' = 16^{\circ} 31' 36''$ the alt. of the Moon's center.

Also Rad : $16^{\circ} \frac{1}{2}$:: $60' 18''$: $57' 15''$ the paral. in alt. by the Gunter.

Now dist. $45^{\circ} 30' 26''$; its $\frac{1}{2}$ is $22^{\circ} 45' 13''$ * $54^{\circ} 20' 0''$ D $16^{\circ} 31' 16''$

*'s ap. alt. $54^{\circ} 20' 0''$ Ref. — 41 — 3 11
 D's ap. alt. 16 31 36 * 54 19 19 16 28 25
 37 48 24 D 17 25 40 Par. + 57 15

Diff. 18 54 12 ap. alts. Diff. 36 53 39 17 25 40
 18 26 50 = $\frac{1}{2}$ diff. true alts.
 L's, 22 45 13 9, 58 746 L's, $22^{\circ} 45' 13''$
 L's, 18 54 12

L's, 1st arc 9, 7 747 L's, 1st arc 9, 32 493 L's, 2d arc 9, 32 493
 0, 23 428 L's, 54' 20' 0, 018 12 L's, 16 31' 36" 18, 90 246 L's, 3d arc 9, 7 6584 L's, 54 19 19
 9, 97 959 L's, 17 25 40 38, 64 789 L's, 4th arc 19, 32 394 L's, 4th arc 9, 500 27 L's, 18 26 50
 9, 83 567 L's, 5th arc 9, 83 567

$\frac{1}{2}$ L, 4th arc 19, 32 394

L's, 5th arc 9, 7 4390

$\frac{1}{2}$ Diff. $22^{\circ} 20' 52''$ 9, 58 064

44 41 44 the true distance.

Again, by the Nautical Almanac 1772, April 13th.

The dist. at 12h. is $44^{\circ} 28' 30'' = A$ and $44^{\circ} 28' 30'' = A$ (109)

at 15h. is $46 13 32 = B$ 44 41 44 at Ship.

Diff. 1 45 2 = $105' 2''$ 0 43 14

Now (L. 40) $105' 02'' = 105,033$; and $13' 14'' = 13,233$

Then (L. 46) As $105,033 : 13,233 :: 3h. : oh. 22m. 46s. time above A$

12 10 11 time A

12 22 40 Greenwich time

9 7 0 Ship time

True diff. Longitude 3 15 40

By acc. diff. long. 3 20 0

Error 4 20 = $1^{\circ} 5'$.

EXAM. IV. Suppose in the year 1773, on September 7th, at 8h. 17½ m. forenoon by a watch, the distance of the nearest limbs of the Sun and Moon should be observed to be 107° 39½'; the altitude of the Sun's lower limb being 15° 5' 4", and of the Moon's 8° 41' 11"; and the Ship by account in latitude 51° 17' S. longitude 66° 30' W. from Greenwich: Required the true longitude of that Ship?

Now Sh. time 20h. 17m. P. M. + 4h. 26m. W. = oh. 43½m. P. M. Gr. t. (57)
At this time, by Naut. Alm. the Sun's declin. 5° 52' 29" N.

the Sun's ½ diam. 15 56 } Sum 30' 45"
the Moon's ½ diam. 14 49 }
D horiz. paral. 52 22

Then 107° 39' 45" + 30' 45" = 108° 10' 30" the apparent distance.

And 15 5 4 + 15 56 = 15 21 the ap. alt. ☉'s center.

And 8 41 11 + 14 49 = 8 26 the ap. alt. ☾'s center.

Also Rad.: 8° 56' : 54' 22" : 53' 30" the par. in alt. by G. scale.

First. To correct the watch.

☉ diff. à pole 95 52½
Co-lat. 38 43½

L's, 0,00228
L's, 0,20379

Co-alt. 74 39
Diff. 57 9½

Sum 131 48½; its ½ = 65 54½

L's, 9,96040

Diff. 17 29½; its ½ = 8 44½

L's, 9,18199

28 11½

19,34846

9,67423

56 22 ½ = 3h. 45m. 29f.

Ap. time 8h. 14m. 31f.

Watch t. 8 17 30f.

W: too fast 2 59

A 24

Secondly:

Secondly. To find the true distance.

Ap. dist. $108^{\circ} 10' 30''$; its $\frac{1}{2} 54^{\circ} 5' 15''$	☉'s alt. $15^{\circ} 21'$	Dalt. $8^{\circ} 56' 0''$
☉ ap. alt. $15^{\circ} 21'$	Ref. —	—
D ap. alt. $8^{\circ} 56'$	☉ $15^{\circ} 17' 36''$	$8^{\circ} 50' 9''$
Diff. $6^{\circ} 25'$	$9^{\circ} 44' 31''$	$+ 54' 22''$
$\frac{1}{2}$ Diff. $3^{\circ} 12\frac{1}{2}'$ of ap. alts.	Diff. $5^{\circ} 33' 5''$	$2^{\circ} 46' 32'' = \frac{1}{2}$ diff. tr. alts.

$\frac{1}{2}$ L, s, $54^{\circ} 05\frac{1}{2}'$	0.09156	9.90844	L, s, $54^{\circ} 05\frac{1}{2}'$
L, s, $3^{\circ} 12\frac{1}{2}'$	8.74792		
L, s, 1st arc	8.83948	9.99896	L, s, 1st arc
		9.90740	L, 2d arc
		9.90740	L, 2d arc
		0.01578	L, s, $15^{\circ} 21'$
		0.00530	L, s, $8^{\circ} 56'$
		19.83588	L, 3d arc
		9.98434	L, s, $15^{\circ} 17' 36''$
		9.99399	L, s, $9^{\circ} 44' 31''$
		39.81391	L, 4th arc
$\frac{1}{2}$ L, s, 4th arc	19.90696	19.90696	$\frac{1}{2}$ L, 4th arc
L, s, 5th arc	9.99922	8.68505	L, s, $2^{\circ} 46' 32''$
$\frac{1}{2}$ Diff. $53^{\circ} 57' 40''$	9.90774	11.22191	L, s, 5th arc.
$107^{\circ} 55' 20''$ the true distance.			

Thirdly. To find the longitude by the Nautical Almanac of 1773.

The dist. at noon is $108^{\circ} 16' 56'' = A$	And $108^{\circ} 16' 56'' = A$	(130)
at 3h. $106^{\circ} 55' 22'' = B$	$107^{\circ} 55' 20''$ at Ship.	
$1^{\circ} 21' 24'' = 81^{\circ} 24''$	$21^{\circ} 36''$	

Now (I. 40) $81^{\circ} 24'' = 81,4'$; and $21^{\circ} 36'' = 2,16'$.
 Then (I. 46) As $81,4' : 21,6' :: 3h. : oh. 47m. 45f.$ time above A

24 time A

Greenwich time	24 47 45
Ship tim.	20 14 31
True diff. longit.	4 33 14
Longit. by acc.	4 26
Error in longit.	7 14 = $1^{\circ} 48'$

METHOD XI. *To find the longitude at sea, by an observation of the distance only.*

P R E C E P T S .

- 1st. Find the Greenwich time: And to which, find (57)
 The right ascension and declination of the Sun and Star,
 The right ascension and declination of the Moon,
 And the horary angles between the meridians, of the ship,
 Moon, and Sun or Star.
- 2d. With the Sun or Star's horary angle, the polar distance, and the
 ship's co-latitude, find the zenith distance. (IV. 253)
 With the Moon's horary angle, her polar distance, and the ship's
 co-latitude, find her zenith distance. (IV. 253)
- 3d. These true altitudes are to be altered, by adding the refractions,
 and subtracting parallax (in the Moon's), to have the ap-
 parent altitudes.
- 4th. With the apparent altitudes, the true altitudes, and the ob-
 served distance, find the true distance. (108)
- 5th. To this distance, find the Greenwich time. (109)
 Then the difference between this time and the ship time
 is the difference of longitude.

R E M A R K S .

The right ascension and declination of the stars are found for the year 1764, in art. 312. Book V. To which are to be applied the yearly variations, taken as many times as years since 1764.

The right ascensions and declinations for the Sun are found (in tables V. 309. 310. or) in the Nautical Almanac, for the noon of each day; and for the intermediate times, by proportional parts. (V. 283. 284. 288).

The right ascensions and declinations of the Moon are given in the Nautical Almanac for every noon and midnight; the parts for intermediate times, may be found nearly accurate, by proportional parts.

The proportional parts for right ascensions and declinations are readily found, near enough by table art. 311. Book V.

The horary angles are found by the following precepts.

The ship time, in degrees, added to the Sun's right ascension, in degrees, gives the right ascension of the mid-heaven; rejecting 360 degrees, when the sum is above that number.

The horary angle at the Sun, is equal to the ship time.

The horary angle at a Star, is equal to the difference between the Star's right ascension, and that of the mid-heaven.

The horary angle at the Moon, is equal to the difference between the Moon's right ascension and that of the mid-heaven.

Note. The declination of an object applied to 90 degrees, gives the polar distance of that object.

EXAM.

EXAM. I. September 3d, 1772, Suppose a Ship in latitude 40° N. Longitude by account 17° W. of Greenwich; at 3 h. P. M. the distance of the nearest limbs of the Sun and Moon was observed to be $82^{\circ} 2' 30''$. Required the Ship's true longitude?

Sep. 3. $\odot$'s semidiameter $15' 56''$; $\odot$'s semid. $16' 1''$; hor. paral. $58' 47''$.
Ship time 3 h. om. P. M. + 1 h. 8 m. W = 4 h. 8 m. Greenwich time.

For $\odot$'s right Ascension.

Sep. 3. $\odot$ R. A. 10 h. 51 m. 30 s.

4 10 55 7

24 h : 4 h. 8 m. : 3 m. : 37 s. : om. 37 s. +

10 51 30

$\odot$ R. ascen. : 10 52 7 = $163^{\circ} 1' 45''$.

For $\odot$'s right Ascension

Sep. 3. noon $\odot$ R. A. $239^{\circ} 56'$

mid-n. 247 7

diff. $7^{\circ} 11'$.

12 h : 4 h. 8 m. : $7^{\circ} 11'$: $2^{\circ} 28' 4''$

239 56

242 24

$\odot$'s R. asc.

Ship time

Sun's rt. asc.

R. A. mid-heaven

$\odot$'s R. asc.

$\odot$'s horary angle

Sh. time or $\odot$'s horary angle

Sh. time or $\odot$'s horary angle

For the Sun's altitude

Given $\odot$'s $\angle = 45^{\circ} 0' 0''$; its $\frac{1}{2} = 22^{\circ} 30'$

$\odot$ pol. dist. = $83^{\circ} 46' 46''$ Diff. = $33^{\circ} 46' 46''$

Co-lat. = $50^{\circ} 0' 0''$ diff. = $16^{\circ} 53' 23''$

$\frac{1}{2} \angle \odot 22^{\circ} 30'$ $2 L, s, \left\{ \begin{array}{l} 9, 58, 284 \\ 9, 58, 284 \end{array} \right.$

Pol. dist. $83^{\circ} 46' 46''$ $L, s, 9, 99743$

Co-lat. $50^{\circ} 0'$ $L, s, 9, 88425$

39, 24736

19, 52368

$L, s, 9, 46319$

9, 87767

10, 06049

26 16 11 9, 64601

52 32 22 = $\odot$'s zen. distance.

37 27 38 = Sun's true altitude

For Sun's declination.

$7^{\circ} 17' 03''$ diff. $22' 14''$

6 54 49

24 h : 4 h. 8 m. : $22' 14''$: $3' 49''$ -

7 17 03

$\odot$'s Declin. 7 13 14 N.

For Moon's declination.

$17^{\circ} 7'$ diff. $0^{\circ} 47'$

17 54

12 h : 4 h. 8 m. : $0^{\circ} 47'$: $0^{\circ} 17' 10''$

17 7

$\odot$'s declin. 17 24 S.

17 7

17 24 S.

$45^{\circ} 0' 0'' = 3$ h.

163 1 45

208 1 45

242 24 0

34 22 15 east of meridian.

45 0 0 west of meridian.

For the Moon's altitude.

Given $\odot$'s $\angle = 34^{\circ} 22' 15''$; its $\frac{1}{2} = 17^{\circ} 11' 7''$

$\odot$'s pol. dist. = $107^{\circ} 24'$ Diff. = $57^{\circ} 24'$

Co-lat. = $50^{\circ} 0'$ diff. = $28^{\circ} 42'$

$\frac{1}{2} \angle \odot 17^{\circ} 11' 7''$ $2 L, s, \left\{ \begin{array}{l} 9, 470, 0 \\ 9, 470, 0 \end{array} \right.$

pol. dist. $72^{\circ} 36'$ $L, s, 9, 97966$

Co-lat. $50^{\circ} 0'$ $L, s, 9, 88425$

38, 80491

19, 40246

$L, s, 9, 68144$

9, 66803

9, 72102

32 51 20 = $\odot$'s zen. distance

65 42 40 = $\odot$'s zen. distance

24 17 20 = Moon's true altitude

To find the true distance of the centers of ☉ and ♀.

Now $82^{\circ} 2' 30'' + 31' 57'' = 82^{\circ} 34' 27''$ apparent dist. of centers.
 Dist. $82^{\circ} 34' 27''$; its $\frac{1}{2} = 41^{\circ} 17' 14''$.

☉ alt. $37^{\circ} 27' 38''$ ♀'s alt. $24^{\circ} 17' 20''$
 Refr. $+ \quad \quad \quad + \quad \quad \quad + \quad \quad \quad$
 ap. alt. ☉ $37^{\circ} 28' 53''$ $24^{\circ} 19' 26''$
 ap. alt. ♀ $23^{\circ} 26' 6''$ paral. $— \quad \quad \quad 53' 20''$
 $\frac{14 \quad 2 \quad 47}{7 \quad 1 \quad 23} = \frac{1}{2}$ diff. ap. alts.,

$L, s, 41^{\circ} 17' 14''$ $0, 18057$
 $L, s, 7 \quad 1 \quad 23$ $9, 08731$
 $L, s, 1st. arc$ $9, 26788$

$9, 81943$ $L, s, 41^{\circ} 17' 14''$

$9, 99242$ $L, s, 1st. arc$

$9, 81185$ $L, 2d. arc$

$9, 81185$ $L, 2d. arc$

$0, 10043$ $L, s, 37^{\circ} 28' 53''$

$0, 03739$ $L, s, 23' 26'' 6$

$19, 76152$ $L, 3d. arc$

$9, 89969$ $L, s, 37^{\circ} 27' 38''$

$9, 95975$ $L, s, 24' 17' 20''$

$39, 62096$ $L, 4th. arc$

$19, 81048$ $\frac{1}{2} L, 4th. arc$

$9, 05953$ $L, s, 6' 35' 9''$

$10, 75095$ $L, s, 5th. arc$

$\frac{1}{2} L, 4th. arc$

$19, 81048$

$L, s, 5th. arc$

$9, 99327$

$\frac{1}{2}$ dist.

$41 \quad 02$

$9, 81721$

$82 \quad 04$ the true distance,

To find the Longitude.

Sept. 3d. at 3h. dist. $81^{\circ} 31' 3''$ $31 \quad 31 \quad 3$
 6h. $83 \quad 7 \quad 19$ $82 \quad 4 \quad 0$

$1 \quad 36 \quad 16 = 96' 16''$ $32 \quad 57$

Now (I. 40) $96' 16'' = 96,266'$; and $32' 57'' = 32,95'$

Then (I. 46) $96,266' : 32,95' :: 3h, : 1h, 1m, 36s.$

Greenwich time for true dist.

Ship time for that dist.

Difference of longitude

Diff. lon. by reckoning

Error in reckoning

Or

$15^{\circ} 24'$ W.

$17 \quad 0$

$1 \quad 36$

EXAM.

EXAM. II. 1772, November 15th, at 21^h 0^m P. M. Suppose in a ship in latitude 30° S. and longitude by account 7° E. of Greenwich, the distance of the nearest limbs of the Sun and Moon was observed to be 112° 26' 10". What was the true longitude of that ship?

Ship's time, 21^h 0^m P. M. — 0^h 28^m E. = 20^h 32^m P. M. Greenwich time.
Nov. 15. ☉'s $\frac{1}{2}$ diam. = 16' 14"; ☉'s $\frac{1}{2}$ diam. 15' 34"; hor. parallax. 57' 7".

For the Sun's right ascension.

Nov. 15. ☉ R.A. = 15^h 25^m 22^s Diff. 4^m 8^s
16. 15 29 30 $\frac{1}{2}$

24^h : 20^h 32^m :: 4^m 8^s 3 32+

☉ right ascension

Or 232° 13' 30"

For the Sun's declination.

18° 43' 37" diff. 14' 54"
18 58 31

24^h : 20^h 32^m :: 14' 54" : 12' 43" +

☉ declination

22 18 56 20 S.

For the ☉'s right ascension.

Nov. 15. mid. ☉'s r.a. 18° 17' diff. 6° 38'
16. noon 124 55

12 : 20 32 :: 6° 38' : 5° 40' +

☉'s right asc. 123 57

For the ☉'s declination.

15° 41' diff. 1° 15'
14 26

12 : 20 32 :: 1° 15' : 1° 4' —

☉'s declination

15 41
14 37 N.

Ship-time 315° ☉ = 21 hours.

☉'s right asc. 232 13 30

547 13 30

360

Mid. heav. 187 13 30

☉'s right asc. 123 57

☉'s horar. ang. 63 14 30

☉'s horar. ang. 45 0

For the Sun's altitude.

Given ☉'s $\angle$ = 45° ☉'s $\frac{1}{2}$ = 22° 30'.

☉ pol. dist. = 71 34 ☉ Diff 11 3 40

Co-latitude = 60 0 ☉ diff. 5 31 50

$\frac{1}{2}$ $\angle$ ☉ 22 20

Pol. dif. 71 3 40

Co-lat. 60 0

$\frac{1}{2}$ diff. 5 31 50

19,53952

9,89380

26 15

52 30 = ☉'s zen. distance.

37 30 = ☉'s true altitude.

For the Moon's altitude.

G. ☉'s $\angle$ = 63° 16' 30"; its $\frac{1}{2}$ = 31 38 15

☉'s pol. dist. = 104 37 Diff. 44 37

Co-lat. = 60 0 ☉ diff. 22 18 $\frac{1}{2}$

$\frac{1}{2}$ $\angle$ ☉ 31 38 $\frac{1}{2}$ 22.55

Pol. dif. 104 37

Co-lat. 60 0

$\frac{1}{2}$ diff. 22 18 $\frac{1}{2}$

19,68140

9,89458

37 44 38

75 29 = ☉'s zenith distance.

14 31 = ☉'s true altitude.

MAXI

T₀

iii

22

22

3

EXAM. III. November 9, 1769, a ship in latitude $24^{\circ} 15' N$. longitude $47^{\circ} 14' W$. of Greenwich; observed the distance of the star *Aquila* to be $58^{\circ} 35'$ from the Moon's near limb, at 6 b. 31 m. 4 s. apparent time: Required the ship's true longitude?

Now $6^h 31^m 4^s P. M.$ + $3^h 8^m 56^s W = 9^h 40^m P. M.$ at Greenwich.

For the Sun's right ascension.

Nov. 9. $\odot R. A. 14^h 59^m 47^s$ diff. $= 4^m 3^s$.

Then $24:9:40::4:3:138+$

14 59 47

15 1 25 = $225^{\circ} 41'$.

For * *Aquila's* right ascension.

1764. * *R. A.* $19^h 39^m 16^s$

5 years variat. + 12

1769 * *R. A.* $19^h 39^m 28^s = 294^s 2'$

For * *Aquila's* declination.

1764 *'s declination $8^{\circ} 15' 41''$

5 years variation. + 42

1769. *'s declination $8^{\circ} 16' 23'' N$.

For D's right ascension.

Nov. 9. n. D *R. A.* $34^h 8^m 16^s$ diff. $= 6^m 15^s$

9. midn. 354 31

Now $12^h:9^h 40^s::6^m 15^s:5^m 6^s +$

348 16

D's *R. A.* $353^m 22^s$

For D's declination.

Nov. 9. n. D's decl. $0^{\circ} 38' N$. diff. $2^{\circ} 36'$.

9. midn. 3 14

$12^h:9^h 40^s::2^{\circ} 36':2^{\circ} 0' +$

0 38

D's declination $2^{\circ} 38' N$.

Ship-time

$97^{\circ} 46' = 6^h 31^m 4^s$.

Sun's right ascension $225^m 21^s$

R. A. mid-heaven $323^m 7^s$

Star's *R. A.* $294^m 52^s$

D west of sh. meridian $28^m 15^s$

To find the Star's altitude.

Given h. angl. $28^{\circ} 15'$; its $\frac{1}{2} = 14^{\circ} 7\frac{1}{2}'$

*'s pol. dist. $81^{\circ} 43' 37''$ diff. $15^{\circ} 48' 37''$

Co-latitude $65^{\circ} 45'$ } $\frac{1}{2}$ 7 59 18

$\frac{1}{2}$ pol. $\angle = 14^{\circ} 7\frac{1}{2}'$ 2 L. $\frac{1}{2}$ { $9:38^m 74^s$ 6

81 43 37 L. $\frac{1}{2}$ { $9:38^m 74^s$ 6

65 45 L. $\frac{1}{2}$ { $9:38^m 74^s$ 6

19, 365 13

9, 933 33

10, 222 21

15 41 9, 43 180

Zen. dist. $31^{\circ} 22'$ and, $58^{\circ} 38' =$ *'s t. alt

To find the Moon's altitude.

Given h. angl. $30^{\circ} 15'$; its $\frac{1}{2} = 15^{\circ} 7\frac{1}{2}'$

D's pol. dist. $87^{\circ} 22'$ diff. $21^{\circ} 37'$

Co-latitude $65^{\circ} 45'$ } $\frac{1}{2}$ 10 48

$\frac{1}{2}$ pol. $\angle = 15^{\circ} 7\frac{1}{2}'$ 2 L. $\frac{1}{2}$ { $9:41^m 69^s$

87 22 L. $\frac{1}{2}$ { $9:41^m 69^s$

65 45 L. $\frac{1}{2}$ { $9:41^m 69^s$

19, 365 13

9, 933 33

10, 222 21

15 41 9, 43 180

Zen. dist. $31^{\circ} 22'$ and, $58^{\circ} 38' =$ *'s t. alt

To find the true distance between *Aquila* and *D*'s center.

Nov. 9. 1769. D $\frac{1}{2}$ diam. = $15' 52''$. Dist. of centers $58^{\circ} 51' 37''$.
Hor. paral. $58' 12''$. Then Rad. : S , $53^{\circ} 2' : 58' 12'' : 34' 30''$, by Gunter.

Dist. $58^{\circ} 51' 37''$; its $\frac{1}{2}$ $29^{\circ} 25' 48''$	$\star$'s alt. $58^{\circ} 38' 0''$	D 's $53^{\circ} 38' 0''$
$\odot$'s alt. $58' 38''$	Refr. + $0' 35''$	+ $0' 42''$
D 's alt. $53' 38''$	$\star$ $58' 38' 35''$	$53' 38' 42''$
Diff. $5' 0''$	D $53' 412''$	— $34' 30''$
$\frac{1}{2}$ diff. $2' 30''$ of true alts.	$5' 34' 23''$	$53' 4' 12''$
	$\frac{1}{2}$ diff. $2' 47' 12''$ app. alts.	

L 's $29^{\circ} 25' 48''$ 0.30860 9.69140 L 's $29^{\circ} 25' 48''$.
 L 's $2' 47' 12''$ 8.68679

L 's 1st arc 8.99539 9.99786

9.68926 L , 2d arc
 9.68926 L , 2d arc
 0.28370 L 's $58' 38' 35''$
 0.22125 L 's $53' 4' 12''$
 19.88347 L , 3d arc
 9.71643 L 's $58' 38''$
 9.77302 L 's $53' 38''$

39.37392 L , 4th arc

19.68646 $\frac{1}{2}$ L , 4th arc
 8.63938 L 's $2' 30''$

L 's 5th arc 11.04708 L 's 5th arc

$29' 11' 40''$ 9.68821

$58' 23' 20''$ the true dist. of $\star$ from D 's center.

To find the Longitude.

Nov. 9. at 9^h P. M. $\star$ à D $58' 1' 37''$ $58' 1' 37''$
 12^h $59' 29' 51''$ $58' 23' 20''$

$1' 28' 14'' = 68' 14''$ $21' 43''$

Now, $(L. 40) 1^h 28' 14'' = 88.23'$; and, $21' 43'' = 21.716$.
And, $(L. 46) 88.23 : 21.716 : 3^h : 0^h 43^m 32^s$

Apparent time at Greenwich
Ship

Difference of times
Longitude by account

Error

$3' 12' 28'' = 48^{\circ} 7'$ diff. longitude.

$3' 8' 56''$

$3.32 = 0^{\circ} 53'$

EXAM.

EXAM. IV. Suppose a Ship in latitude 20° N. and by account in longitude 28° W. of Greenwich, should on December 30th, at 7h. P. M. apparent time, find, by observation, the distance of the star Aldebaran to be $69^{\circ} 1' 30''$ from the Moon's farther limb: Required the Ship's true longitude?

Dec. 30. the Moon's semidiam. $= 14' 56''$; and hor. parallax $= 54' 49''$.

Ship time 7h. om. P. M. + 1h. 32m. W. = 8h. 52m. P. M. at Greenwich.

For the Sun's R. Ascension.

Dec. 30. $\odot$ R. A. $18^{\circ} 41' 14''$ diff. 4 26. Then 24h. : 8h. 52m. :: 4 26 : 1 38 + 31.

$18^{\circ} 45' 40''$

$18^{\circ} 41' 14''$

For * Aldebaran's r. ascen.

1764 Al. R. A. = 4h. 22 24

8 years variat. + 27

*'s R. Ascen. $4^{\circ} 22' 51'' = 65^{\circ} 42' 45''$

For $\odot$'s r. ascension.

Dec. 30. noon $353^{\circ} 48'$ diff. $5^{\circ} 43'$

Mid-n. $359^{\circ} 31'$

12h. : 8h. 52m. :: $5^{\circ} 43' : 4^{\circ} 14' +$

$353^{\circ} 48'$

$\odot$'s r. asc.

$358^{\circ} 2'$

Ship time $10^{\circ} 0' = 7^{\circ}$

$\odot$ r. ascen. $280^{\circ} 43' = 18^{\circ}$

rt. asc. mid-h. $385^{\circ} 43'$

*'s rt. asc. $65^{\circ} 42' 54'$

$320^{\circ} 0' 15''$

360

* E. of merid. $39^{\circ} 59' 45''$

For the Star's altitude.

Given * $\angle = 39^{\circ} 59' 45''$; its $\frac{1}{2} 19^{\circ} 59' 53''$

* po. dist. $= 73^{\circ} 57' 53''$ diff. $3^{\circ} 57' 53''$

Co-latit. $= 70^{\circ} 0' 0''$ diff. $1^{\circ} 58' 56''$

$\frac{1}{2} \angle$ * $19^{\circ} 59' 53''$ } $9^{\circ} 53401$

pol. dist. $73^{\circ} 57' 53''$ } $9^{\circ} 53401$

Co-lat. $70^{\circ} 0' 0''$ } $9^{\circ} 98277$

$39^{\circ} 02378$

19,51189

diff. 1 58 56

9,99755

$19^{\circ} 4' 40''$ 9,51434

$38^{\circ} 9' 20''$ *'s zenith dist.

$51^{\circ} 50' 40''$ *'s true altitude.

For * Aldeb. declination.

1764 declin. $16^{\circ} 1' 0''$ N.

8 y. variat. + 1 7

*'s Decl. $16^{\circ} 2' 7''$ N.

For $\odot$'s declination.

$0^{\circ} 39'$ S. diff. $1^{\circ} 56'$

$1^{\circ} 17'$ N

12h. : 8h. 52m. :: $1^{\circ} 56' : 1^{\circ} 30'$

$0^{\circ} 39'$

$\odot$'s declination.

$0^{\circ} 51'$ N.

For the Moon's altitude.

Given $\odot$'s $\angle = 27^{\circ} 41'$; its $\frac{1}{2} 13^{\circ} 51'$

$\odot$'s pol. dist. $= 89^{\circ}$ diff. 19°

Co-latit. $= 70^{\circ} 0'$ diff. $9^{\circ} 34\frac{1}{2}$

$\frac{1}{2} \angle \odot$ $13^{\circ} 51'$ } $9^{\circ} 37909$

pol. dist. 89° } $9^{\circ} 37909$

Co-latit. $70^{\circ} 0'$ } $9^{\circ} 99995$

$38^{\circ} 73112$

19,36556

diff. 9 34 $\frac{1}{2}$

9,90996

$16^{\circ} 35' 20''$ 9,45560

$33^{\circ} 10' 40''$ $\odot$'s zenith dist.

$56^{\circ} 49' 20''$ $\odot$'s true altitude.

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To find the true distance between *Aldebaran* and *D's* center.

Now $69^{\circ} 1' 30'' - 14' 56'' = 68^{\circ} 46' 34'' = \text{ap. dist. of their centers.}$

ap. Dist. * $68^{\circ} 46' 34''$; its $\frac{1}{2} = 34^{\circ} 23' 17''$
 Alt. * $51^{\circ} 50' 40''$ alt. D $56^{\circ} 49' 20''$
 Refr. + $46''$ + $37''$
 ap. alt. * $51^{\circ} 51' 26''$ $56^{\circ} 49' 57''$
 ap. alt. D $56^{\circ} 20' 10''$ Par. — $29' 47''$
 $4^{\circ} 28' 44''$ $56^{\circ} 20' 10''$
 $2^{\circ} 14' 22'' = \frac{1}{2}$ diff. ap. alts.

D 's true alt. $56^{\circ} 49' 20''$
 *'s true alt. $51^{\circ} 50' 40''$
 Diff. $4^{\circ} 58' 40''$
 $\frac{1}{2}$ diff. true alts. $2^{\circ} 29' 20''$

$L, s, 34^{\circ} 23' 17''$ $0, 24810$
 $L, s, 2^{\circ} 14' 22''$ $8, 59200$
 $L, s, 1st \text{ arc}$ $8, 84010$

$9, 75190$ $L, s, 34^{\circ} 23' 17''$

$9, 99896$ $L, s, 1st \text{ arc}$
 $9, 75086$ $L, s, 2d \text{ arc}$
 $9, 75086$ $L, s, 2d \text{ arc}$
 $0, 20928$ $L, s, 51^{\circ} 51' 26''$
 $0, 25024$ $L, s, 56^{\circ} 20' 10''$
 $19, 96724$ $L, 3d \text{ arc}$
 $9, 73818$ $L, s, 56^{\circ} 49' 20''$
 $9, 79085$ $L, s, 51^{\circ} 50' 40''$
 $39, 49627$ $L, 4th \text{ arc}$
 $19, 74813$ $\frac{1}{2} L, 4th \text{ arc}$
 $8, 63775$ $L, s, 2^{\circ} 29' 20''$
 $11, 11038$ $L, s, 5th \text{ arc.}$

$\frac{1}{2} L, 4th \text{ arc}$

$19, 74813$
 $8, 59200$
 $8, 84010$

$L, s, 5th \text{ arc}$

$9, 99870$
 $9, 74943$

$34^{\circ} 10'$
 $68^{\circ} 20'$ the true distance.

To find the Longitude.

Dec. 30. at 6h. dist. $69^{\circ} 49' 54''$
 gh. $68^{\circ} 19' 39''$

$69^{\circ} 49' 54''$
 $68^{\circ} 20' 0''$

$1^{\circ} 30' 15'' = 90' 15''$

$1^{\circ} 29' 54'' = 89' 54''$

Now (L. 40) $90' 15'' = 90, 25'$; and $89' 54'' = 89, 9$
 Then (L. 46) $90, 25' : 89, 9 :: 3h. : 2h. 59' 16''$
 6h.

Greenwich time for true dist.

$8^{\circ} 59' 16''$

Ship time for that distance

$7^{\circ} 0' 0''$

Difference of Longitude

$1^{\circ} 59' 16''$

Diff. by dead reckoning

Or

$29^{\circ} 49'$

Error in reckoning

$1^{\circ} 49'$

SECTION XI.

Of a Ship's reckoning.

113. A SHIP'S reckoning is that account, whereby at any time it may be known where the ship is, and on what course or courses she is to steer to gain her port.

DEAD RECKONING, is that account deduced from the occurrences wrote on the log-board,

The LOG-BOARD is a painted black board, whereon is wrote in chalk whatever is thought worthy of note from day to day; and for a proper disposition of such notes, the board is usually divided into five columns: The left-hand one contains the 24 hours from the noon of one day to the noon of the next; in the second and third columns are the knots and fathoms the ship is found to run per hour, set against the hours when the log was hove; the fourth column contains the courses the ship steers; and in the fifth, or right-hand column, is wrote the winds, the alteration of the sails, the business doing aboard, and what other remarks the officer of the watch thinks proper to insert: For it should be observed, that it is usual to divide a ship's company into two parts, called the starboard and larboard watches, who alternately do the duty of the ship for four hours and four hours.

The LOG-BOOK is a book ruled like the log-board, in order to contain the daily copies of the remarks wrote on the log-board, which is the only authentic record of the ship's transactions; and these are by the persons who keep *Journals* transcribed every day at noon into their log-books, from whence they make the necessary deductions relative to the ship's place.

A SEA JOURNAL is a book wherein is truly entered the most remarkable daily occurrences relating to the ship during her voyage outward and homeward.

There are various ways of keeping sea journals, according to the different notions of mariners concerning what articles are to be entered.

Some writers direct the keeping such a kind of journal, which is only an abstract of each day's transactions, specifying the weather, what ship's or lands were seen, accidents on board, the latitude, longitude, meridional distance, course, and run: These particulars are to be drawn from the ship's log-book, or that kept by the person himself, for most good mariners keep private log-books.

Other authors recommend the keeping but of one account, including the log-book and all the work of each day, with the deductions drawn therefrom: This method is used in this treatise, on account of representing to the beginner the whole of each day's work: but when he is well versed in this method, he may abridge what part of it he pleases: Although it is conceived, that a journal neatly kept, with all the particulars of the work commodiously ranged, would give more satisfaction to those who might have occasion afterwards to suspect it, than a journal containing the heads only, and all the intermediate parts suppressed.

The method here chose to introduce the young mariner into this capital part of navigation, is, by shewing him first how to work a few
separate

separate days works independent one of the other, and then to proceed to a continued journal of several successive days works; for the doing of which it was judged necessary to premise the following observations.

114. I. Mariners reckon time by the civil account used on shore: But they keep the reckoning for the Ship's place in the astronomical manner; beginning at noon and counting thence 24 hours to the next noon: The first 12 hours, from noon to mid-night, they mark with P. M. signifying after mid-day; and the second 12 hours from mid-night to noon, they mark with A. M. signifying after midnight; ending their days work on the noon of the neminal day; whereby their ship account is 12 hours earlier than their shore account of time; and consequently the Sun's declination, and other daily astronomical matters fitted for noon, must be sought for on the day according to their account of time.

Thus, a day's account marked Sunday the 9th of October, began on Saturday noon, and ends on Sunday noon; so the Sun's declination for the 9th of October is used for the noon of Sunday.

II. If there is an observation for an amplitude or an azimuth; let that amplitude or azimuth be worked as shewn between article 22, and 28; and then find the variation as shewn at article 29; taking care in these operations, that the declination of the day be fitted to the proper time, and longitude, as shewn in Book V. article 286.

III. Correct each course by the variation thus found, as at article 30; also correct these corrected courses by the proper allowance for leeway, as shewn at articles, 31, 32.

IV. Write these corrected courses in a Traverse Table, sum up the knots and fathoms, allowing ten fathoms to a knot, belonging to those hours the ship kept on the same course, and write the several sums as distances in the Traverse Table against their respective courses.

V. Complete the Traverse Table as shewn in Book VII. 36, find the present latitude and longitude as shewn in articles 72, 73, 74. Book VIII. and the following examples, and then will the ship's place be obtained by dead reckoning.

VI. The present longitude each day is found by middle latitude; but the bearings and distances are found by Mercator's sailing; the one being rather preferable, for its ease of working, in the former case, the other in the latter.

VII. As the greatest trouble in using the Traverse Table is the finding therein a corresponding difference of latitude and departure, or meridional difference of latitude and difference of longitude, nearly equal to two given numbers, it may perhaps be found most convenient, as the Traverse Table to the points of the compass, VII. 66, lies in two openings, or folios, first to find the nearest corresponding numbers in one of those folios, marking what course they stand with, and then to turn to the degrees nearest that course in the large Traverse Table, VII. 67, and near that place the given numbers will be found more accurate with but little trouble. Thus to find the course to 672 meridional difference of latitude, and 353 difference of long. In the Traverse Table, page 85 the nearest numbers 67,05 and 35,82 stand with $2\frac{1}{2}$ points, or about 28 degrees. Then turning to 28 degrees, and not finding them there, turn to 27° , and the numbers 67,26 and 35,39 answer to $27^{\circ} 45'$, the course.

115. EXAM. I. August 25th, 1772. Yesterday noon we were in the latitude of $46^{\circ} 28' N$, longitude $22^{\circ} 18' W$, and have sailed till this day noon as by the log-board, the tide having all the time set S. b. E. $2\frac{1}{2}$ miles an hour: Required the ship's place, and the direct course and distance made good?

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Courses	Dif	N.	S.	E.	W.
1	0	3	NNE.	West	NNE.	31	28,6		11,9	
2	6	2			ENE.	35	13,4		32,3	
3	6	5			E. b. S.	36		7,0	35,3	
4	6	4			SSE.	51		47,1	19,5	
5	6	0			S. b. E.	60		58,8	11,7	
6	6	1	ENE.	NW.			42,0	112,9	110,7	Dep.
7	6	6						42,0		
8	5	8					D. lat.	70,9		
9	5	6								
10	5	4								
11	5	5								
12	5	3	E. b. S.	North						
1	5	9								
2	6	2								
3	6	0								
4	6	3								
5	6	4								
6	7	0	SSE.							
7	6	8								
8	7	3								
9	7	5								
10	7	1								
11	7	9								
12	7	3								

Depart. lat.	= $46^{\circ} 28' N$.	M. P. = 3156
Diff. lat.	= $1^{\circ} 11' S$.	
Pres. lat.	= $45^{\circ} 17' N$.	M. P. = 3054
Sum lats.	= $91^{\circ} 45'$	M. D. lat. = 102
Middle lat.	= $45^{\circ} 52'$	
Co-middle lat.	= $44^{\circ} 08'$	
Dep. longitude	= $22^{\circ} 18' W$.	
Difference long.	= $2^{\circ} 39' E$.	Or $2^{\circ} 40'$
Present long.	= $19^{\circ} 39' W$.	
Direct course	= $S. 57^{\circ} 15' E$.	
Distance	= 131 miles.	

The courses and winds in the log-board being examined, it appears that the ship goes large, and has no leeway; therefore the several courses from the log-board are entered in the Traverse Table without alteration.

Next, the fathoms and knots belonging to each course are summed up, and the results are put in the column of distances in the Traverse Table; and to these courses and distances the whole difference of latitude and departure are found.

Then the present lat. is found by VI. 51; the co-mid lat. by VIII. 45; the present long. by VIII. 64; and the course and distance by VII. 33.

Or by Mercator's. Having found the present lat, find the meridional diff. lat. = 102 by VIII. 52; and the course and distance by VII. 33.

Then the diff. long. being found by VIII. 65, will be $2^{\circ} 40'$, differing by one mile from that found by middle latitude.

116. EXAM. II. May 24, 1771. Being yesterday noon in the latitude of $49^{\circ} 27' N.$, and longitude $16^{\circ} 02' W.$, we have sailed till this day noon, as by the log-board, in a current setting $N.W.$ $2\frac{1}{4}$ miles an hour: Required the ship's place, together with her direct course and distance sailed?

LOG-BOARD.			Winds.
H.	K.	F.	
1	5	2	SW.bW
2	5	3	NW.bW
3	5	2	
4	4	4	
5	4	3	
6	4	2	
7	5	5	SE. b. S.
8	5	1	SW.b.S.
9	6	4	
10	6	2	
11	7	2	
12	6	3	
1	6	2	
2	7	4	NE.
3	7	4	ESE.
4	7	6	
5	6	4	
6	6	3	
7	6	5	
8	5	2	NNE.
9	5	3	East
10	5	4	
11	5	5	
12	4	2	

TRAVERSE TABLE.					
Courses	Dif.	N.	S.	E.	W.
S.W.	29		10,5		20,5
S.E.	43		30,4	30,4	
NE.b.N	42	34,9		23,3	
N.b.E.	26	25,5		5,1	
N.W.	54	38,2			38,2
		98,6	50,9	58,8	58,7
		50,9		58,7	
		47,7		0,1	Dep.

Dep. latitude = $49^{\circ} 27' N.$ Mer. pts. = 3423
 Diff. latitude = $0^{\circ} 48' N.$

Prof. latitude = $50^{\circ} 15' N.$ Mer. pts. = 3498

Sum latitudes = $99^{\circ} 42'$ M.D. lat. = 75°

Middle lat. = $49^{\circ} 51'$

Co.mid. lat. = $40^{\circ} 09'$

Dep. long. = $16^{\circ} 02' W.$

Diff. long. = $00^{\circ} 00' W.$

Prof. long. = $16^{\circ} 02' W.$

Course is due north,
 Distance 48 miles.

As each course on the log-board is 6 points distant from the wind, therefore one point leeway is allowed to each, and the courses thus corrected (art. 32) are put in the column of courses in the Traverse Table.

Then the Traverse Table is completed to the distances belonging to the respective courses, and the whole diff. lat. is found to be $47,7$ miles of north, and but $\frac{1}{2}$ th part of a mile of departure; therefore it may be concluded, that the ship would have got to her present place by a course due north, and consequently she has not at all altered her longitude.

In these two examples, and the following ones, it was not thought necessary to take any more than one fractional place in the several differences of latitudes and departures, although the Traverse Table gives two such places.

B b 3

114. EXAM.

117. EXAM. III. June 27, 1771. Being yesterday noon in latitude $25^{\circ} 30' S.$, and longitude $10^{\circ} 15' E.$, we have sailed till this day noon, as per log board, in a current setting S. $2\frac{1}{2}$ miles an hour, the variation $1\frac{1}{2}$ point west: Required the ship's place?

LOG-BOARD.				TRAVERSE TABLE.						
H.	K.	F.	Courses	Winds	Courses	Dif.	N.	S.	E.	W.
1	6	0	SW.	WNW.	S.b.W $\frac{1}{2}$ W	30		28,7		8,7
2	6	2			S.b.E. $\frac{1}{2}$ E.	32		30,6	9,3	
3	6	4			S $\frac{1}{2}$ E.	30		29,9	2,9	
4	6	0			SE.bE $\frac{1}{2}$ E.	39		18,4	34,4	
5	5	3			S.b.E. $\frac{1}{2}$ E.	60		57,4	17,4	
6	6	0	S, b.W.	W.b.S.			D. lat.	165,0	64,0	8,7
7	5	1							8,7	
8	5	4							55,3	Dep.

Diff. lat. = $2^{\circ} 45' S.$

Dep. lat. = $25^{\circ} 30' S.$ Merid. pts. 1583

Present lat. = $28^{\circ} 15' S.$ Merid. pts. 1768

Sum of lats. = $53^{\circ} 45'$ M. diff. lat. 185

Mid. lat. = $26^{\circ} 52'$

Co-mid. lat. = $63^{\circ} 08'$

Dep long. = $10^{\circ} 15' E.$

Diff. long. = $1^{\circ} 02' E.$ Or, $1^{\circ} 1\frac{1}{2}' E.$

Present long. = $11^{\circ} 17' E.$

Course is S. $18^{\circ} 30' E.$

Distance is 174 miles.

The courses and winds in the log-board being examined, it appears that the ship is close hauled on each tack, and one point leeway being allowed (32) reduces the course to SW. b. S., S. b. W., SE., SSE., and take a course for the current S.; these several courses being corrected by art. 30 for the variation $1\frac{1}{2}$ point west, give those in the Traverse Table; to which the whole difference of latitude and departure is to be found as before.

And hence the present latitude and longitude may be found either by middle latitude, or Mercator's sailing; For as the ship is so near the equator, the difference will be almost insensible.

If a greater accuracy is required, which is very rarely wanted, especially in such small latitudes, the diff. of long. may be found to each course, as shewn in Book VIII. 77, 78, and thence the whole diff. long. may be collected; but there will be no material difference gained by such operation.

115. EXAM.

I.
8,7
3,7
P.P.

118. EXAM. IV. Yesterday noon we were in lat. $14^{\circ}40'N$, long. $57^{\circ}25'W$, we have sailed till this day noon, as by the log-board, in a current setting $WSW \frac{1}{2} W$, $\frac{1}{2}$ mile an hour: the variation was north $\frac{1}{2}$ point west: Required the ship's place, and her course and distance to the island of Barbadoes?

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Course	Dif.	N.	S.	E.	W.
1	5	0	SW $\frac{3}{4}$ W.	WNW $\frac{1}{2}$ W.	SSW.	21		19,4		8,0
2	5	2			SW.	52		36,8		36,8
3	5	2			S.b.W $\frac{1}{2}$ W.	41		39,2		11,9
4	6	0			SE. b.S	44		16,6	24,5	
5	6	3	SW $\frac{1}{2}$ W.	NW. $\frac{1}{2}$ W.	W.b.S.	12		4,6		11,1
6	6	4					D lat.	136,6	24,5	67,8
7	6	3								24,5
8	7	0								43,3
9	7	3							Dep.	

Diff. latitude = $2^{\circ}17'S$.
Departed latitude = $14^{\circ}40'N$.

Present latitude = $12^{\circ}23'N$.

Sum of latitudes = $27^{\circ}03'$

Middle latitude = $13^{\circ}31'$

Co-middle latitude = $76^{\circ}29'$

Departed longitude = $87^{\circ}25'W$.

Difference of longitude = $0^{\circ}45'W$.

Present longitude = $58^{\circ}10'W$.

Here the ship's place is found; namely, in lat. $12^{\circ}23'N$, and long. $58^{\circ}10'W$.

Now to find her course and distance to Barbadoes.

Barbadoes's latitude = $13^{\circ}00'N$. Barbadoes's long. = $59^{\circ}50'W$.
Present latitude = $12^{\circ}23'N$. Present longitude = $58^{\circ}10'W$.

Difference of latitude = $0^{\circ}37'N$.

Sum of latitudes = $25^{\circ}33'$

Middle latitude = $12^{\circ}41'$

Co-mid. lat. = $77^{\circ}19'$

Diff. longitude = $1^{\circ}40' = 100m$.

The departure = $97,5$.

The course is $N. 69^{\circ}15'W$.

The distance 104 miles.

In this example, the manner of correcting the courses and finding the ship's place is the same as in the preceding ones.

But the course and distance she has still to go to arrive at her port, is found by the Traverse Table, according to the directions, VIII. 56. Or, when the latitudes concerned are above 40° , the operation may be best performed by the directions for the use of the Traverse Table, in VIII. 57.

B b 4

116. EXAM.

119. EXAM. V. Yesterday noon we were in lat. $33^{\circ}40' N.$, long. $16^{\circ}20' W.$, the Sun was observed to set $50^{\circ}18'$ from the north point of the compass; we have sailed till this day noon, as per log-board, in a current setting WSW. $1\frac{1}{2}$ mile an hour: Required the ship's place, and her course and distance to the west end of the Island of Madeira, June 4th, 1772?

LOG-BOARD.			Winds	
H.	K.	F.	Courses	
1	6	2	S. b. W.	West
2	6	0		
3	6	3		
4	7	0		
5	7	2		
6	7	3		
7	7	2	SW b. S.	W. b. N.
8	7	2		
9	7	4		
10	7	6		
11	7	4		
12	8	1		
1	8	0		
2	8	5		
3	8	2		
4	7	5	SW b. W.	NW.
5	7	3		
6	6	6		
7	6	4		
8	6	0		
9	6	2		
10	6	1		
11	6	3		
12	6	1		

LOG-BOARD.

TRAVERSE TABLE.

Courses	Dist.	N.	S.	E.	W.
S. $01^{\circ} E$	40		40.0	0.7	
S. $10^{\circ} W$.	70		68.9		12.2
S. $44^{\circ} W$.	58		41.7		40.3
S. $55^{\circ} W$.	36		20.6		29.5
		D. lat.	171.2	0.7	82.0
				Dep.	81.3

Before the courses can be corrected to put in the Traverse Table, the variation of the compass must be found from the Sun's true amplitude; which may be found by article 22, or by working the proportion in article 23, either by logarithms or the Gunter.

June 4th, 1772, the declination is $22^{\circ}32' N.$

As cos. $33^{\circ}40' : \text{Rad.} :: \sin. 22^{\circ}32' : \sin. 27^{\circ}22'.$

So that the true amplitude = $62^{\circ}38' W.$
Mag. amplitude = $50^{\circ}18'$

(19)

Variation

= $12^{\circ}20'$

The courses in the log-board being corrected by this variation and the leeway, will give the courses fitted for the Traverse Table.

Depart. latitude	= $33^{\circ}40' N.$
Diff. latitude	= $2^{\circ}51' S.$
Present latitude	= $30^{\circ}49' N.$
Sum of latitudes	= $64^{\circ}29'$
Middle latitude	= $32^{\circ}14'$
Co-middle latitude	= $57^{\circ}46'$
Depart. longitude	= $16^{\circ}20' W.$
Diff. longitude	= $1^{\circ}36' W.$
Present longitude	= $17^{\circ}56' W.$

Depart. latitude = $33^{\circ}40' N.$
Diff. latitude = $2^{\circ}51' S.$
Present latitude = $30^{\circ}49' N.$

Sum of latitudes = $64^{\circ}29'$
Middle latitude = $32^{\circ}14'$

Co-middle latitude = $57^{\circ}46'$

Depart. longitude = $16^{\circ}20' W.$

Diff. longitude = $1^{\circ}36' W.$

Present longitude = $17^{\circ}56' W.$

In the work for the amplitude, the lat. at Sun-set was taken the same as at noon; for although there were about 46 m. of southing in that time, and so the lat. at Sun-set was about $32^{\circ}54'$, yet the amplitude being only about 15 less, the alteration in variation would scarcely affect the diff. lat. and dep. found from the courses to be corrected.

In the work for the amplitude, the lat. at Sun-set was taken the same as at noon; for although there were about 46 m. of southing in that time, and so the lat. at Sun-set was about $32^{\circ}54'$, yet the amplitude being only about 15 less, the alteration in variation would scarcely affect the diff. lat. and dep. found from the courses to be corrected.

Madeira's lat.	= $32^{\circ}26' N.$
Present latitude	= $30^{\circ}49' N.$
Diff. latitude	= $1^{\circ}37'$
Sum of latitudes	= $63^{\circ}15'$
Middle latitude	= $31^{\circ}37'$
Co-mid. latitude	= $38^{\circ}23'$
Madeira's long.	= $17^{\circ}21' W.$
Present long.	= $17^{\circ}56' W.$
Diff. longitude	= $0^{\circ}35' W.$
The course (57)	= $N. 17^{\circ}15' E.$
The distance	= 101 miles.

M. P.	
Madeira's lat.	= $32^{\circ}26' N.$
Present latitude	= $30^{\circ}49' N.$
Diff. latitude	= $1^{\circ}37'$
Sum of latitudes	= $63^{\circ}15'$
Middle latitude	= $31^{\circ}37'$
Co-mid. latitude	= $38^{\circ}23'$
Madeira's long.	= $17^{\circ}21' W.$
Present long.	= $17^{\circ}56' W.$
Diff. longitude	= $0^{\circ}35' W.$
The course (57)	= $N. 17^{\circ}15' E.$
The distance	= 101 miles.

120. EXAM. VI. Being yesterday noon in lat. $19^{\circ}30'S$, long $0^{\circ}10'E$; this forenoon we observed the Sun's altitude to be $10^{\circ}40'$, when he was 80° from the north point of the compass, and his declination $17^{\circ}27'N$; we have sailed till this day noon, as per log-board, in a current setting by the compass WNW . $\frac{1}{2}$ mile an hour: Required the ship's place, with her direct course and distance to the Island of St. Helena?

LOG-BOARD.			
H. K.	F.	Courses	Winds.
1	6	7	N. b. E. N. b. E
2	0	2	
3	6	4	
4	6	3	
5	0	1	
6	6	0	
7	5	8	North
8	5	4	
9	5	0	
10	5	3	
11	5	6	
12	5	9	
1	5	7	
2	6	4	NNW.
3	6	8	
4	7	0	
5	7	3	
6	7	6	
7	7	5	
8	7	0	
9	7	2	
10	7	4	
11	6	3	
12	6	0	

TRAVERSE TABLE.

Courses.	Dif.	N.	S.	E.	W.
N. $13^{\circ}30'W$.	38	36.9			8.9
N. $24^{\circ}45'W$.	38	34.5			15.9
N. $47^{\circ}15'W$.	70	51.7			55.6
N. $81^{\circ}00'W$.	12	1.9			11.8
		125.0			92.2
		D. lat.			Dep.

Before the courses can be corrected, an azimuth must be worked as in articles 26, 27.

Decl.	$17^{\circ}27'$	Lat.	$19^{\circ}30'$	Alt.	$10^{\circ}40'$
A	$= 107^{\circ}27'$	Co-lat.	$70^{\circ}30'$	Co-alt.	$79^{\circ}20'$
D	$= 8^{\circ}50'$			Co-lat.	$70^{\circ}30'$
Sum	$116^{\circ}17'$		$58^{\circ}08' = \frac{1}{2}$ sum	D	$= 8^{\circ}50'$
Dif.	$98^{\circ}37'$		$49^{\circ}19' = \frac{1}{2}$ dif.		

Then by the Gunter, the extent from $58^{\circ}8'$ to $79^{\circ}20'$ on the fines, will reach from $70^{\circ}30'$ to some point beyond 90° ; rest one foot there, and extend the other to $49^{\circ}19'$; then this extent will reach from the beginning of the verfed fines to the verfed line of 67° , the azimuth sought, as the latitude is south. Hence the variation will be $13^{\circ}30'W$; with which the courses in the log-board are to be corrected.

For the bearing and dist. of Helena.

St. Helena's lat.	$= 16^{\circ}00'S$.	M. P.	973
Present latitude	$= 17^{\circ}25'S$.		1061
Dif. latitude	$= 1^{\circ}25'$		18

Depart. latitude	$= 19^{\circ}30'S$.
Difference of latitude	$= 2^{\circ}05'S$.
Present latitude	$= 17^{\circ}25'S$.
Sum of latitudes	$= 36^{\circ}55'$
Middle latitude	$= 11^{\circ}27'$
Co-middle latitude	$= 71^{\circ}33'$
Depart. longitude	$= 0^{\circ}10'E$.
Dif. longitude	$= 1^{\circ}36'W$.
Present longitude	$= 1^{\circ}26'W$.

Here the azimuth is worked with yesterday's lat.; but had it been worked with this day's, the variation would have been $12^{\circ}30'$, the diff. lat. 126 m. and the depart. 90 m.

Distance 270 miles.

Course N. $71^{\circ}30'W$. (57)

In the following Journal are introduced almost all the circumstances that can attend a day's work in any voyage: The ship is taken from the river *Thames*, conducted through the *Downs*, down the *English Channel*, and from thence over the *Ocean*, till she is brought within sight of the port she is bound to, which is *Funchal* in the island of *Madeira*. In this navigation, care has been taken to point out the usual business done in like circumstances in wind and weather; the courses and distances run down the Channel are such as produce the bearings and distances of the places mentioned, and it is not unusual for many vessels to keep a track not very different from what is here assigned. Indeed the runs inserted in the log-boards, after the ship has taken her departure from the Lizard, are rather somewhat greater than can be expected from any vessel in like weather: But such long days runs were used to save four or five days works, which must have been added to bring the ship to her port, had the usual runs been introduced. All the astronomical observations are fitted to the times and places; and it is apprehended that the whole is conducted in such a manner as it is possible to have happened.

In most Journals, the week days are marked by the planets characters,

Thus	☉	stands for	Sunday
	♂		Monday
	♂		Tuesday
	♂		Wednesday
	♂		Thursday
	♂		Friday
	♂		Saturday.

123. *A JOURNAL of a voyage in the year 1772. In the Hope-
well of London, from England towards Madeira. LUCUS
TRUSTY, Commander. Kept by FRANCIS DRAKE, Mate.*

Winds.	Remarks on board.
SSW.	♂. 27th September. At 5 A. M. the pilot came on board; then weighed and failed from Cherry-Garden. At 11 came to with the best bower * at Blackwall.
From SSW. to NNW.	♀. 28th Septem. Fresh gales and cloudy weather with rain. At 5 A. M. weighed and failed; and at 9 came to an anchor at Gravesend, and cleared ship.
W. b. S.	♂. 29th September. At 4 P. M. weighed and failed, moderate weather: At 9 came to with the best bower at the Nose in $9\frac{1}{2}$ fathoms; fresh gales: At 4 A. M. weighed and failed. At 11 came to anchor in the Downs in 7 fath. Deal-castle bearing W. $\frac{1}{2}$ S. distant 3 miles,
W. b. S.	♀. 30th September. At 1 P. M. sent the pilot ashore. These 24 hours, the first and middle parts moderate and fair, the latter part strong gales and cloudy.
Variable from S. b. W. to West.	♂. 1st October. Strong gales and cloudy. At 2 P. M. veered out the long service † of the best bower; got top-gallant yards down. At 4 P. M. struck yards and top-masts. These 24 hours had very hard gales of wind.
SW.	♂. 2d October. These 24 hours, for the most part fresh gales. At 4 A. M. hove up best bower, and let go small bower*. At 9, hove up small bower, and let go best bower again.
W. b. N.	♂. 3d. October. At 6 P. M. strong gales with heavy rain. At 8, veered out long service, and let go the sheet anchor * [under foot. At 9 A. M. hove up sheet anchor.
NW.	♂. 4th. October. These 24 hours, for the first and middle parts moderate and fair; the latter part strong gales.
NW.	♀. 5th October. These 24 hours, fresh gales and fair. At 3 P. M. got top-gallant masts down. At 10. A. M. got yards and top-masts up.

* NOTE, It is customary in most ships to have three kinds of anchors, namely: The small bower, used to fix the ship in still water and calm weather.

The best bower, a larger anchor, for riding in rough weather.

The sheet anchor, or largest, for fixing the ship in stormy weather.

† The first 30 fathoms of cable next the anchor is called *short service*; and when the remaining part is veered out, it is called the *long service*.

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H.	K.	F.	Courfes	
2	4	6	S.b. W $\frac{1}{4}$ W	24. 6th October. At 2 P. M. hove short and hoisted in the long-boat.
4	6	8		At 4, weighed and sailed in company with a 20 gun man of war and 40 sail of merchantmen.
6	8	10		At 6, South-Foreland bore NNW. dist. 4 miles.
8	10	12	W.	At 2 A. M. Fairlee bore N. dist. 6 miles.
10	12		SW $\frac{1}{4}$ W.	At 6, Beachy bore N. b. W. 6 miles.
12				At 8, Beachy bore NE. b. E. 9 miles.
			WN. W $\frac{1}{2}$ W.	Fresh gales and clear. Several ships standing up Channel. Close reef both top-fails.
			WSW.	At 12, Bembridge point bore WNW.
				27. m. Still in company with the fleet.

LOG-BOARD.				Remarks on board.
H.	K.	F.	Courfes	
2	4	6	WSW.	2. 7th October. Fresh gales and clear.
4	6	8	W. $\frac{1}{4}$ S.	At 4 P. M. parted with the fleet, they being bound to Spithead. Dunnofe bearing WNW. distance 21 miles.
6	8	10		At 5, let out one reef of each top-fail.
8	10	12		At 7 A. M. Portland lights bore NNW, 9 miles.
10	12			At 10 ditto bore NE. 12 miles.
12				Fourteen sail in fight,
			S.b. W $\frac{1}{2}$ W	
			NNE.	

LOG-BOARD.				Remarks on board.
H.	K.	F.	Courfes	
2	4	6	S.b. W $\frac{1}{4}$ W	5. 8th. Octob. Cloudy weather, fresh gales
4	6	8		At 4 P. M. Start bore W. b. N. dist. 12m.
6	8	10	WbN. $\frac{1}{2}$ W	At 8 ditto bore E. b. N. 9m.
8	10	12		At 4 A. M. Edystone bore NE. b. E $\frac{1}{2}$ E. 12m.
10	12			At 6, got up both top-gallant masts and yds.
12				At 7 $\frac{1}{2}$, Lizard bore W. b. N.
				Several ships standing up Channel.
				At noon Lizard bore N. b. W. dist. 21m.
				For, From 7 $\frac{1}{2}$ h. to 12h. is 4 $\frac{1}{2}$ hours.
				4 $\frac{1}{2}$ hours, at 5, 4 miles an hour, is 24 miles.
				Between SW. $\frac{1}{4}$ W. and W. b. N. is 4 $\frac{3}{4}$ points.
				Between SW. $\frac{1}{4}$ W. and N. b. W. is 5 $\frac{1}{4}$ points.
				The other angle is 6 points.
				Then 6 pts.: 4 $\frac{3}{4}$ pts.: 24m.: 21m. (III. 45)

Journal from England towards Madeira.

LOG-BOARD.

H. K. R.	Courses	Winds
—	Foggy & lay by	WSW.
2	WSW. $\frac{1}{4}$ W.	NW. b. N.
4	—	[$\frac{1}{4}$ W.]
6	—	—
8	—	—
10	—	—
12	—	—
2	—	—
4	—	—
6	—	—
8	—	—
10	—	—
12	—	—

Remarks on board.

○. 9th October. These 24 hours, first part foggy, the latter part moderate gales and clear weather.

At 4 P. M. fog cleared up. Shifted the Channel compass, where the variation was allowed for, and put another into the bitacle. Lizard bore N. $\frac{1}{2}$ E. distance 21 miles. Variation $1\frac{1}{2}$ point to the west. Leeway $\frac{1}{2}$ point.

Unbent our cables and stowed anchors. At 2 A. M. set top-gallant sails.

Observed ○ mer. zen. dist. = $54^{\circ} 33' S.$
Declination = $6^{\circ} 35' S.$

Lat. by observ. = $47^{\circ} 58' N.$
Differing 4 min. from the latitude by account $48^{\circ} 02'.$

To diff. lat. 114.6 and dep. 99.6 .
The course is S. 41° W.

Distance 152 miles.

Dep. lat. $49^{\circ} 57'.$ M. P. 3470

Prof. lat. $47^{\circ} 58'.$ M. P. 3289

Mer. diff. lat. 181

To course 41° and M. d. l. 181 m.

The diff. long. is $157 = 2^{\circ} 37' W.$

Dep. longit. = $5^{\circ} 14' W.$

Prof. long. = $7^{\circ} 51' W.$

TRAVERSE TABLE.

Courses	Diff.	N.	S.	E.	W.
S. b. E.	21	—	—	—	—
SW $\frac{1}{4}$ W.	140	—	20,6	4,1	103,7
—	—	—	94,0	—	—
—	—	D. l.	114,6	4,1	103,7
—	—	—	—	—	Dep. 99,6

Here the latitude by account agreeing nearly with the latitude by observation, it is judged that the ship's place is truly ascertained.

Ship's latitude = $47^{\circ} 58' N.$ Meridional parts = 3288,6

Madeira's latitude = $32^{\circ} 38' N.$ Meridional parts = 2073,4

Diff. latitude = $15^{\circ} 23'$ Merid. difference lat. = 1215,2

Or 920 miles.

Ship's longitude = $7^{\circ} 51' W.$

Madeira's long. = $16^{\circ} 51' W.$

Diff. longitude = $9^{\circ} 00' = 540$ miles.

Island of Madeira bears S. $23^{\circ} 45' W.$ Distance 1010 miles.

Here the ship is supposed to sail from the Lizard; and as the Lizard bore N. $1^{\circ} E.$, the course must be on the opposite point, which is S. $\frac{1}{2} W.$; and this course corrected by the variation, as at article 30, gives S. b. E.; also the other course WSW. $\frac{1}{4} W.$ being corrected both for the variation and leeway (articles 30, 32); the Traverse Table, and other work, for the present latitude and longitude, is performed by the observations in art. 115.

Then the bearing and distance to Funchal in the island of Madeira is found, as in VIII. 57, working by the Traverse Table.

Journal

Journal from England towards Madeira.

LOG-BOARD.			Remarks on board.
H.	K.	F. Courses	
		West	
12	7	6	D. 10th October. These 24 hours stiff gales with drizzling rain.
4	8	4	Variation $1\frac{1}{2}$ point west.
6	8	6	At 4 P. M. spoke with the Charming Molly, bound from Carolina to London.
8	8	9	At 8, took in top-gallant sails.
10	9	0	At 4 A. M. took single reef in each T. S.*.
12	8	6	People employed in making points.
2	8	3	Observed Sun's merid. zen. dist. = $54^{\circ}09'S$.
4	8	4	Declination = $6^{\circ}57'S$.
6	8	1	
8	8	6	Latitude by observation = $47^{\circ}12'N$.
10	8	3	
12	8	0	

TRAVERSE TABLE.

Courses	Diff.	N.	S.	E.	W.
WSW. $\frac{1}{2}$ W.	202		58,6		193,3

Dep. lat. = $47^{\circ}58'N$. or last day's lat.
 Diff. lat. = $0^{\circ}59'S$. as by Trav. Tab.
 Pref. lat. = $46^{\circ}59'N$. as by account.

Here the latitude by observation differs 13 minutes from that by account.

Therefore, before the longitude can be found, the dead reckoning must be corrected by article 82, as the course is nearer the parallel than the meridian.

Then to the diff. lat. $58,6$, and departure $193,3$, the distance is 202 .
 And to the dist. 202 , and diff. lat. by observ. 46 , the dep. is $196,5$.
 And course $S. 76^{\circ}45'W$.

Now dep. lat. = $47^{\circ}58'M.P.$ = 3288
 Pref. lat. = $47^{\circ}12'$ M.P. = 3229
 Mer. diff. lat. = 68

Hence (VIII. 59) diff. lon. = $4^{\circ}49'$

M. P.
 Lizard's latitude = $49^{\circ}57'N$. 3470
 Ship's latitude = $47^{\circ}12'N$. 3229

Diff. latitude = $2^{\circ}45'$ M.d.l. 241

Or 165 miles.

Lizard's long. = $5^{\circ}14'W$.
 Ship's long. = $12^{\circ}40'W$.

Diff. longitude = $7^{\circ}26'$ = 446 m.

Lizard bears $N. 59^{\circ}45'E$.
 Distance 324 miles.

M. P.
 Madeira's lat. = $32^{\circ}38'N$. 2073
 Ship's latitude = $47^{\circ}12'N$. 3229

Diff. latitude = $14^{\circ}34'$ 1156

Or 874 miles.

Madeira's long. = $16^{\circ}81'W$.
 Ship's longitude = $42^{\circ}40'W$.

Diff. longitude = $4^{\circ}11'$ = 251 m.
 Madeira bears $S. 13^{\circ}30'W$.
 Distance 903 miles.

* Mariners in their Journals use abbreviations of words; such as T. S. for top-sails, B. T. S, both top-sails; and many others.

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board,
H.	K.	F.	Winds	
2	9	2	S. $\frac{1}{2}$ E.	9. 11th October.
4	10	0	NNW.	These 24 hours hard gales and clear weather.
6	9	5		At 6 P. M. close reefed the top-sails.
8	10	0		At 6 A. M. a sail standing to northward.
10	10	6		At noon Cape Finisterre bore N. 85° E.,
12	10	2		distant 25 miles.
2	10	4		Variation $1\frac{1}{2}$ W.
4	9	6		Observed Sun's merid. zen. diff. $50^{\circ} 33'S.$
6	10	0		Declination $7^{\circ} 20 S.$
8	10	4		
10	9	8		Latitude by observation $43^{\circ} 13' N.$
12	10	0		Dep. latitude $47^{\circ} 12'$

TRAVERSE TABLE.

Course	Diff.	N.	S.	E.	W.
SSE.	240		221,8	91,8	

Latitude by observation $43^{\circ} 13' N.$
Dep. latitude $47^{\circ} 12'$

Diff. latitude $3^{\circ} 59' = 39m.$
Departed lat. $= 47^{\circ} 12' N.$
Diff. latitude $= 3^{\circ} 42' S.$

Pref. latitude $= 43^{\circ} 30' N.$ by acct.
Pref. latitude $= 43^{\circ} 13' N.$ by observ.

Here the latitude by acct differs 17 m. from the latitude by observation. Therefore before the longitude can be found, the account of departure must be corrected by article 81, as the course is only two points from the meridian.

Now to diff. lat. 221,8 and departure 91,8, the course is SSE.
To course SSE. and diff. lat. 239, the correct departure is 99,5.

Then, Sum of latitudes $= 90^{\circ} 25'$
Middle latitude $= 45^{\circ} 12'$
Co-middle lat. $= 44^{\circ} 48'$

Dep. longitude $= 12^{\circ} 40' W.$
Diff. longitude $= 2^{\circ} 21' E.$
Pref. longitude $= 10^{\circ} 19' W.$

M. P.
Lizard's latitude $= 49^{\circ} 57' N.$ 3470
Ship's latitude $= 43^{\circ} 13'$ 2881
Diff. latitude $= 6^{\circ} 44'$ M.d.l. 589

M. P.
Madeira's lat. $= 38^{\circ} 31' N.$ 2073
Ship's latitude $= 43^{\circ} 13'$ 2881
Diff. latitude $= 10^{\circ} 35'$ 808

Or
Lizard's long. $= 40,4$ miles.
Ship's longitude $= 5^{\circ} 14' W.$
 $= 10^{\circ} 19' W.$

Or
Madeira's long. $= 6,34$ miles.
Ship's long. $= 16^{\circ} 51' W.$
 $= 10^{\circ} 19'$

Diff. longitude $= 5^{\circ} 05' = 305m.$
Lizard bears N. $27^{\circ} 15' E.$
Distance 455 miles.

Diff. longitude $= 6^{\circ} 32' = 392m.$
Madeira bears S. $26^{\circ} 0' W.$
Distance 710 miles.

Journal from England towards Madeira.

LOG-BOARD.			Remarks on board.
H. K. F.	Courses	Winds	8 12th October.
2 10	6 Wb.S. $\frac{1}{2}$ W.	SbW $\frac{1}{2}$ W.	These 24 hours very hard gales and
4 10	2 Leeway	1 point	equally with rain.
6 11	0 SSW.	W.	Top-fails clofe reef.
8 10	3 Leeway	1 point	At 10 P.M. spoke with the Princeffa
10 9	7 WbN. $\frac{1}{2}$ W.	SSW $\frac{1}{2}$ W.	now bound for London.
12 9	2 Leeway	1 point	Obfer. the Sun to fet 66' from N. westward.
4 10	5 SW. $\frac{1}{2}$ W.	NWbW $\frac{1}{2}$ W.	Hence find the true amplitude. (art. 24)
6 10	8 Leeway	1 point	The lat. 43° 13' N. Declin. 7° 43' S.
8 10	0 Leeway	1 point	To course 7° 45' and dist. 100, the dep. is 13.49.
10 10	1		To course 43° 15' and d. lat. 13.49, the dist. is 19.
12 9	6		To dist. 100 and dep. 19, the course is 11°.
			Therefore, the true ampl. is N. 79° 00' W.
			Mag. ampl. 66 00

TRAVERSE TABLES.				
Courses	D.	N.	S.	W.
W b. S.	42		8.2	41.2
S. $\frac{1}{2}$ E.	62		61.9	3.0
W b. N.	61	12.1		60.8
SSW. $\frac{1}{2}$ W.	83		73.2	39.1

Dep. latitude = 43° 17' N.
 Diff. latitude = 2 11 S.
 Latitude by account = 41° 04' N.
 Lat. by observation = 41° 16' N.
 Sum latitudes = 84 29
 Middle latitude = 42 14
 Dep. longitude = 10 19 W.
 Diff. longitude = 3 14 W.
 Present longitude = 13 33 W.

Here the lat. by acct. having out-run the lat. by obs. the dep. must be corrected by article 83, as diff. lat. and dep. are nearly equal.

Then diff. lat. 131.2, and dep. 138.1, the distance will be 190.
 To dist. 190, and true diff. lat. 117, find the departure 149.7.
 The sum of both dep. 138.1 and 149.7 is 287.8.
 The half sum 143.9 = corrected departure.

M. P.
 Lizard's lat. = 49° 57' N. 3470
 Ship's latitude = 41° 16' N. 2723
 Diff. latitude = 8 41 747
 Or = 521 miles.
 Lizard's long. = 5° 14' W.
 Ship's long. = 13 33 W.
 Diff. longitude = 8 19 W = 499m.
 Lizard bears N. 33° 45' E.
 Distance 624 miles.

M. P.
 Madeira's lat. = 32° 38' N. 2073
 Ship's latitude = 41° 16' N. 2723
 Diff. latitude = 8 38 650
 Or = 518 miles.
 Madeira's long. = 16° 51' W.
 Ship's long. = 13 33 W.
 Diff. long. = 3 18 W. = 198.
 Madeira bears S. 17° W.
 Distance 542 miles

Journal

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H	K.	W.	Courses	
2	9	6	NW $\frac{1}{2}$ W	24. 13th October. These 24 hours hard gales and cloudy weather, accompanied with hard rains. Handed fore-top sails.
4	9	2	Leeway	
4	9	5	S. $\frac{1}{2}$ W.	
8	8	5	Leeway	
10	8	7	WSW $\frac{1}{2}$ W.	
12	7	2	Leeway	At 10 P. M. heard a gun at a distance.
12	7	3	NNE $\frac{1}{2}$ E.	At 2 A. M. saw to the NNE. $\frac{1}{2}$ E. the flash of a gun, and after 56 beats of the pulse at the wrist we heard the sound, by which we judged the distance to be about 11 miles (VII. 59), and a ship in distress. Bore away to speak with her.
4	6	7	Leeway	At 6, having greatly over-run the distance, and finding nothing, concluded she was gone down, so hauled our wind, and stood to the SW. No observation this day. Variation $1\frac{1}{2}$ point west.
4	7	6	SW.	
8	7	2	Leeway	
40	7	0		
12	6	4		

TRAVERSE TABLE.

Courses	D.	N.	S.	E.	W.
NW $\frac{1}{2}$ W.	37	22,0			29,7
S $\frac{1}{2}$ E.	36		35,8	3,5	
W. b. S.	30		6,6		33,6
N. b. E.	31	33,6		6,6	
SW $\frac{1}{2}$ W.	56		37,6		41,5
		55,6	80,0	10,1	104,8
		D. I.	24,4	Dep.	94,7

Depart. latitude = $41^{\circ} 16' N.$
 Difference of latitude = $0^{\circ} 24' S.$
 Present latitude = $40^{\circ} 52' N.$
 Sum of latitudes = $82^{\circ} 08'$
 Middle latitude = $41^{\circ} 04'$
 Depart. longitude = $13^{\circ} 33' W.$
 Difference of longitude = $2^{\circ} 06' W.$
 Present longitude = $15^{\circ} 39' W.$

M. P.
 Lizard's latitude = $49^{\circ} 57' N.$
 Ship's latitude = $40^{\circ} 52' N.$

M. P.
 Madeira's lat. = $32^{\circ} 38' N.$
 Ship's latitude = $40^{\circ} 52' N.$

Diff latitude = $9^{\circ} 05'$
 Or = $54\frac{1}{2}$ miles.
 Lizard's long. = $5^{\circ} 14' W.$
 Ship's longitude = $15^{\circ} 39' W.$

Diff. latitude = $8^{\circ} 14'$
 Or = 494 miles.
 Madeira's long. = $16^{\circ} 51' W.$
 Ship's longitude = $15^{\circ} 39' W.$

Diff. longitude = $9^{\circ} 25' = 565$ m.
 Lizard bears N. $37^{\circ} 45' E.$
 Distance 685 miles.

Diff longitude = $1^{\circ} 12' = 72$ m.
 Madeira bears S. $6^{\circ} 30' W.$
 Distance 505 miles.

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H. K. F.	Courses	Winds		
2	7	S. $\frac{1}{2}$ W.	E.	9. 14th October.
4	8	Leeway	o point	These 24 hours variable winds and hard
6	8	E. $\frac{1}{2}$ S.	SSE.	gales, with cloudy and foggy weather.
8	9	Leeway	1 point	Set fore-top sails.
10	8	W. b. N.	SW. b. S.	A great sea setting to the northward, for
12	8	Leeway	1 point	which allow half a mile an hour.
2	8	South	W.	At 6 P. M. handed fore top sail.
4	9	Leeway	o point	No observation to-day.
6	9	E. b. S. $\frac{1}{2}$ E.	S. b. E. $\frac{1}{2}$ E.	Variation $1\frac{1}{2}$ point west.
8	8	Leeway	2 points	Half a mile an hour, or 12 miles being al-
10	8	S. $\frac{1}{2}$ E.	W.	lowed for the sea setting northward, reduces
12	9	Leeway	o point	the difference of latitude 58,7 to 46,7.

TRAVERSE TABLE.

Courses	D.	N.	S.	E.	W.
S. b. E.	29		28,4	5,7	
ENE.	36	13,8		33,3	
W b. N $\frac{1}{2}$ W	31	4,6			30,7
S. b. E. $\frac{1}{2}$ E.	35		33,9	8,5	
NE b. E $\frac{1}{2}$ E	36	18,5		30,9	
SSE.	36		33,3	13,8	
		36,9	95,6	92,2	30,7
			58,7	61,5	

Tried the log line and glaſs uſed this day, and find only $45\frac{1}{2}$ feet on a mean for the length of each knot, and $28\frac{1}{2}$ ſeconds for the run of the glaſs.

Now by art 6. $28,5 \times 5 = 142,5$

and $45,5 \times 3 = 136,5$

Then by logs. or by Gunter's ſcale,

As $142,5 : 136,5 :: 46,7 : 44,7$

And $61,5 : 58,9$

So the true diff. lat. is $44,7$ S.

departure $58,9$ E.

Dep. latitude $= 40^{\circ} 52' N.$

Diff. latitude $= 0^{\circ} 45' S.$

Preſent latitude $= 40^{\circ} 07' N.$

Sum of latitudes $= 80^{\circ} 59'$

Middle latitude $= 40^{\circ} 29'$

Depart. longitude $= 15^{\circ} 39' W.$

Diff. longitude $= 1^{\circ} 17' E.$

Preſent. longitude $= 14^{\circ} 22' W.$

M. P.

Lizard's lat.

Ship's latitude $= 49^{\circ} 57' N.$

Diff. latitude $= 40^{\circ} 07' N.$

Or $= 9^{\circ} 50' Md. l.$

Lizard's long.

Ship's long. $= 5^{\circ} 14' W.$

Diff. longitude $= 9^{\circ} 08' = 548 m.$

Lizard bears N. $33^{\circ} 15' E.$

Distance 705 miles.

M. P.

Madeira's lat.

Ship's latitude $= 32^{\circ} 38' N.$

Diff. latitude $= 40^{\circ} 07' N.$

Or $= 7^{\circ} 29' Md. l.$

Madeira's long.

Ship's long. $= 16^{\circ} 51' W.$

Diff. longitude $= 14^{\circ} 22' W.$

Madeira bears S. $15^{\circ} W.$

Distance 464 miles.

Journal

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H. K.	F.	Course	Winds	
			SE. E.	
2	9	4 S. 61° W	W. 0 pts.	15th October. Then 24 hours, hard, but steady gales, and cloudy weather. At 4 P.M. set fore-top sails. At 8 A.M. observed the alt. of ☉'s center to be 17° 5', and bearing S. 49° 30' E. At 10 h. obs. alt. ☉'s center 35° 15'. The dist. run to 8 h. A.M. is 193 m. Yesterday's variat. of 14° alters the course S. 61° W., to S. 47° W., this and 193 dist. gives a diff. lat. 132 m. = 2° 12'. Hence lat. at 1st observ. is 37° 55' N.
4	9	6		
6	9	8		
8	9	8		
10	9	6		
12	9	0		
2	9	4		
4	10	0		
6	10	0		
8	10	0		
10	10	0		With lat. 37° 55', alt. 17° 5', and polar dist. 98° 50' the ☉'s true azimuth is N. 116. 18 E. Mag. azim. is N. 130° 30' E. and variation 14° 12' W. Hence the correct course is S. 46° 48' W., with which, and the whole dist. 233, the depart. is 170 m. W. and the diff. lat. 160 m. S. = 2° 40'. Then (40 07—2° 40' =) 37° 27' N. is the ship acct. of lat. at noon. The time between the observ. is 2 h. and the distance run is 20 m. The course 46° 48' and dist. 20, gives a diff. lat. 14 m. S. And the lat. in at the 2d observ. is 37° 41' N. The diff. of ☉'s rh. N. 130° 30' E. and ship's rh. N. 61° E. is 69° 30'. A course 69° 30' and dist. 20, gives a diff. lat. 7', subtractive. And 17° 5'—0° 7' = 16° 58' for the first alt. corrected (75.) Now with lat. 37° 41' N. ☉ decl. 8° 50' S.; interval of time 2 h. or 30°; the 1st alt. 16° 58'; 2d alt. 35° 15'; the mid-day lat. comes out 38° 07'. And (40° 07'—38° 07' = 2° =) 120 m. is the true diff. lat. Hence 185 m. is taken for the true departure And 18° 22' W. is the ship's longitude. Now a parallel of lat. through 38° 07', in the variation chart, cuts the variation lines of 14° in the long. of 18° W. which confirms the ship's account of longitude.
10	10	0		
12	10	0		

Liz. lat. = 49° 57' N. M.P. 3470 Mad. lat. = 32° 28' N. M.P. 2073
 Ship's lat. = 38 07 N. 2472 Ship's lat. = 38 07 N. 2472
 Diff. lat. = 11 50 998 Diff. lat. = 5 39 399
 Or = 710 miles. Or = 339 miles.
 Liz. long. = 5° 14' W. Mad. long. = 16 51
 Ship's long. = 18 22 W. Ship's long. = 18 22
 Diff. long. = 13 08 W. = 788 m. Diff. long. = 1 31 = 91
 Lizard bears N. 38° 15' E. Madeira bears S. 12° 45' E.
 Distance 905 miles. Distance 348 miles.

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H. K. F.	Courses	Winds		
2	6 S. b. W. $\frac{1}{2}$ W.	W. b. S. $\frac{1}{2}$ W.		D 17th October.
4	Leeway	1 point		These 24 hours, stiff gales, clouly and variable winds.
6				Variation $1\frac{1}{2}$ point west.
8				Out 1st and 2d reef of both top-sails.
10	W. b. S. $\frac{1}{4}$ W.	South		At 13h. P.M. Observed the distance of the star Pollux from the Moon's farthest limb, to be $51^{\circ} 36'$.
12	Leeway	0 point		At 8 A.M. the Porto Sancto bore SE. distance 10 leagues. And Madeira bore SSW. $\frac{1}{4}$ W., distance 15 leagues, by estimation.
2	SSW.	W. b. S.		
4	Leeway	1 point		
6				
8				
10				
12				

Observed $\odot$'s mer. zen. dist. $42^{\circ} 16' S.$ Declination $9^{\circ} 34' S.$ Lat. by observation $32^{\circ} 42' N.$
Departed latitude $35^{\circ} 26'$ Diff. of latitude $2^{\circ} 44' = 164m.$

TRAVERSE TABLE.

Courses	D.	N.	S.	E.	W.
S. $\frac{1}{2}$ E.	70		69.7	6.9	
WSW.	71		27.2		65.6
S. $\frac{1}{4}$ E.	55		54.9	2.7	
			151.8	9.6	65.6
					56.0

At 13 h. I reckon my lat. $34^{\circ} 1' N.$ long. $16' W.$

Then 13h. + 1h. 4m. W. = 14h. 4m. the time at Greenwich (57.)

At this time $\odot R. A. = 203^{\circ} 21' 30''$; $\mathcal{D}$ diam. = $15' 57''$ hor. par. = $56' 07''$.* Pol. lux $R. A. = 112^{\circ} 50' 15''$; and decl. = $28^{\circ} 33' 34'' N.$ Moon's $R. A. = 102^{\circ} 27'$; and decl. = $17^{\circ} 42' N.$ The horary $\angle$ at * = $74^{\circ} 28' 45''$; at $\mathcal{D} = 64^{\circ} 15' 30''$.With *'s hor. $\angle = 74^{\circ} 28' 45''$, the pol. dist. $61^{\circ} 26' 26''$, and the co-lat. $55^{\circ} 59'$, the *'s alt. will be $27^{\circ} 33' 20''$.With $\mathcal{D}$'s hor. $\angle = 64^{\circ} 15' 30''$, the polar dist. $72^{\circ} 18'$, and the co-latitude $55^{\circ} 59'$ the $\mathcal{D}$'s altitude will be $30^{\circ} 52'$.Now the observed dist. lessened by $\mathcal{D}$'s $\frac{1}{2}$ diameter, gives $51^{\circ} 20' 43''$.And the Moon's parallax in alt. is $48'$, by the Gunter.Then with the true alts. of the * = $27^{\circ} 33' 20''$, of the $\mathcal{D} = 30^{\circ} 52'$;their apparent alts., and the apparent dist. $51^{\circ} 20' 43''$, the true distancewill be found $51^{\circ} 11' 4''$.Hence the long. at 13 h. is $16^{\circ} 21' W.$ The last 11 hours we have made $65' S.$, and $22' W.$ Hence at noon our lat. is $32^{\circ} 56' N.$ and long. $16^{\circ} 48' W.$ Our lat. by observation is $32^{\circ} 42' N.$ The error in lat. is $14'$, too little.Then with diff. lat. $65'$ and dep. $22'$, the course is 19° .With course 19° and error $\frac{1}{2}$ lat. $14'$, the error in dep. is $5'$.So the correct dep. is $27 m.$ and the diff. long. is $32' W.$ Hence at noon the ship's lat. is $32^{\circ} 42'$ and long. $16^{\circ} 53' W.$ which isvery near Funchal, in lat. $32^{\circ} 38' N.$ long. $16^{\circ} 51' W.$

C c 3

APPEN-

Journal from England towards Madeira.

LOG-BOARD.			Remarks on board.
H. K. F.	Courses	Winds	
2 10	S. b. W $\frac{1}{2}$ W	W. b. S $\frac{1}{2}$ W	☉ 16th October. These 24 hours very hard gales and variable, with clouds and rain. Variation 14 degrees west. Intending to take an observation of the Moon's distance from the Sun, or Star, we this day, before and after noon, took equal altitudes of the Sun, and set a good second watch to the apparent time. Observed Sun's merid. zen. dist. = $44^{\circ} 38' S.$ Declination = $9^{\circ} 12' S.$ Lat. by observation = $35^{\circ} 26' N.$
4 9	Leeway	1 point	
6 10	ESE.	South	
8 10	Leeway	1 point	
10 10	ESE.	South	
12 9	S. b. W $\frac{1}{2}$ W	ESE.	
2 10	Leeway	0 point	
4 10	ESE.	South	
6 11	Leeway	1 point	
8 11	ESE.	South	
10 11	Leeway	1 point	
12 11	ESE.	South	

TRAVERSE TABLES.

Courses	D.	N.	S.	E.	W.
S $8^{\circ} 30' E$	60	—	—	—	—
S $87^{\circ} 15' E$	61	2,9	59,3	8,9	—
S $3^{\circ} 0' E$	89	—	88,9	60,9	4,7
S $70^{\circ} 15' E$	46	—	15,5	43,3	—
—	—	2,9	163,7	113,1	4,7
—	—	D. 1.	160,8	108,4	Dep.

Here the latitude as by account agrees with the latitude by observation.
 Therefore the different longitude is found with the departure by account.

M. P.	M. P.
Lizard's lat. = 49° 57' N.	Madeira's lat. = 32° 38' N.
Ship's latitude = 35° 26' N.	Ship's latitude = 35° 26' N.
Diff. latitude = 14 31	Diff. latitude = 2 48
Or = 871 miles.	Or = 168 miles.
Lizard's long. = 5° 14' W.	Madeira's long. = 16° 51' W.
Ship's long. = 16 07 W.	Ship's long. = 16 07 W.
Diff. longitude = 10 53 = 653 m.	Diff. long. = 0 44
Lizard bears N. 28° 30' E.	Madeira bears S. 12° 15' W.
Distance 990 miles.	Distance 172 miles

Journal

Journal from England towards Madeira.

LOG-BOARD.				Remarks on board.
H. K. F.	Courses	Winds		
—	—	—	—	17th October.
2 8	S. $\frac{1}{2}$ W. $\frac{1}{2}$ W.	WbS. $\frac{1}{4}$ W	1 point	These 24 hours, stiff gales, cloudy and variable winds.
4 9	Leeway	—		Variation $1\frac{1}{4}$ point west.
6 9	—	—	South 0 point	Out 1st and 2d reef of both top-sails.
8 8	—	—		At 13h. P.M. Observed the distance of the star Pollux from the Moon's farthest limb, to be $51^{\circ} 36'$.
10 9	WbS. $\frac{1}{4}$ W	—	SSW. 1 point	At 8 A.M. the Porto Sancto bore SE. distance 10 leagues. And Madeira bore SSW. $\frac{1}{4}$ W., distance 15 leagues, by estimation.
12 8	Leeway	—		Observed $\odot$'s mer. zen. dist. $42^{\circ} 16' S.$

TRAVERSE TABLE.

Courses	D.	N.	S.	E.	W.
S. $\frac{1}{2}$ E.	70	—	—	—	—
WSW.	71	—	69,7	6,9	65,6
S. $\frac{1}{2}$ E.	55	—	27,2	2,7	—
—	—	—	54,9	—	—
—	—	—	151,8	9,6	65,6
—	—	—	—	—	56,0

At 13 h. I reckon my lat. $34^{\circ} 1' N.$ long. $16^{\circ} W.$
 Then 13 h. + 1 h. 4 m. $W. = 14 h. 4 m.$ the time at Greenwich (57.)
 At this time $\odot R.A. = 203^{\circ} 21' 30''$; $\mathcal{D}$ diam. $= 15' 57''$ hor. par. $= 56' 07''$.
 $\ast$ Pollux $R.A. = 112^{\circ} 50' 15''$; and decl. $= 28^{\circ} 33' 34'' N.$
 Moon's $R.A. = 102^{\circ} 27'$; and decl. $= 17^{\circ} 42' N.$
 The horary $\angle$ at $\ast = 74^{\circ} 28' 45''$; at $\mathcal{D} = 64^{\circ} 15' 30''$.
 With $\ast$'s hor. $\angle = 74^{\circ} 28' 45''$, the pol. dist. $61^{\circ} 26' 26''$, and the co-lat. $55^{\circ} 59'$, the $\ast$'s alt. will be $27^{\circ} 33' 20''$.
 With $\mathcal{D}$'s hor. $\angle = 64^{\circ} 15' 30''$, the polar dist. $72^{\circ} 18'$, and the co-latitude $55^{\circ} 59'$ the $\mathcal{D}$'s altitude will be $30^{\circ} 52'$.
 Now the observed dist. lessened by $\mathcal{D}$'s $\frac{1}{2}$ diameter, gives $51^{\circ} 20' 43''$.
 And the Moon's parallax in alt. is $48'$, by the Gunter.
 Then with the true alts. of the $\ast = 27^{\circ} 33' 20''$, of the $\mathcal{D} = 30^{\circ} 52'$; their apparent alts., and the apparent dist. $51^{\circ} 20' 43''$, the true distance will be found $51^{\circ} 11' 4''$.
 Hence the long. at 13 h. is $16^{\circ} 21' W.$
 The last 11 hours we have made $65' S.$, and $22' W.$
 Hence at noon our lat. is $32^{\circ} 56' N.$ and long. $16^{\circ} 48' W.$
 Our lat. by observation is $32^{\circ} 42' N.$ The error in lat. is $14'$, too little. (81.)
 Then with diff. lat. 65 and dep. 22, the course is 19° .
 With course 19° and error 41 lat. 14, the error in dep. is 5.
 So the correct dep. is 27 m. and the diff. long. is $32' W.$
 Hence at noon the ship's lat. is $32^{\circ} 42'$ and long. $16^{\circ} 53' W.$ which is very near Funchal, in lat. $32^{\circ} 38' N.$ long. $16^{\circ} 51' W.$

C c 3

APPEN-

A P P E N D I X,

Containing the principles upon which some of the preceding precepts are founded.

124. I. *The Theory of Davis's Quadrant.* Plate XIV. Fig. 9.

To shew that the arcs $DF + HC$ make the zenith distance.

The line AH is horizontal.

Therefore the arc $FBH =$ height of the Sun above the horizon.

Now the arc $FBH = \text{arc } FE + BH$.

Also the arc $d/c = 90^\circ = \text{alt. and zen. dist. taken together.}$

Therefore the zen. dist. $= \text{arc } fd + HC = \text{arc } DF + HC$.

125. *Allowing the Sun's semidiameter in using the Shade vane.* Fig. 10.

If AB represents the Sun, whose center is C , and its rays pass over the top of an upright wall DE ; then it is evident the ray cc only, gives the true altitude HC , or zenith distance zc : Also, that the sun must descend from the ray Aa , to come to cc ; and ascend from the ray bb to come to cc .

Therefore the Sun's semidiameter must be added in zenith distances, and subtracted in altitudes.

126. II. *Of the Theory of Hadley's Quadrant.*

To what has already been said of this most excellent instrument, it has been judged proper to shew also upon what principles it is constructed.

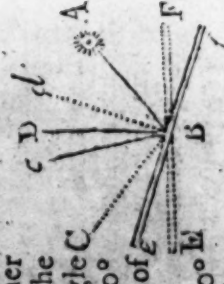
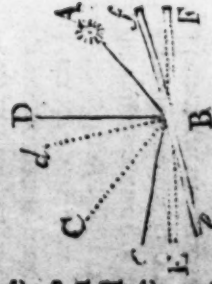
A ray of light AB falling on a mirror, or polished plane, EF , will be reflected in the line BC , so that the angle of reflection DBC , shall be equal to the angle of incidence DEA , DB being perpendicular to EF : or $\angle FBA$ is equal to $\angle EBC$.

Let the angle of incidence DBA be 30° , the angle of reflection DBC will be 30° also; and the like in other inclinations.

If while the radiant A remains in the same place, the mirror EF , by revolving round B , moves into the position fe ; then the alteration in the angle between the incident and reflected ray, will be double to the angle shewing the change of position in the mirror.

Suppose the mirror changes 10° from the radiant, then the perpendicular BD goes 10° farther from the radiant A , into the position bd ; and the incident angle dBA is 40° , and the reflected angle C dBC is 40° ; so the angle ABC is altered from 60° to 80° ; a difference of 20° , for 10° change of position in the mirror.

Suppose the mirror to change, in position, 10° towards the radiant A , then the perpendicular is changed into bd , 10° nearer to the radiant. And the angle DEA of 30° becomes dBA of $20^\circ = \angle dBC$; so that the angle ABC is 40° , whereas



whereas ABC was 60° ; that is, by altering the position of the mirror 10° the angle APC was altered 20 degrees.

The following proposition was received from a friend*.

127. With a Hadley's quadrant, the angle of elevation under which an object is seen, is equal to twice the arc the index has passed over.

The instrument, or angle ACB , is 45 degrees. ab is a reflecting glass fixed parallel to CB . CD is the index, with a reflecting glass at C , so fixed as to be parallel to ab , when CD coincides with CB . H

$\odot$ the Sun, $\odot G$ a solar ray reflected from G to F , and from F to E , the place of the eye.

HFE the horizontal line at the time of the observation.

Then is the angle of elevation 10° equal to twice the angle BCD .

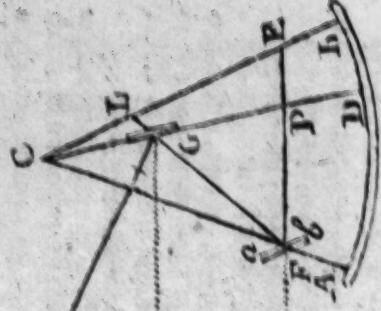
For the reflected $\angle DGF =$ incident $\angle OGC = \angle CGL$. (I. 93)

And the reflected $\angle bFE =$ incident $\angle aFG = \angle FLE = \angle FEL$. (I. 95)

Now $\angle IGC = (\angle HPC =) \angle PCE + (\angle PEC = \angle FLE =) \angle GCL + \angle CGL$ (I. 96)

Or $(\angle IGC =) \angle IGQ + \angle OGC = 2\angle BCD + \angle CGL$:

Then $\angle IGQ = 2\angle BCD$.



128. III. Of the Dip of the Horizon. Plate XIV. Fig. 12.

The eye being raised above the level of the sea, observes an horizon below the level of the true one.

Thus let asf be the surface of the sea, es the height of the eye above that surface, and AF the observer's horizon: Then the eye looking along ea for the horizon, it dipping below the true one, by the arc Aa , the apparent altitude as of an object B must evidently be lessened by the arc Aa : But if the observer has his back towards the object B ; then looking along the line ef , the apparent horizon now becoming the line fe , and dipping below the true one by the arc Ff , or Aa ; it is plain the apparent altitude es is too little, and must be augmented by the arc $Aa = Ff$.

Consequently in the fore observations, the altitude must be lessened by the horizontal dip; and in the back observations, the altitude must be increased by the dip; but in zenith distances, add the dips forwards, and subtract them backwards.

129. To compute the Horizontal Dip.

Let c be the earth's center, E the eye, and the tangent Ef meeting the surface in f , where the sky and water seem to meet; then $csfE$ is a triangle right-angled at f , wherein are known $cs = 20899059$ feet, the Earth's semidiameter, and $ce = (cs + se)$ the height of the eye added to the Earth's radius: Then as $CE : Rad :: cf : fm. \angle cEf$, whose complement is the $\angle fEf$, the dip, sought.

• Mr. William Payne.
C c 4

IV. Of

130.

IV. *Of Refraction.*

Celestial objects, the nearer they are to the horizon, are seen through a larger portion of the atmosphere, and consequently have a greater quantity to be allowed for refraction, than when seen with greater altitudes.

Tables of refraction have been given by many astronomers, but that which is esteemed the most correct, was formed from the observations of the late celebrated Dr. Bradley; who also found a rule for correcting common tables of refraction: which is,

Increase any given altitude by thrice the refraction on that alt.

Then to the log. cotangent of that sum, add the log. 1.75587.

The sum lessened by 10 in the index, is the log. number, of seconds in the refraction; which is more correct than the table.

Here 1,75587 is the log. of $57''$ the mean refraction on an alt. of 45° .

This rule is fitted to a mean warmth and weight of the atmosphere, viz. to the height 50 in Fahrenheit's Thermometer, and to the height 29.6 of the Barometer: For other states of the air, Dr. Bradley gave a rule for a further correction: But it is apprehended that neither of these corrections need be applied to the table of refractions, for any observations but such where is wanted a precision not to be obtained at sea.

131.

V. *Demonstrations of the rules at articles 60, 61, 62.*

See Plate XIV. Figures 13, 14, 15.

Let the circle $HZRN$ represent the meridian of the place, HR the horizon, and z its zenith or pole: Also let PS represent the axis, P being the north, and s the south pole; the line EQ the equator, $\odot$ the place of the Sun, the arc $z\odot$ the zenith distance, and $E\odot$ the declination.

Now the latitude is expressed either by the height of the pole above the horizon, or by the distance of the equator from the zenith; consequently the arc RP ($=HS$), or the arc zE , being known, will give the latitude.

But the arc zE is ever equal to the sum of the arcs $z\odot$ and $E\odot$ when of contrary names; or to their difference when of the same name.

When the object does not set, then its least altitude $R\odot$ added to the arc $P\odot$, or co-declination, gives RP , the height of the pole above the horizon, or the latitude sought: And the same may be understood, was the least altitude above the south point of the horizon. Fig. 15.

132. Hence it will be easy to construct any of these cases. Thus: Set the zenith distance from z , to the left if south, but to the right if north; and this gives $\odot$ the place of the Sun.

Set the declination from $\odot$, to the right if south, but to the left if north; and this gives E the place of the equator.

Then drawing the diameter EQ , and the diameter PS at right angles to EQ , the latitude will be north or south, according as P or s falls above the horizon.

VI. *Of*

133.

VI. Of Facio's Problem.

In the year 1728, Mr. *Nicholas Facio Duillier*, F. R. S. published a pamphlet, intitled, *Navigation Improved*, wherein is considered more at large, than had before been done, the problem for finding the latitude from two observations of the Sun's altitude out of the meridian, with the time between them, which he states in the following terms.

134.

PROBLEM.

To find the latitude, by two altitudes of the Sun, and the time between them; supposing the observations to have been made from the same place, or from different places, whether at sea or land. And likewise to find the hours, and the Sun's azimuths, at the times of both observations.

Thus he proposes expressly to distinguish between the case, when the two observations are made in the same place, and when, by the motion of the ship, in the interval between them, they are made under different zeniths; which do not appear to have been attempted before: He also extends the interval of time between the observations farther than is usually done; not confining them within the compass of one day, but so remote, that the change in the Sun's declination may be necessary to be taken notice of.

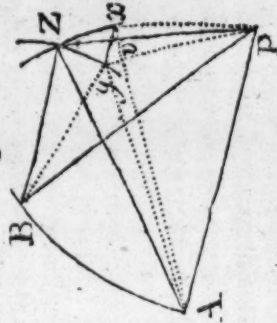
His method of proceeding is thus.

Supposing P the elevated pole; A and B the two places of the Sun, and AP , PB , AB , arcs of great circles passing through the points P , A , and B : Then if PA be the distance of the Sun from the Pole P in the first observation, and PB its distance in the second, the angle APB corresponding to the time between the observations, whether that angle represent the whole interval of time, or only its difference from one or more intire days, Z also denoting the zenith of the place, when both the observations are made under the same.

Here, if the interval of time be large, A , AP may be different from PB . Therefore in the triangle APB , from AP , BP , and the angle APB , he finds the side AB ; then from the three sides AP , BP , and AB , either of the angles PAB , PBA may be known; also from AZ , ZB and AB in the triangle AZB , either of the angles ABZ , BAZ ; and in the last place, from these angles with ABP , or BAP , may be deduced the angle ZAP or ZBP , whence in either of the triangles $P AZ$ or $P BZ$ may be computed PZ , the distance of the zenith Z from the pole.

He likewise farther proposes to compute the angle APZ , BPZ for assigning the distance of each observation from noon, in order to determine the time of the day when they were respectively made.

All these trigonometrical operations he performs by one axiom (See art. 251. Book IV.) in the doctrine of spherical triangles, which had



been applied by others also, not only to the finding an angle from three sides given, but likewise to the computing the third side from two sides, and an angle between them.

But when the ship is in motion, z is not to be considered as the zenith of the ship in either observation. However in the first observation the ship's zenith will always be in a circle described on A , as a pole, and with the interval Az , suppose in x , and in the other observation will be in a circle described to the pole B with the interval Bz , suppose in y , the arch xy denoting the ship's course, making with the meridian of the first observation the angle pxy .

This angle pxy , and the length of the arch xy are to be assigned by the compass and log-line.

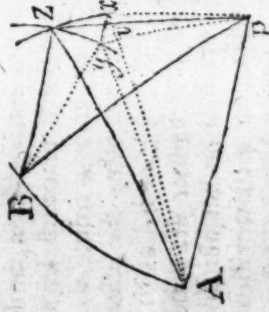
Now when the ship has been in motion during the interval between the observations, and these two additional *data* are added to the altitudes of the Sun, and the interval of time, *Facile* proceeds thus.

Having found the point z , as before, he computes in the triangle AzP , from AP , Az , Pz the angle AzP ; also in the triangle BzP from BP , Bz , Pz the angle BzP , and from the angles AzP , BzP , the angle AzB , which (the angles Azx and Bzy being both right) will be equal to yzx .

Then supposing the distance run xy so small an arch, that the triangle xyz may be considered as rectilinear, without any sensible error; and also, that the angle xpz is so small, that xy meeting Pz in v , the angle pvx may be assumed, as not sensibly differing from the angle of the ship's course; whence the angle BzP having been computed, and thence the angle yzv known (Bzy being a right angle) from the angles yzv and zvy , the angle zyx , which is the complement of the angle which the ship's course makes with Bz , will be known; then from the angles xyz , zyx , together with the side yx , he finds the side zx ; and lastly, in the triangle pxz from xz , zp , and the angle xzp , the side px is found for the complement of the ship's latitude at the first observation; as also the angle zpx , and thence the angles APx , the distance of the observation with the Sun in A from the meridian, in which that observation was made.

And farther, *Facile* considering this computation, though it come very near the truth, as an approximation only, from the assumption of the angles yxp , and zvx , as equal, and the triangle zxy as rectilinear, he proceeds to correct the latitude, and the angle xpz thus found, as follows.

He assumes at pleasure two values for zx (or rather for the arch of a great circle passing through z and x) one somewhat greater and the other somewhat less than the value of zx now computed. Then for each value of zx , he finds in the triangle pxz from the sides pz , zx , and the angle pxz , the side px and angle xpz ; and in the triangle pxy from px now found with xy , and the angle pxy given from the course and distance run by the ship from x to y , he computes py , and the angle xpy . Then



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Book IX.

DAYS WORK.

395

from the angles $\angle PZ$, $\angle P Y$ finding the angle $\angle P Z$, and thence $\angle P B$, at length in the triangle $B P Y$ from $\angle P B$, $\angle P Y$, and the angle $\angle B P Y$ computes $B Y$.

If $B Y$, thus computed, comes out equal to $B Z$, then is the value of z , whence it is deduced, rightly assumed; but if the value in both the computations differ from the truth, $P x$ and the angle $\angle Z P x$ are to be corrected by the simple rule of false position.

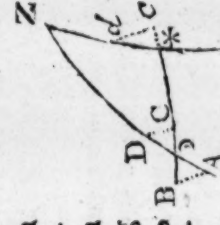
But this method of Facio is attended with an operose calculation, even though the second correction, which will always be but small, should be neglected. And the expedient proposed by Mr. Graham (Philosophical Transactions, N^o 435) of observing by an azimuth compass the angle, which the ship's course makes with the azimuth circle of the Sun in one of the observations, which has been explained above (Book IX. article 76) is much more simple.

135.

VII. Of Mr. de la Caille's Scales.

As the preceding methods of computing the true distance of the Moon from the Sun, or from a fixed star, requires a calculation, by some, supposed too operose for mariners in general to be easily reconciled to; an instrument, accompanied with three compound scales, is proposed by the late able astronomer, the Abbe de la Caille. These scales are not described with sufficient distinctness to enable a workman to make them with accuracy. But the first is no other than two scales of natural sines set at right angles, to one another, and is made necessary only from his method, which requires, besides the distance of the Moon from the star, the altitudes of that star to be observed a little time before the distance is taken, and both the altitude and azimuth of the Moon to be observed some little time after, (supposing all the observations to be taken by one person) The other two scales are the principal, and are contrived for assigning the effect of the Moon's parallax, and the refraction both of the Moon and star upon their apparent distance; and are grounded on this principle, that the effect of the Moon's parallax on the observed distance is to the parallax in altitude, as the cosine of the angle between the Moon's azimuth circle and her distance from the star, to the radius; and that the like proportion obtains, in regard to the refraction of the Moon and the star.

If z be the zenith of any place, $z A$ the azimuth circle in which the Moon is at $\angle$; but by her parallax appearing in A , and $z *$ be the azimuth wherein the star is placed at $*$; a perpendicular, as $A B$, being let fall on $\angle *$ from A ; the parallax in altitude, $\angle A$, being will be to $\angle B$, the effect of that parallax on the distance $\angle *$, as radius to the cosine of the angle $A \angle B$. In like manner if $\angle$ be the place of the Moon, $\angle D$ the effect of the refraction on her altitude, and $\angle c$ be the effect of that refraction on the distance $\angle *$; $\angle D$ will be to $\angle c$, as radius to the cosine of the angle $D \angle c$; and the refraction of the star will have a similar effect on the distance $\angle *$.



Now

136. VIII. *Of the times of the Moon's phases, and the time of High-water.*

The methods given in Book VI, at articles 130, 135, 136, for computing the Moon's age, southings, and time of high-water, frequently give those times many hours wide of the truth, and thereby may occasion fatal accidents: It has therefore been judged proper to shew how the times of high-water are to be found more accurately; unless when the tides are made irregular by strong winds.

In the Nautical Almanac, the first page of every month contains the Moon's phases at Greenwich; which are easily transferred to any other place, whose longitude is known, by the following

RULE. To the Greenwich time of the proposed phase, apply the difference of longitude in time (57), will give the time of that phase at the place proposed.

EXAM. I. *At what times in May 1773, will the several phases of the Moon happen at Canton in China?*

Canton is E. of Greenwich ($113^{\circ}3' =$) 7h. 32m.

Then the times at Canton for the Moon's phases in May 1773, will be

Full Moon	6d. 12h. 34m.	(=6d. 5h. 2m.+7h. 32m.)
Last Quarter	13	7 0
New Moon	21	10 15
First Quarter	29	4 55

EXAM. II. *At what times in March 1773, will the several phases of the Moon happen at Halifax in Nova Scotia?*

Halifax is west of Greenwich ($63^{\circ}31' =$) 4h. 14m.

Then at Halifax, the times of the phases required will be.

First Quarter	1d. 0h. 17m.	(=1d. 4h. 31m.-4h. 14m.)
Full Moon	8	6 41
Last Quarter	14	22 15
New Moon	22	12 8
First Quarter	30	17 45

137.

Of the time of high-water.

The times of high-water at the syzgies are nearly the same in most places; but the times of high-water from day to day between the syzgies, does not alter 48 minutes, as is usually reckoned in pilorage; the difference being sometimes more, sometimes less, according as the waters are acted on, either by the sum of the attractive forces of the Sun and Moon, or by their difference, considered in respect to the places of the Earth and Moon relative to the Sun.

The following table * being used with the time of either phase of the Moon, will, in general, shew the time of high-water within a quarter of an hour of the truth at any known port, when the tides are not greatly influenced by the winds.

138.

TABLE

Of the correction to be applied to the time of high-water on the syzgies, to give the time of high-water on any other day.

Interv. of time.	D.	H.	After n. moon		Before 1st qu.		After 1st qu.		Before f. moon		After f. moon		Before 3d qu.		After 3d qu.		Before n. moon		Interv of time.	
			Addit.	H. M.	Addit.	H. M.	Addit.	H. M.	Addit.	H. M.	Addit.	H. M.	Addit.	H. M.	Addit.	H. M.	Subtr.	H. M.	Subtr.	D. H.
0	0	0	0	0	5	6	5	6	0	0	0	0	5	6	5	6	0	0	0	0
6	0	8	0	8	4	51	5	22	0	9	0	8	4	51	5	22	0	9	0	6
12	0	17	0	17	4	37	5	40	0	18	0	17	4	37	5	40	0	18	0	12
18	0	26	0	26	4	23	6	0	0	27	0	26	4	23	6	0	0	27	0	18
1	0	0	0	36	4	9	6	20	0	37	0	36	4	9	6	20	0	37	0	1
6	0	45	0	45	3	56	6	39	0	47	0	45	3	56	6	39	0	47	0	6
12	0	54	0	54	3	44	6	58	0	57	0	54	3	44	6	58	0	57	0	12
18	0	1	0	2	3	32	7	18	0	1	0	2	3	32	7	18	0	1	0	18
2	0	11	1	11	3	21	7	37	1	17	1	11	3	21	7	37	1	17	1	0
6	1	19	1	19	3	11	7	56	1	28	1	19	3	11	7	56	1	28	1	6
12	1	28	1	28	3	1	8	14	1	39	1	28	3	1	8	14	1	39	1	12
18	1	37	1	37	2	50	8	31	1	51	1	37	2	50	8	31	1	51	1	18
3	0	1	1	46	2	40	8	47	2	4	1	46	2	40	8	47	2	4	3	0
6	1	54	2	54	2	30	9	2	2	16	1	54	2	30	9	2	2	16	2	6
12	2	3	2	21	2	21	9	17	2	29	2	3	2	21	9	17	2	29	2	12
18	2	12	2	12	2	12	9	31	2	44	2	12	2	12	9	31	2	44	2	18
4	0	2	2	21	2	3	9	44	2	58	2	21	2	3	9	44	2	58	2	0

If the time to be sought in this table is between the hours in the outside columns, the equation is to be corrected by proportional parts.

* From M. de la Caille.

138. PROPOSITION I.

The time of high-water in any port on the syzigies, or days of new and full moon, being known; to find the time of high-water in that port on any other day.

SOLUTION.

1st. In the Nautical Almanac, seek in the given month, or in that immediately preceding or following it, for the time of that phase which happens nearest to the given day; and take the difference of those times.

2d. In table (138.) this diff. of time gives an equation, which, as its title directs, being applied to the time of the syzigie high water for the given port (IV. 137) gives the time of the afternoon high-water in that port.

But if the time of the morning high-water is required.

1st. Do as directed in the 1st article above, and increase the said diff. by 12 hours, if the given day falls before the phase.

Or diminish it by 12 hours, when after the phase.

2d. The equation to this time (138.) applied to the syzigie time (IV. 137) gives the morning time of high-water,

Ex. I. At Portsmouth, where it is high-water on the Syzigies at 11 h. 15 m. Required the afternoon and morning times of high-water on the 13th of January, and on the 2d of February, in the year 1773?

Here the nearest phase to January 13, is Jan. 15 d. 10 h. 10 m.

So the time proposed is 2 d. 10 h. 10 m. before the 3d, or last, quarter.

To 2 d. 10 h. and before 3 d. 9 h. the equation is 3 h. 5 m.

Then 11 h. 15 m. + 3 h. 5 m. = 14 h. 20 m. or high-water at 2 h. 20 m. P. M.

Again 2 d. 10 h. 10 m. + 12 h. = 2 d. 22 h. 10. the eq. to which is 2 h. 44 m.

Then 11 h. 15 m. + 2 h. 44 m. = 13 h. 59 m.; or high water at 1 h. 59 m. A. M.

To Feb. 2d. the nearest phase is the 1st quarter, on Jan. 30 d. 7 h. 53 m.

And as Feb. 2d, may be called

Their diff. or time after the first quarter, is 2 d. 16 h. 7 m.

The equation to which is 8 h. 26 m.

Then 11 h. 15 m. + 8 h. 26 m. = 19 h. 41 m. Or high-water at 7 h. 41 m. P. M.

Again 2 d. 16 h. 7 m. - 12 h. = 2 d. 4 h. 7 m; the equ. to which is 7 h. 50 m.

Then 11 h. 15 m. + 7 h. 50 m. = 19 h. 5 m.; Or high-water at 7 h. 5 m. A. M.

Ex. II. On the 11th. of March 1773, Required the times of high-water at Halifax in Nova Scotia, where the syzigie time of high-water is at 7 h. 30 m.?

Halifax is W. of Greenwich ($61^{\circ} 31' =$) 4 h. 14 m.

The nearest phase to March 11, is full Moon, at 8 d. 10 h. 55 m.

Which fitted to Halifax, becomes

March 11, is after full Moon, 2 d. 17 h. 19 m. its equation is 1 h. 35 m.

Then 7 h. 30 m. + 1 h. 35 m. = 9 h. 5 m. P. M. the time of high-water.

Again 2 d. 17 h. 19 m. - 12 h. = 2 d. 5 h. 19 m.; its equation is 1 h. 18 m.

Then 7 h. 30 m. + 1 h. 18 m. = 8 h. 48 m. A. M. the time of high-water.

140.

PROPOSITION II.

The time of high water in any Port being observed on any day: To find the time of high water in that port on the syzigies.

SOLUTION.

1st. By the difference of longitude, reduce the given time to Greenwich time.

2d. Take the difference between this time and that of the Moon's nearest phases.

3d. The equation to this diff. (in table art. 138.) applied to the said Greenwich time, contrary to what its column directs, give the syzigie time of high water at that port.

EXAM. I. Suppose the time of high water was observed in a port to be at 5 h. P.M., when it wanted 3d. 17 h. of the time of new moon; Required the time of high water there on the day of new moon?

Now 3d. 17 h. before new moon, gives an equation of 2 h. 41 m.

The column directs subtraction, the contrary is addition.

Then 5 h. + 2 h. 41 m. = 7 h. 41 m. P.M. the syzigie time of high water.

Also 3d. 17 h. + 12 h. = 4 d. 5 h. the equation to which is 3 h. 10 m. +

Then 5 h. + 3 h. 10 m. = 8 h. 10 m. A.M. the time of new-moon high water.

EXAM. II. Suppose on the 19th of March, 1773, at 8 h. 11 m. P.M. it was observed to be high water in the river Senegal? What is the time of high water on the full and change days?

The river Senegal is west of Greenwich ($16^{\circ} 20' =$) 1 h. 5 m.

The nearest phase to March 19th is the third quarter, at 15 d. 2 h. 29 m. which fitted to Senegal, makes 15 d. 1 h. 24 m.

The 19th day is after the 3d quarter, 3 d. $22\frac{1}{2}$ h.; its equation is 9 h. 41 m. which is subtractive, contrary to its title in the table.

Then 8 h. 11 m. - 9 h. 41 = 10 h. 30 m. the time of high water on the days of the full and change of the Moon in the river Senegal.

141. Of the apparent rising of the Moon at nearly the same time for several days together.

There are few mariners who have not observed at certain times, that the Moon appears to rise or set for several days together at the same time nearly, notwithstanding that she is known to be retarded nearly 48 minutes in each day's motion.

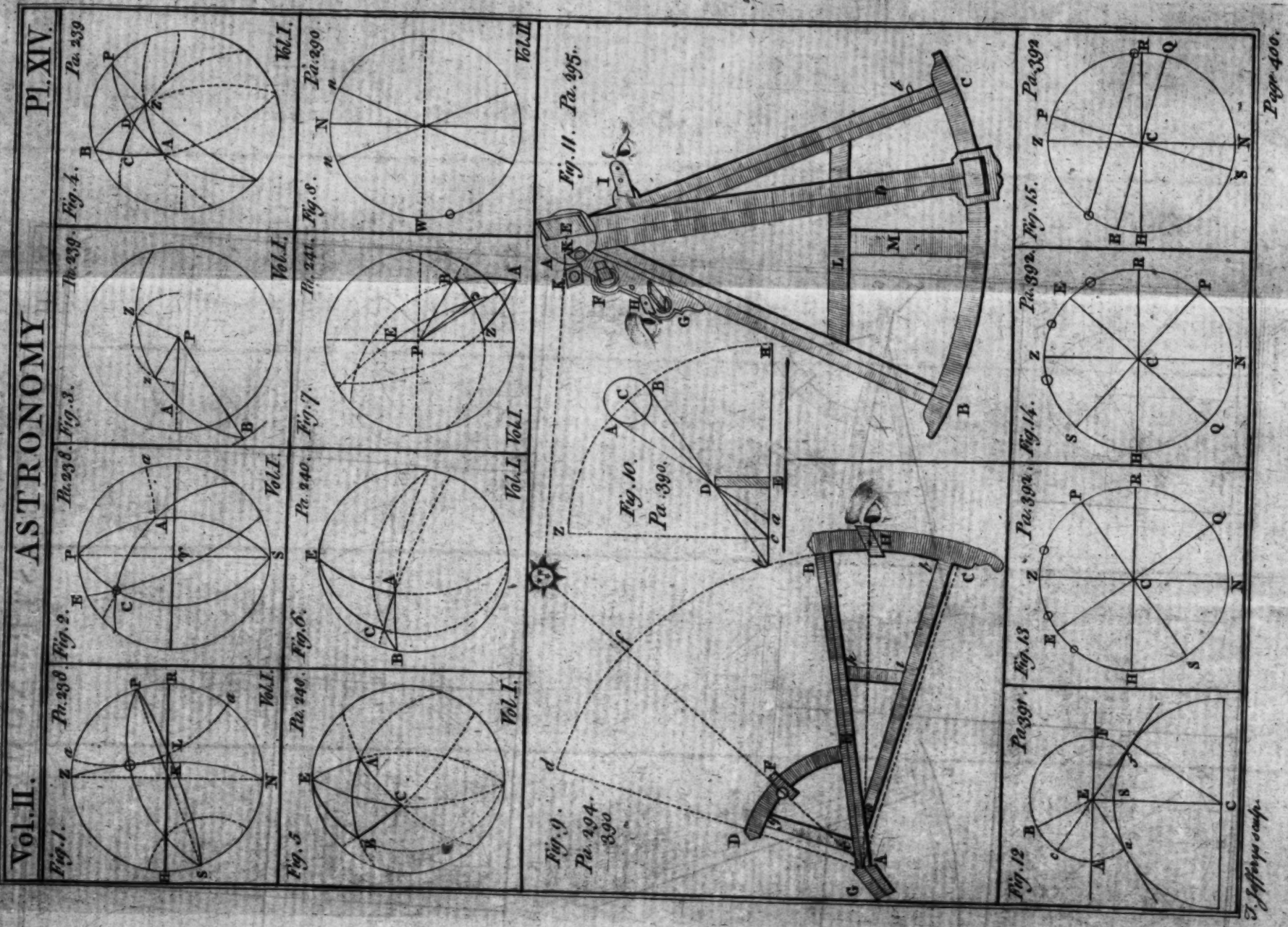
This appearance arises chiefly from the daily alteration of 5 or 6 degrees in the Moon's declination, when she is near the celestial equator: Now a change in the declination of celestial bodies, will cause them to rise or set sooner or later, as well as when the observer changes his latitude.

Thus in the latitude of 50 degrees, when the Sun has 6° of declination, he rises at 5 h. 31 m.; but when he has no declination, he rises at 6 o'clock; so that a change of 6° in declination near the equinox, accelerates the rising 29 minutes.

By a like change in the Moon's declination, her rising or setting may be quickened or delayed half an hour; but half an hour's acceleration, diminishes a great part of the 48 minutes of daily retardation which she ought to have; and if to this be joined the apparent acceleration which may be caused by a less velocity in the motion of the Moon near her apogee, and also that which may arise from an alteration of 3 or 4 degrees in the ship's latitude in 24 hours; it will follow, that the rising of the Moon will not appear to be sensibly retarded for several days together.

Now what accelerates the rising, does retard the setting; so when the Moon appears to rise for several days together about the same hour, its setting will appear to be retarded nearly the double of 48 minutes, or about an hour and half: On the contrary, if its setting is nearly about the same hour for several days, its rising will appear to be retarded almost double of its known daily motion.

Articles

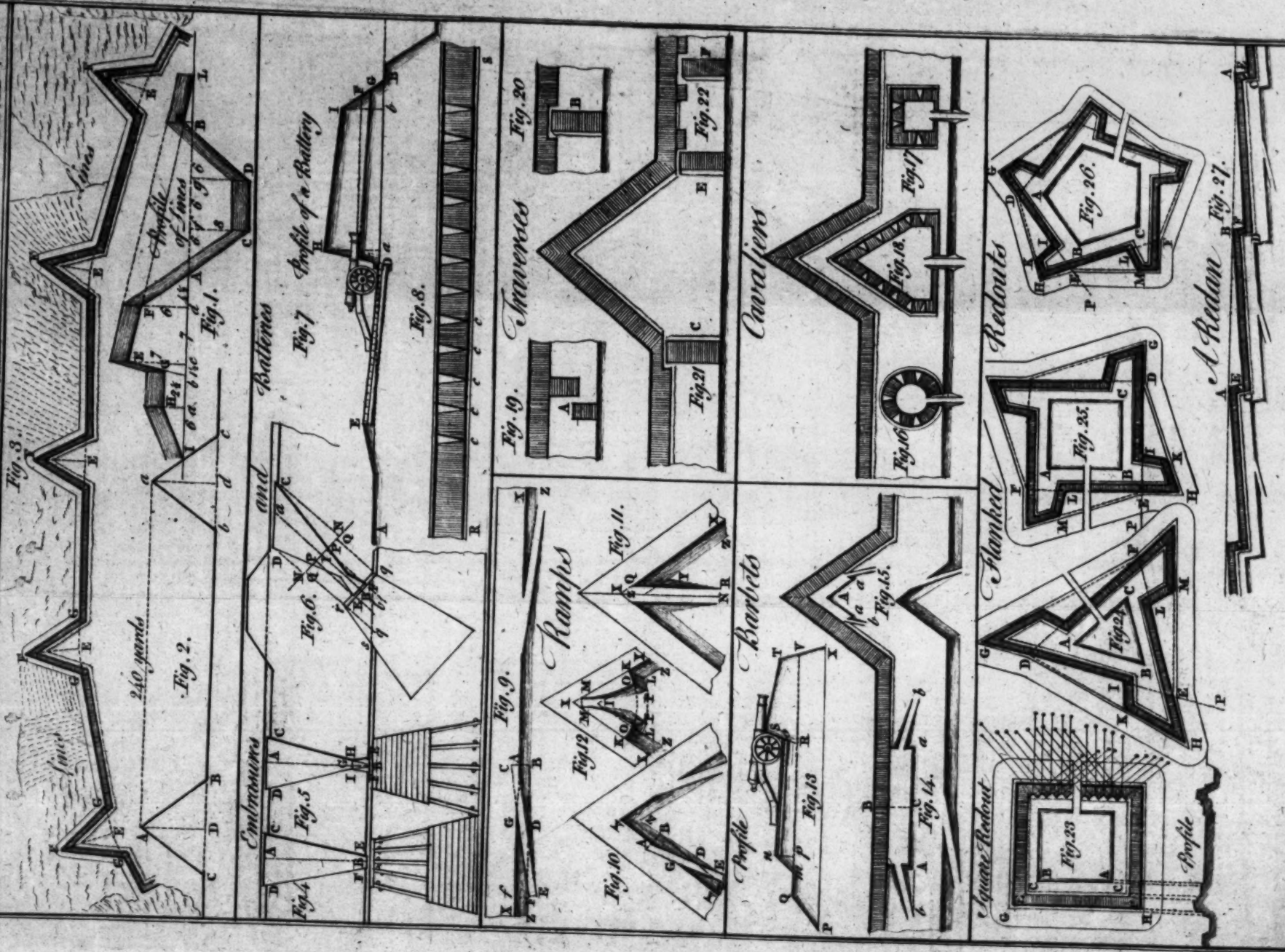


J. Jefferys sculp.

449 TABLE OF SINES AND TANGENTS. 145.

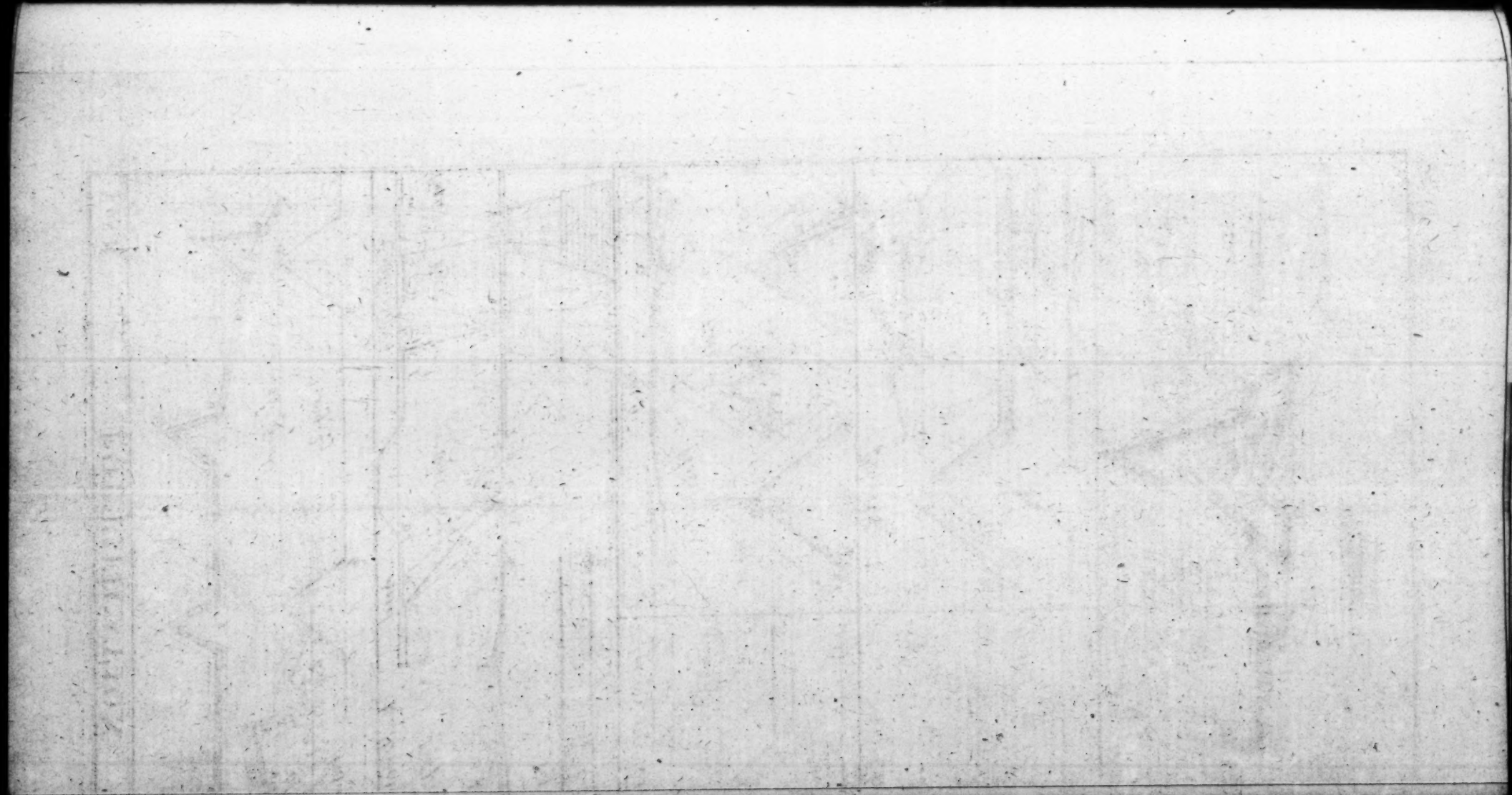
44 Degrees					45 Degrees				
M	Sine	Co-fine	Tang.	Co-tang.	M	Sine	Co-fine	Tang.	Co-tang.
0	9,84177	9,85693	9,98484	10,01516	60	9,84177	9,85693	9,98484	10,01516
1	9,84190	9,85681	9,98509	10,01491	59	9,84190	9,85681	9,98509	10,01491
2	9,84203	9,85669	9,98534	10,01466	58	9,84203	9,85669	9,98534	10,01466
3	9,84216	9,85657	9,98560	10,01440	57	9,84216	9,85657	9,98560	10,01440
4	9,84229	9,85644	9,98585	10,01415	56	9,84229	9,85644	9,98585	10,01415
5	9,84242	9,85632	9,98610	10,01390	55	9,84242	9,85632	9,98610	10,01390
6	9,84255	9,85620	9,98635	10,01365	54	9,84255	9,85620	9,98635	10,01365
7	9,84268	9,85608	9,98661	10,01339	53	9,84268	9,85608	9,98661	10,01339
8	9,84281	9,85596	9,98686	10,01314	52	9,84281	9,85596	9,98686	10,01314
9	9,84295	9,85583	9,98711	10,01289	51	9,84295	9,85583	9,98711	10,01289
10	9,84308	9,85571	9,98736	10,01263	50	9,84308	9,85571	9,98736	10,01263
11	9,84320	9,85559	9,98762	10,01238	49	9,84320	9,85559	9,98762	10,01238
12	9,84334	9,85546	9,98787	10,01213	48	9,84334	9,85546	9,98787	10,01213
13	9,84346	9,85534	9,98812	10,01188	47	9,84346	9,85534	9,98812	10,01188
14	9,84359	9,85522	9,98838	10,01162	46	9,84359	9,85522	9,98838	10,01162
15	9,84372	9,85510	9,98863	10,01137	45	9,84372	9,85510	9,98863	10,01137
16	9,84385	9,85497	9,98888	10,01112	44	9,84385	9,85497	9,98888	10,01112
17	9,84398	9,85485	9,98913	10,01087	43	9,84398	9,85485	9,98913	10,01087
18	9,84411	9,85473	9,98939	10,01061	42	9,84411	9,85473	9,98939	10,01061
19	9,84424	9,85460	9,98964	10,01036	41	9,84424	9,85460	9,98964	10,01036
20	9,84437	9,85448	9,98989	10,01011	40	9,84437	9,85448	9,98989	10,01011
21	9,84450	9,85436	9,99014	10,00985	39	9,84450	9,85436	9,99014	10,00985
22	9,84463	9,85423	9,99040	10,00960	38	9,84463	9,85423	9,99040	10,00960
23	9,84476	9,85411	9,99065	10,00935	37	9,84476	9,85411	9,99065	10,00935
24	9,84489	9,85398	9,99090	10,00909	36	9,84489	9,85398	9,99090	10,00909
25	9,84502	9,85386	9,99116	10,00884	35	9,84502	9,85386	9,99116	10,00884
26	9,84515	9,85374	9,99141	10,00859	34	9,84515	9,85374	9,99141	10,00859
27	9,84528	9,85361	9,99166	10,00834	33	9,84528	9,85361	9,99166	10,00834
28	9,84540	9,85349	9,99191	10,00809	32	9,84540	9,85349	9,99191	10,00809
29	9,84553	9,85337	9,99217	10,00783	31	9,84553	9,85337	9,99217	10,00783
30	9,84566	9,85324	9,99242	10,00758	30	9,84566	9,85324	9,99242	10,00758
31	9,84579	9,85312	9,99267	10,00733	29	9,84579	9,85312	9,99267	10,00733
32	9,84592	9,85299	9,99292	10,00707	28	9,84592	9,85299	9,99292	10,00707
33	9,84605	9,85287	9,99318	10,00682	27	9,84605	9,85287	9,99318	10,00682
34	9,84617	9,85274	9,99343	10,00657	26	9,84617	9,85274	9,99343	10,00657
35	9,84630	9,85262	9,99368	10,00632	25	9,84630	9,85262	9,99368	10,00632
36	9,84643	9,85250	9,99393	10,00606	24	9,84643	9,85250	9,99393	10,00606
37	9,84656	9,85237	9,99419	10,00581	23	9,84656	9,85237	9,99419	10,00581
38	9,84669	9,85225	9,99444	10,00556	22	9,84669	9,85225	9,99444	10,00556
39	9,84682	9,85212	9,99469	10,00531	21	9,84682	9,85212	9,99469	10,00531
40	9,84694	9,85200	9,99495	10,00505	20	9,84694	9,85200	9,99495	10,00505
41	9,84707	9,85187	9,99520	10,00480	19	9,84707	9,85187	9,99520	10,00480
42	9,84720	9,85175	9,99545	10,00455	18	9,84720	9,85175	9,99545	10,00455
43	9,84733	9,85162	9,99570	10,00429	17	9,84733	9,85162	9,99570	10,00429
44	9,84745	9,85150	9,99596	10,00404	16	9,84745	9,85150	9,99596	10,00404
45	9,84758	9,85137	9,99621	10,00379	15	9,84758	9,85137	9,99621	10,00379
46	9,84771	9,85125	9,99646	10,00354	14	9,84771	9,85125	9,99646	10,00354
47	9,84784	9,85112	9,99671	10,00328	13	9,84784	9,85112	9,99671	10,00328
48	9,84796	9,85100	9,99697	10,00303	12	9,84796	9,85100	9,99697	10,00303
49	9,84809	9,85087	9,99722	10,00278	11	9,84809	9,85087	9,99722	10,00278
50	9,84822	9,85074	9,99747	10,00253	10	9,84822	9,85074	9,99747	10,00253
51	9,84834	9,85062	9,99773	10,00227	9	9,84834	9,85062	9,99773	10,00227
52	9,84847	9,85049	9,99798	10,00202	8	9,84847	9,85049	9,99798	10,00202
53	9,84860	9,85037	9,99823	10,00177	7	9,84860	9,85037	9,99823	10,00177
54	9,84873	9,85024	9,99848	10,00152	6	9,84873	9,85024	9,99848	10,00152
55	9,84885	9,85012	9,99874	10,00126	5	9,84885	9,85012	9,99874	10,00126
56	9,84898	9,84999	9,99899	10,00101	4	9,84898	9,84999	9,99899	10,00101
57	9,84911	9,84986	9,99924	10,00076	3	9,84911	9,84986	9,99924	10,00076
58	9,84923	9,84974	9,99949	10,00050	2	9,84923	9,84974	9,99949	10,00050
59	9,84936	9,84961	9,99975	10,00025	1	9,84936	9,84961	9,99975	10,00025
60	9,84948	9,84948	10,00000	10,00000	0	9,84948	9,84948	10,00000	10,00000

END OF BOOK IX.





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A
T R E A T I S E
O F
M A R I N E F O R T I F I C A T I O N .

BY MARINE FORTIFICATION, is meant the art of raising works fit for the defence of a harbour against the attacks of any kind of shipping.

As the works proper for this purpose depend in a great measure on the principles employed in the fortification of towns; therefore it will be necessary to give some general notions of the usual methods of fortifying, previous to the measures that seem most effectual to the securing of harbours.

It is not intended in this tract to give a complete system of fortification; and consequently the reader will not be perplexed with the different systems, as they are called, of *such* and *such* Engineers; many of them being the productions of dabblers in the art, who, having diverted themselves with conundrums and whimsical scratchings on paper, published their conceits for a like amusement to others; and gravely compared their *pretty drawings* with the approved works of experienced Engineers: It is high time that most of their names and notions were forgot, and that those who hereafter may write on this subject, should confine themselves to the description of such works only, as are most in use, and which by experience have been found to be most conducive to the expected defence of the place, and retardation of the approaches of the enemy. And this is what is briefly attempted in the following pages.

MARINE FORTIFICATION.

PART I.

Of Land Fortifications.

2. **FORTIFICATION** is the art of making certain works about a place, whereby the persons within may be able to defend themselves against the attacks of a much greater number from without, more effectually and longer than if the place was not furnished with such works.
3. Defensive works usually consist of moats and banks, covering the place which they defend.
4. A place is said to be covered, either when the enemy is prevented from coming to that place by a body of troops or other hindrance; or, when a sufficiently strong bank, raised before that place, prevents those without from seeing them within.

5. Defensive works are of various kinds, the most simple are usually called lines.

SECTION I.

Of Lines or Intrenchments.

6. A **LINE**, or an **INTRENCHMENT**, consists of a bank of earth and a ditch, and is of two sorts.

First, Such as the trenches carried on in a siege, where the bank is between the ditch and the enemy or town; and here the earth is thrown up against the town, because the workmen are thereby better covered in carrying on the works.

Secondly, Such as serve to inclose a town or camp, secure a pass, cover the entrance into a country, or other place, &c.; in these the ditch is between the bank and the enemy.

The first fort are usually called trenches or approaches; and the second are generally called lines; which are the fort that will be here more particularly treated on.

7. Lines are generally made to shut up an avenue or entrance to some place; the sides of that entrance being covered by rivers, woods, mountains, morasses, or other obstructions not easy to be passed by an army: Though there have been cases where lines were constructed in plain countries; but then they were carried quite round the place they were to defend, as the lines surrounding a camp, called lines of circumvallation. Lines are likewise thrown up to stop the progress of an army, as those made by Marshal Villars and others. However, in what follows it will be supposed that the line is to cover a pass that can only be attacked in front.

8. The making of lines greatly depends on the nature of the place, and the time that can be taken to construct them; those done at leisure being very different from those done in haste.

The

The works usually thrown up in haste are called *temporary*; they serving chiefly on a present emergency: Those made at leisure are generally intended to last many years, and are kept in repair, under a probability of their being frequently attacked. But whether the work be temporary or lasting, it is necessary that it be adapted to the materials at hand, or that can be procured with the least expence and trouble.

A road or pass may be advantageously held by few men against a much greater number, many ways; as by turning a water-course, planting of stakes, felling of trees, digging of ditches, making a fence of stones, &c. But the way now most usually practised is, by making a line after some such like method as contained in the following precepts.

9. *To throw up a Line across a pass.*

1st. Having chose the place judged most convenient for the line, let a rope, or haybands, or twisted straw or grafs, be run quite across the way along the intended place of the line, pegging it to the ground at every 4 or 5 yards; and at the distance of about 10 or 12 feet before the line, towards the enemy, let such another line, or a row of stakes, be carried in a parallel position to the first rope.

2d. Range the labourers or troops within those limits, either single, in pairs, or treble, &c. according to the number there are to employ, or the time in which the work is to be done, each being furnished with proper digging tools.

3d. Let the earth, as they dig it up in this breadth, be thrown on the other side of the first rope, until a bank of about 5 or 6 feet thick and 6 or 7 feet high be raised; observing that the sides of the ditch be sloped in about the same manner as the earth naturally takes in rolling down the bank; and continue the digging till the ditch is about 5 or 6 feet deep, the breadth of the bottom being about $\frac{1}{3}$ of the breadth flaked out at top: And while the ditch is digging, if the earth as it is thrown up be trod down, the bank will be more compact; or a few hand-rammers (such as the paviours use) kept constantly going, will render the work more firm or durable.

4th. Let the inner side of the bank be pared with the spade into such a slope, as a man standing upright may easily touch, with his arm-extended straight before him; and at the foot of this bank, let a *foot-bank* or step be raised, of such a height, as a man standing on it may easily fire his musket over the bank; that is, let the top of the foot-bank be about four feet and a half lower than the top of the bank or *breast-work*.

5th. If there is time and opportunity, let the workmen throw up the loose and scattered earth upon the bank, on both sides, and slap or beat it with the flats of their shovels to make it the more neat and compact: Also make a gentle slope to the top of the *foot-bank*, that the troops may ascend it the more easily; and let the *crown* or *top* of the *breast-work* be

* A 2

sloped

4 MARINE FORTIFICATION.

sloped so, that a musket laid flat on it may strike the ground with its shot about 5 or 6 feet beyond the ditch.

When the soil is clay, sand, turf, or mould, the workmen will need only spades and shovels: when gravelly or stony, then one man should break the ground with a pickaxe or mattock, while another throws it upon the bank, where a third should spread it level and tread it down.

10. A work of this kind being completed, the troops behind the *line* may wait the coming of an enemy with less concern than without it: For 1st, the bank or breast-work secures them from the enemy's fire; 2d. And when they stand on the foot-bank, they are more than 3 covered, while the enemy are wholly exposed; and consequently the troops within may make three of their shots tell for one of the enemy's. 3d. They can, by going off the foot-bank, be quite covered while they load again. And on the whole, they are in no great danger of being forced from the lines while they have courage enough to keep possession of them, except the enemy are greatly superior in numbers and cannon.

The magnitude of lines are generally proportioned to the little time that can be spared to throw them up, and to the strength of the enemy who are to attack them. The following forts are what have been practiced on several emergencies, and the rate at which men ordinarily work on such occasions.

II. A TABLE of the *dimensions of Lines.*

Breast-work			Ditch		Expence	
Thickness at top	Height within	Height without	Upper breadth	Lower breadth	Solid content	Days works
4 feet	7½ feet	6 feet	8 feet	2½ feet	4½ ft.	1
5	7½	6	10	3½	6½	1½
6	7½	6	12	4	8	2
7	7½	6	14	4½	10	2½
8	7½	6	16	5½	12	3
9	7½	6	18	6	15	3½

The day's work here, is for one yard in length.

In the first, 100 men will complete 100 yards in length of this kind of lines in one day; 200 men in half a day; 400 men in a quarter of a day, and so on.

12. When lines are thrown up at leisure, then the ditch is usually 18 feet broad at top, 7 or 8 feet deep, and the sides of the ditch are to sloped, as to leave only 6 feet breadth at bottom; the breast-work, or parapet, is about 7 feet thick on the top or crown, and 7 or 8 feet high.

In Plate XV. fig. 1. shews the heights, depths, and breadths of the several parts of a line well designed and finished.

When

Where

IL represents the ground-line, or surface of the place.

AB, the breadth of the ditch at the top.

CD, the breadth of the ditch at bottom.

FAC, is the slope or scarp of the parapet and ditch.

DBK, is the counterescarp.

EF, the top or crown of the parapet or breast-work.

EG, the inner slope of the parapet.

HG, the top of the foot-bank.

HI, the slope of the foot-bank.

BKL, a small sloping bank called the glacis.

13. *This figure is called a section or profile, and may be thus drawn.*

1st. In the ground-line, lay off, from any scale of equal parts, the distances $1a=6$ feet, $ab=4$ ft. $bc=1\frac{1}{2}$ ft. $cd=7$ ft. $dA=4\frac{1}{2}$ ft.

$Af=6$ feet, $fg=6$ ft. $gB=6$ ft. $BL=5$ ft.

2d. Through a, b, c, d, e, f, g, B , draw lines perpendicular to IL.

3d. Make $aH=2\frac{1}{2}$ ft. $=bG$; $cE=7$ ft. $dF=6$ ft. $fC=8$ ft. $=gD$.

4th. Draw IH, HG, GE, EF, FAC, CD, DB, which continue till it meets the line FL, and the profile is constructed.

14. When lines are made to cover a camp, or a large tract of land where a considerable body of troops are posted, the work is not made in one straight, or uniformly bending line: But at certain distances, the lines project in salient angles toward the enemy. These projections, or salient angles, are called REDENTS, or FLANKERS; because the fire, from them, takes the enemy in flank or sideways as they march to attack the lines.

The distance between the salient angles of the flankers is usually between the limits of 200 and 260 yards; the ordinary flight of a musket-ball, point blank, being generally within those limits; although muskets, a very little elevated, will do effectual service at the distance of 360 yards.

15. A flanker consists of two lines, called faces, meeting in a point, and forming an angle called the *salient angle*, or *flanked angle*.

That part of the line between two adjacent flankers, is called the *curtin*.

The *gorge* of a flanker is the distance between its faces taken at their intersection with the curtains.

16. In Plate XV. fig. 2. shews the forms of the usual lines; where the figures, CAB, cab , are redents, or flankers.

AC, AB, ac, ab , the faces.

CB, cb , the gorges.

AD, ad , the capitals.

Bb, the curtain.

The angle CAB, cab , the salient, or flanked, angle.

The distance of the salient angles is about 240 yards on a mean.

The length of the capital is usually between 40 and 50 yards.

The length of the gorges is also about 60 or 70 yards.

17. To make a Plan of lines with redents. Plate XV. Fig. 3.

1st. Let the line $EEEE$, $\mathcal{E}c$. be so drawn, if possible, that wherever there is a bend or angle, it may either be at once, or twice, or thrice, $\mathcal{E}c$. the length of about 240 yards from one another, whereby a redent will always be where there is an angle.

2d. In this line lay off distances of 240 yards each, as from E to E , E to E , $\mathcal{E}c$. reckoning from the bends toward each end, whether it happens that the line will, or will not be exactly measured by a repetition of the 240 yards.

3d. At each point E draw the capital EF in a perpendicular position to the direction of the line in that point, and make the capitals about 40 or 50 yards long.

4th. On each side of E take the half gorge EG , EG , each of about 30 or 35 yards, and draw the faces FG , FG ; and the out-line, or master-line of the curtains and redents are formed.

5th. Parallel to each curtain and face draw lines, within, at the distances from the master-line of 7 feet, 8 feet, 12 feet, and 18 feet.

Then the breadth of 7 feet, represents the plan of the parapet.
 of 1 foot. its inner slope.
 of 4 feet the top of the foot-bank.
 of 6 feet the foot-bank slope.

6th. On the outside of the master-line, draw lines at the distances of $10\frac{1}{2}$, $16\frac{1}{2}$, and $22\frac{1}{2}$ feet, parallel to each curtain and face; and these will represent the plans of the scarp, ditch, and counter-scarp; observing that the salient angles of the counter-scarp are rounded before the angles of the redents.

When plans are drawn from a large scale, all the lines before mentioned are drawn: But when the plan is drawn from a small scale, as of 20 yards or more to an inch, then the plan is usually represented by four parallel lines; one without the master-line, representing the counter-scarp or out-line of the ditch; and two within, representing the breadths of the parapet and foot-bank.

Sometimes a short line is hastily formed by a *CHEVAUX DE FRISE*; which is a beam of 6 or 8 inches square, and 10 or 12 feet long; the sides being bored through with holes about an inch and half diameter, and 6 inches distant from one another; through each hole goes a staff of about 5 or 6 feet long, armed at both ends with an iron spike, so that whichever way it is laid along, one row of pikes is presented against the enemy: A line is formed of these pieces by chaining together a sufficient number of them end to end.

In countries where is plenty of wood, a line may be formed by laying in a position pointing to the enemy, the stems of trees and their larger branches, piled one on another to a sufficient height, and the interstices filled with earth: Such a work, called an *ABBATIS*, affords an excellent defence; as was experienced at Ticonderoga.

SECTION II.

18. *Of Batteries.*

When lines are to be vigorously defended against a potent enemy, it is proper to raise *batteries*; that is, works from whence cannon may play on the troops as they advance to the attack: These batteries are either *open* or *covered*.

19. An **OPEN BATTERY** is nothing more than a number of cannon, generally *field-pieces* (that is, such as carry a ball not exceeding 9 pounds weight) ranged in a line or row abreast of one another, on some natural small elevation of the ground; or an artificial bank of about a yard or two high: These cannon are ranged at about 15 or 16 feet distant from one another, having their shot and loading utensils lying by their side; and the powder lodged in a hole at some distance behind the battery. A less distance than about 15 feet from gun to gun, might occasion danger, considering the hot service for which such batteries are erected.

20. A **COVERED BATTERY** is when the cannon and gunners are covered by a bank made of brushwood, faggots, and earth; of about 18 or 20 feet thick, and 7 or 8 feet high. The cannon used in such batteries are generally from 9 to 18 pounders: and sometimes 24 pounders are used in them.

21. The faggots, or, as they are most usually called, the **FASCINES**, are made of the middling-sized loppings of trees, under an inch diameter, and are of three different lengths, 6, 9, and 12 feet, according to the work they are intended for: They are usually bound or tied round with wyths; the shortest in two places, the next in three, and the longest have four bands. These fascines are commonly 8 or 9 inches thick throughout, the ends being cut so as not to be tapering.

The fascines are fixed to the ground, and to one another by flakes called **PICKETS**, like birch broom-sticks, but stronger; they are about an inch, or inch and half thick, and from 3 to 6 feet long: The 6 feet ones are in number about $\frac{2}{3}$ of the 3 feet pickets, but much stronger.

All *field batteries* consist of four chief parts, namely, the *ditch*, the *parapet*, the *platform*, and the *magazine*.

22. *Of the Ditch.*

The ditch is usually dug in the front and sides, and sometimes quite round; and serves not only to furnish the earth necessary for the parapet, but it also prevents the enemy from entering the battery so readily as he might otherwise do. The ditch in front is commonly 18 or 20 feet broad, and the depth about 7 or 8 feet; the sides sloping towards the bottom, which is about 6 feet wide: But the ditches on the sides are about 10 feet wide, and 6 feet deep. These are the dimensions usually given; but the general practice seems to be, only to get earth sufficient for the work, and not mind the regularity of the ditch: For as these works in attacks are usually done by night, the keeping strictly to the aligned dimensions is hardly practicable.

* A 4

23. *Of*

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23.

Of the Parapet.

The parapet is generally raised about 3 or 4 feet distant from the brink of the ditch, the space left between, called the BERM or FORELAND, serving to lodge the rubbish beat down by the enemy's shot, that it may not fall into the ditch.

The thickness of the parapet should be about 18 or 20 feet, in order to be cannon proof, and about 7 or 8 feet high, when the enemy has no command above the battery: But if they have a command over this height, the parapet should be raised high enough to cover the men while they load the guns.

The length of the parapet depends on the number of guns to be employed in the battery.

24. Thus for one gun, allow 8 yards in length.

And 6 yards more for every other gun.

So 2 guns has 14 yards; 3 guns 20 yards; 4 guns 26 yards, &c.

25. There should be great care taken that the battery be not *enfiladed* or *raked* by the enemy's cannon: Neglects of this kind are often fatal to the officers and gunners, as was well known at the siege of Carthage, anno 1741. But this is easily prevented by raising at one, or both ends of the parapet, a bank across the battery called an *epaulement*, of the same thickness and height with the parapet, and about 15 or 18 feet long.

26. The parapet consists of two parts, namely, the *wall* and the *merlons*.

The WALL is that part of the parapet which is contained in one piece from end to end, and is about $2\frac{1}{2}$ or 3 feet high.

The MERLONS are detached pieces of the parapet, leaving openings called EMBRASURES, through which the cannon deliver their shot.

The embrasures should as much as is possible be cut perpendicular to the parapet; therefore the battery should be parallel, or nearly so, to the object to be battered: For the direct shots have most force; and oblique embrasures weaken the merlons, or parts of the parapet standing between the embrasures.

27. The embrasures are usually about 2 or $2\frac{1}{2}$ feet wide on the inside, and about 9 or 10 feet on the outside; whereby the cannon may be traversed from the right to the left, so as to command a pretty large extent in front. The distance from the middle of one embrasure to the middle of the next should be about 18 feet, in order to leave sufficient room for the working of the guns, and the stowage of the shot and other necessaries.

28.

Of the Platform.

The PLATFORM is a floor made to facilitate the rolling of the carriage wheels, and to prevent them from sinking into the ground by the weight of the cannon, especially in wet weather.

Platforms are generally laid sloping towards the parapet 9 or 10 inches; this carries off the rain, prevents the gun from recoiling so much when fired as it would do if laid level; and when loaded, it is more easily brought to the embrasure.

29. In

29. In temporary batteries, the platforms are made of planks laid across some ground timbers or sleepers: There is usually a platform made to each gun; it is about 18 feet long, 8 feet broad next the parapet, and about 14 feet broad at the tail, the intermediate spaces between the platforms serving for the shot and other necessities.

30. When a platform is to be laid on marshy ground; first lay a floor or two of fascines; cover these with *burdles* of 12 or 15 feet long, and 6 or 7 broad; on these lay a floor of 3 or 4 inches of earth, and therein lay the sleepers, and over them the planks.

31. When a battery is built of stone or brick, the platform is generally a flat-stone pavement ranging the whole length of the battery: This, on account of its resisting the injuries of the weather for a long time, is to be preferred to planks; but in case of a bombardment such a platform is to be avoided, because the shells will not only break the pavement, but also, by driving about the broken stones, do the troops much mischief.

32. *Of the Magazine.*

The magazine to a field battery is usually made about 50 or 60 yards behind the platform: It is a cavity dug in the ground about 4 feet deep, and the earth thrown between the pit and the platform; the sides of the pit are sometimes planked round to keep it dry, and prevent the earth from crumbling in; and the powder barrels placed herein are covered with hurdles and earth, or tanned hides, to preserve the powder from wet and fire.

The communication to the magazine is by a sloping trench beginning to descend about 5 or 6 yards behind the platform; and the earth thrown on that side where it will most conveniently cover the persons who remove the barrels of powder from the great magazine to the battery, or small magazine.

33. When there are many cannon in battery, and the service is quick, it is customary to have to every two pieces a small magazine to hold 20 or 30 barrels of powder: This is placed about 15 or 20 yards behind the platform, and against the merlon between the cannon; and as these barrels are used, they are replaced by others from the great magazine.

At each magazine a sentinel is placed to prevent accidents.

And to prevent persons coming into the battery and magazines who have no business there, a trench is sometimes dug behind the magazine, and carried into the trenches which communicate between the magazine and battery.

34. *To construct a Fascine battery.*

For one gun provide fascines of 9 ft. 600; of 6 feet, 100; of 3 ft. 120.

And for every other piece 400; 100; 100;

each fascine being about 9 inches thick; and let there be three or four pickets for every fascine, for many will break in driving.

Trace the limits of the parapet in two parallel lines about 18 feet distant, allowing 8 yards in length for one gun, and 6 yards for every other gun; and along these lines cut a trench about half spade deep.

Lay

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Lay a row of 9 feet fascines along one trench, observing that their ends be well jammed one into the other, and let each be pegged down with two pickets, drove into the ground till the head is sunk into the fascine.

Close to this row lay another, letting the two end ones be of 6 feet, and the rest 9 feet, whereby the joinings in this row will not fall against the joinings in the first row: Let these be also staked down as before. Close to the second row lay a third all of 9 feet. Close to the third lay a fourth, the two end ones being 6 feet, and the rest 9 feet. In this manner lay the rows of fascines within the limits staked out, which will be covered with 24 rows; then over this floor of fascines throw earth to fill up the hollows, and let the whole be trod or rammed down.

On the first floor lay a second, observing, that as in the first floor every second row ended with 6 feet fascines, so in the second floor let every odd row end with six feet ones, whereby the joinings of no two fascines in this floor will fall over the joining of the fascines below them. Let the outside rows in the second floor be so placed over those in the lower, that there be a little sloping preserved, the work narrowing upwards. The pickets which stake down this floor are to be drove up to the head, and the channels or hollows in the floor are to be filled up with earth. In this manner lay four floors, which will raise the work to about $2\frac{1}{2}$ or 3 feet, and that part of the breast-work called the wall will be completed.

35.

To stake out the Merlons.

Measure from each end of the wall 12 feet, there stick a stake; and plant other stakes at every intermediate 18 feet; This being done on the inside of the wall, let other stakes be planted on the outside, either directly opposite the former, or in the line towards the place where the gun is more particularly intended to deliver its shot. Plant other stakes on the inside, one a foot distant on each side the former, and this will leave spaces of two feet each for the inner opening of the embrasures: Then on the outside, plant other stakes at 5 or 6 feet distance from the former ones, one on each side, and spaces of 10 or 12 feet will be marked out for the outside openings of the embrasures.

In the direction of the pickets which limit the inner and outer openings of the embrasures, let single rows of fascines be staked down across the wall, and these will be the sides of the embrasures: Fill the intermediate spaces or merlons with rows of fascines laid lengthwise to the wall, and this will be the first floor of the merlon, which is to be picketted down, and the hollows filled with earth as before directed. Let other floors be raised in like manner, until the merlons are carried up to about 5 or 6 feet, or more if necessary; and on the top of each let a bed or floor of earth be laid of about 8 or 12 inches thick.

Across the top of the inner opening of the embrasure, either lay a bundle of 10 or 12 fascines bound together, or as many stufed into a kind of basket open at both ends, and the bundle well staked to the merlons.

Provide a *blind* made of planks 3 or 4 inches thick, to fit the inner opening of the embrasure, which is to be put up while the gunners are loading the piece, to preserve them from the fire of the musketry. The

The cheeks or sides of the embrasures are to slope so, as to have the top wider than the bottom; which not only leaves a greater opening for the blast of the gun, but these slopes, in works of this kind, give a greater degree of strength to support themselves against the thrust of the upper parts.

36.

To construct the Platforms.

Against each embrasure lay on the ground five pieces of timber, called sleepers, of about 18 feet long, and 6 or 8 inches square; let the ends next the embrasure be placed at such a distance from one another, that the distance from out to out be about 6 feet, the other ends spreading about 12 or 14 feet from out to out; drive two stakes of about 3 inches square into the ground on both sides of each end of every sleeper, whereby they will be kept steady in their places, and let the earth be well rammed and beat up close to them: But observe to let the tails of the sleepers be raised about 6 or 8 inches higher than the ends next the wall, to prevent the cannon from recoiling too much when fired.

Cover the sleepers with planks of about two inches thick laid cross-wise; that next the embrasures being about 8 feet long, the rest gradually increasing, so that the plank at the tail be about 14 feet long, and let the planks be nailed to the sleepers.

On this floor fix a piece of timber of about 6 or 8 inches square next the embrasure, to serve as a spur for the carriage wheels to knock against (and is therefore usually called the *knocker*), when the gun is run up to the wall; observing that the spur be laid parallel to the object to be battered.

The whole platform should be of oak, if it can be had.

37. If there is any danger of the battery's being raked by the enemy's cannon, let an *epaulement* or *screen* be raised at one or both ends of the parapet, joining to it, and constructed by floors of fascines, laid as before shewn: Then all the straggling twigs of the fascines being cut off, the battery is constructed.

A battery thus made will do in case of haste, and where earth enough is not easily had: But as it is subject to be fired when the wood is dry, it is not so much in use as another fort called a coffer battery; especially where wood is scarce and earth plenty.

A COFFER BATTERY is that where the sides of the wall and merlons only are formed of fascines, and all the cavities or included spaces filled with earth.

38.

To make a Coffer battery.

The place of the battery being determined, mark out with a line of some kind, the limits of the parapet of about 18 or 20 feet thick; and 3 or 4 feet before the parapet, mark out with lines or stakes the limits of the ditch, of about 10 or 12 feet broad, or more if earth is wanted; allowing 8 yards in length for one gun, and 6 yards more for every other gun.

On the outlines of the parapet, cut a trench of about 5 or 6 inches wide and deep, and therein lay a row of fascines, the ends being jammed one into the other; let these be staked down: Lay on them another row,

so that the joinings of these be not directly over the joinings of the lower one, and the knots of all the bands turned inwards, stake these down; and on them lay in like manner a third and fourth row, &c. until the height be about 3 feet. The same kind of work being done at the ends, and for the epaulement if wanted, the coffer for the wall will be made.

Then let the men be disposed along the place intended for the ditch, and with the proper tools break the ground and throw it into the coffer; where another set of men are, as the earth is thrown in, to spread it and stamp it down with rammers; and this work is to be continued until the coffer is filled.

39. When the wall is finished, let the embrasures be staked out as before (35), and a coffer formed in like manner, as for the wall, for each merlon, which is also to be filled with earth and rammed down.

The other articles in the preceding battery are to be followed in this, the only difference being in the making of the parapet.

When a proper place for a temporary battery is desitute both of dry earth and wood, then materials must be carried to the place; these usually are *gabions* and *earth-sacks*.

The EARTH-SACKS are only bags to carry earth in.

A GABION is a large basket without a bottom: They are of various sizes, as from 3 feet high and $2\frac{1}{2}$ or 3 feet diameter, to 6 or 8 feet high and 6 or 7 feet diameter.

40.

To make a Gabion battery.

Along the line marked out for the battery, let the gabions be planted in the places where the merlons are to be: The gabions used are of 5, 6, and 7 feet diameter, and 8 feet high. Each merlon must have 7; that is, 3 within of six feet diameter, next two of 7 feet diameter, and on the outside two of 5 feet diameter; observing to leave proper openings for the embrasures, of about 2 feet on the inside, and 9 or 10 on the outside.

Or thus: Let the merlons be made of gabions 5 feet diameter; then put four within, three in the middle, and two on the outside; this construction being rather stronger than the former.

The ends and epaulements are also formed by three rows of gabions.

The floors or bottoms of the embrasures are to be filled with gabions of about three feet high, and of a proper diameter to fill up the spaces between the merlons.

The gabions being placed, they are to be filled with earth brought from the nearest places that will afford it, in the earth-sacks; or else they may be filled with dung mingled with sand; and in cases of necessity, they may be filled with large faggots, or billet wood, observing that the avoids between the gabions are also filled.

Batteries of this construction are usually made on marshy or rocky ground.

41. The *Sieur Ramy*, in his *Memoirs of Artillery*, has given a table for the ready finding of all the requisites for the construction of temporary batteries, and for their daily service, the pieces being 24 pounders; and although these batteries are calculated only for sieges, and are of the coffer-kind, yet from this table may be derived such notions as will greatly help young artists on other occasions.

It

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It is also proper to mention the number of fascines and pickets that is usually expected every day from the labour of each man employed in that service.

Fascines of 5 or 6 feet long, and 5 or 6 inches thick, bound with two wyth bands each, one man will make 16 or 18 in a day, with two pickets to each.

Fascines of 8 or 9 feet long by 8 or 9 inches thick, and two pickets to each, one man usually makes 10 or 12 in a day.

Fascines of 12 feet long by 9 inches thick, with three pickets to each, 8 or 10 such are usually expected for one man's day's work.

The days here understood are such wherein the men may work about 12 hours.

In the following table C stands for hundred.

42. A TABLE for constructing of Batteries.

Number of 24 pounders in battery.	Length of the parapet in yards.	Number of workmen to construct the battery.	Number of workmen's tools used at the battery, viz.	Fascine-makers, each a bill and hatchet.	Fascines of 8 or 9 feet by 8 or 9 inches.	Fascines of 12 feet by 8 or 9 inches.	Fascines of 5 or 6 feet by 5 or 6 inches, made by the cavalry.	Pickets from 3 to 6 feet long, and about 1½ inch thick.	Mallets to drive the pickets.	Handbills, 2 for each embrasure, beside spare hatchets.	Planks for the platforms, of 2 or 2½ inches thick.	Gunners to serve in the battery.	Soldiers to assist the gunners.	Powder for 100 rounds, at 12 lb. each per day.	Shot of 24 pounds for 100 rounds.
2	14	50	70	15	120	40	2 C	520	10	4	32	4	12	24	2 C
3	20	60	85	20	165	60	3 C	740	14	6	48	6	18	36	3 C
4	26	70	100	25	210	80	4 C	960	18	8	64	8	24	48	4 C
5	32	80	115	30	255	100	5 C	1180	22	10	80	10	30	60	5 C
6	38	90	130	35	300	120	6 C	1400	26	12	96	12	36	72	6 C
7	44	100	145	40	345	140	7 C	1620	30	14	112	14	42	84	7 C
8	50	110	160	45	390	160	8 C	1840	34	16	128	16	48	96	8 C
9	56	120	175	50	435	180	9 C	2060	38	18	144	18	54	108	9 C
10	62	130	190	55	480	200	10 C	2280	42	20	160	20	60	120	10 C
11	68	140	205	60	525	220	11 C	2500	46	22	176	22	66	132	11 C
12	74	150	220	65	570	240	12 C	2720	50	24	192	24	72	144	12 C
13	80	160	235	70	615	260	13 C	2940	54	26	208	26	78	156	13 C
14	86	170	250	75	660	280	14 C	3160	58	28	224	28	84	168	14 C
15	92	180	265	80	705	300	15 C	3380	62	30	240	30	90	180	15 C
16	98	190	280	85	750	320	16 C	3600	66	32	256	32	96	192	16 C

43. When

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43. When batteries are erected at leisure, and wanted to stand for some years, then it is best to make them of stone or brick, or good loamy earth, according as the materials are nearest at hand, or can be most easily procured.

When made of stone, the thickness of the parapet is usually about 4 or 5 feet; But this thickness is varied according to the quality of the stone, or to the size of the cannon that may be brought by an enemy to fire on it.

Parapets of brick, are of 5, 6, 8, or even 10 feet thickness, according to the goodness of the materials, or the service it is to endure; and the angles are generally of stone.

A parapet made of good earth, and 18 or 20 feet thick, with the surface covered with green turf, will resist the injuries of the weather for several years, and stand the shock of an enemy's cannon the best of any.

44. As these perpetual batteries are generally raised somewhat above the level of the adjacent land or water which they are to command; so it is useful to make the floors of the embrasures sloping, the drip being on the outside, about a foot or foot and half below the sill of the inner opening, whereby the cannon may be pointed downwards.

When the embrasures are cut through an earthen parapet, they are commonly lined with brick; or at least the angles are made of masonry, these parts being the most subject to be injured by the weather.

The best embrasures are those made with a neck, whereby a part of the opening becomes more contracted, and consequently is better adapted to preserve the gunners and cannon from the enemy's shot, than when the cheeks of the embrasures are made straight.

45. Although the axis or middle line of an embrasure should, if possible, be at right angles to the length of the parapet, yet it may sometimes be necessary to have them oblique: When this happens, the inside of the parapet is notched, or so cut away, to leave proper room for the front of the platform, that the wheels of the carriage may be drawn up against the inner opening for the axis of the gun to lie in the axis of the embrasure: But then the outside of the parapet is to be strengthened, to compensate for what is cut away within.

The foregoing particulars, relating to embrasures, will be sufficiently illustrated by the following precepts for their construction.

46. To construct the Plans of embrasures.

FIRST. For the Plain embrasure. Plate XV. Fig. 4.

1st. Draw the axis AB perpendicular to the parapet.

2d. For the front opening, make $AC = AD$, each from 4 to 5 feet.

3d. For the inner opening, make $BF = BE$, each from 1 to $1\frac{1}{4}$ feet.

4th. Draw FD , EC , for the cheeks of the embrasure required.

47. Se-

47. **SECONDLY.** *For the necked embraasures.* Plate XV. Fig. 5.
1st. Having drawn the cheeks for a plain embraasure as before; then from B, the middle of the inner opening, draw BC, BD.
2d. In the axis AB, take $BC = 2\frac{1}{2}$ or 3 feet; through G draw IH at right angles to BA, cutting BC, BD, in H, I, and join EH, FI.
Then DIF, CHE, are the angular cheeks, IH being the neck or narrowing of the embraasure.

48. **THIRDLY.** *For an oblique embraasure.* Plate XV. Fig. 6.
1st. Draw the axis *ab* in the direction where the gun is to command, and through any convenient point I, taken in *a b*, draw NN at right angles to *ab*.
2d. Make IN = 4 or 5 feet for the front opening, IP = 1 or $1\frac{1}{4}$ feet for the inner opening, and IQ = 4 feet for the front of the platform.
3d. Through N, P, Q, and parallel to *ab*, draw NC, ND, cutting the front of the parapet in C, D; and Pp, Qq, cutting the inside of the parapet in q, q.
4th. Through q draw qr parallel to NN, cutting the former lines in r, B, E, r; join ED, EC, and draw rs parallel to a line through B and C. Then the space qrs being cut away, the wheels of the carriage can knock against the line qr, and the axis of the gun will lie in the line Ba intended for the axis of the embraasure.

It is evident that the parapet is considerably weakened by the cuts for these oblique embraasures, and therefore should be strengthened on the outside.

49. *When the embraasures are to be expressed in the parapets of a plan drawn from a small scale, it is done thus.* Plate XV. Fig. 8.

On the inside of the parapet RS, lay off as many times 6 yards, in the points c, c, c, &c. as the length of the parapet will admit, unless restrained by particular circumstances; and from the points c, c, c, &c. draw lines to the front of the parapet, dividing it into parts of 3 yards each; then taking the parts opposite the points c for the fronts of the embraasures, the other parts will represent the fronts of the merlons.

Shade all the merlons, leaving white angular spaces to express the embraasures and strengthen the inner line of the parapet and the front lines of the merlons.

50. *Of the Profiles of batteries* Plate XV. Fig. 7.

Let the ground line of the battery be AB, BD that of the parapet, whose inner slope DH, is formed by making Da = $1\frac{1}{2}$ feet, and the perpendicular aH = 6 or 7 feet; the crown of the parapet HI is formed by making bI a foot or two lower than aH; and the front of the battery IB is found by making bB = $\frac{2}{3}$ bI, when of earth, or $\frac{1}{2}$ of bI when of masonry. Make DC = $2\frac{1}{2}$ or 3 feet, gives c the fill of the embraasure, whose floor cG is to drip a foot or two below the level line cF.

The platform DE is 18 or 20 feet; the tail E rising about 6 inches above the level line AB, the lower double line represents the sleeper laid lengthwise, and the upper double line shaded with the lines across, expresses the ends of the planks laid on the sleepers.

A gun on its carriage, with the wheels against the knocker at D, is annexed to the figure, to strengthen the learner's notions.

SECTION

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SECTION III.

Of additional works.

51.

Of Ramps.

When a battery is raised above the level of the ground it is built on, there must be made gentle slopes for the cannon to be drawn up and down by, and also for the easy communication of the troops posted there; these slopes, to distinguish them from the slope usually given to elevated works for their better security of standing, are called *Ramps*.

The base of the slopes commonly given to earth-banks for their support, is on the inside about a foot base for a foot of height, and on the outside is about 8 inches on a foot, or $\frac{2}{3}$ of the height; that is, they rise 12 inches, on 8 inches of base: But for the slopes of ramps, the rise is only about 2 inches on 12, or the length of the ramp's base is six times the height, and this is general, for the draught of carriages: But foot-ways need not be of so gentle a slope, a rise of one foot in three may do well enough; or, instead of ramps, stairs may be, and commonly are, used for the passage of the foot.

The breadth of a carriage-ramp is usually about 9 or 10 feet, that breadth being sufficient for the carriage and foot-way beside: But those for foot-passage only, need not be above 3 or 4 feet wide.

Ramps may either rise on the side of an elevated work, or against a salient angle of that work, or on each side of an entering angle: their representations in plans may be expressed as in the following constructions.

52. *To construct a Ramp on the side of a bank.* Plate XV. Fig. 9

Let xx , zz , be lines expressing in a plan the slope of a bank equal to its height, which suppose to be 12 feet.

1st. Through A , the point assigned where the ramp is to end, draw CB perpendicular to xx , and make AC of about 10 feet.

2d. In zz take $BD = 4$ times AB , or 16 yards; through D draw AE , and through C draw CF parallel to AE , cutting xx in G ; and zz in F .

3d. Through F draw, at right angles to GF , the line FE , cutting xx in f , and AE in E ; join BE , and the lines GF , EA , EB express the ramp.

It is sometimes proper to have ramps rise on either hand for the convenience of carriages coming from the right or left; but they must then be so far asunder, that should a carriage be drawing up each at the same time, the horses of one may pass clear of those of the other

To construct a Ramp in a salient angle of a bank.

Plate XV. Fig. 10.

53. FIRST METHOD. This is done in the same manner as in the side of a bank; observing that the end A is taken 6 or 7 yards from the salient angle, when a ramp is to be constructed on both sides; otherwise the ramp may end at the salient angle x .

54. SECOND METHOD. Fig. 11. 1st. From x draw a line xn bisecting the salient angle; make $xn = 6$ times the height, or equal to 12 yards, and on both sides draw a parallel line qr at the distance from xn of half the breadth of the way; suppose at 5 feet distance from xn .
2d. Bisect the angle xqr with the line qx , cutting the line zz in y ; join yr , and the plan is formed.

55. THIRD METHOD. Fig. 12. From x the salient angle, with a radius of 6 times the height, suppose of 12 yards, describe the arc il ; and with 5 times the height, or 10 yards, describe the arc kk , cutting zi in o , and draw op parallel to kl .

Make $xm = 2$ or 3 yards, and draw the curved lines mo , mp , which in plans are best done by hand; or else draw right lines from m to o , and from m to p ; and the plan of the ramp is made.

56. Of Barbets.

It often happens that by raising the floor of part of a battery, the guns placed thereon have an advantageous command over some part of the neighbourhood; and when the guns thus raised fire over the crown of the parapet without any embrasures, it is called an OPEN BATTERY, or *Battery en barbe*, or plainly a BARBET.

These barbets may be made either in a curtain, or at the salient angle of a flanker: They always should be $2\frac{1}{2}$ or three feet lower than the crown of the parapet, be about 8 or 9 yards broad at top, with a proper slope to the base, of a length proper for the number of guns to be mounted on them, allowing about 5 or 6 yards for each, and ascended at each end by a proper ramp.

57. To give a farther illustration to this article, here is added an explanation of the plan, and of the profile through the line bc . Plate XV. Fig. 13, 14, 15.

Let pqr be a common bank of a line, whose parapet is $rstv$; the inner slope rs being about 6 or 7 feet higher than qr : Then the bank mno , raised so high that the cannon can fire over the crown of the parapet st , is the *barbet*, whose height np is about 3 or 4 feet.

On the top of the barbet a platform is raised as in other batteries.

Let the figs. 14, 15, represent part of the plan of a line, and one of its flankers, or of a battery constructed in such a form; where A is the length of the barbet or raised battery, suited to the number of guns to be used, which are to be drawn up the ramps put at the ends; the breadths being about 8 or 9 feet, and the length ab about 7 or 8 yards.

58.

Of Cavaliers.

A cavalier is a battery raised above the other works, and covered by a parapet with embrasures.

Cavaliers are of various shapes, such as square (Plate XV.) fig. 17, or round, fig. 16, or like a horse-shoe, or a plane line; but when they are constructed in flankers, as is usually the case, fig. 18, they are of a like

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like figure to that flanker; leaving a space of about 8 or 10 yards, or more, between the parapet of the flanker and the outline of the cavalier.

They should be faced with earth, or plank or brick, but not with stone, because the splinters knock'd off by the enemy's cannon would greatly annoy the troops that may be posted in the line below the cavalier.

Any other particulars will be sufficiently known from the figures; observing, in the plans, to represent their outside limits by a double line, to express the sloping of the earth, and to put a ramp in the gorge.

59.

Of Traverses.

A traverse is a bank of earth thrown perpendicularly across a line, or other work, to prevent the enemy's cannon from raking that line.

These traverses may be from 12 to 18 feet thick, in order to be cannon proof, and their height about 6 or 7 feet; or more, if the line is exposed to any eminence from whence the enemy has a command.

And to preserve a communication, a passage of about 5 or 6 feet wide must be left at one end of the traverse.

60. The different ways of constructing these works are readily seen by the figures in Plate XV. fig. 19, 20, 21, 22.

Thus at A, fig. 19, it is formed by two equal short banks, whose ends jut one beyond the other about 3 or 4 feet, leaving a passage of about 6 feet.

At B, C, and figs. 20, 21, the end of the traverse juts about 2 or 3 feet into the parapet, beside the passage: But as this greatly weakens the parapet, it is to be practised only in such places where there is sufficient thickness left; and where nothing can be spared from the parapet, the traverse may be made as at E and F, fig. 22.

If any part of a work, thus shut in by one or more traverses, is likely to be defended by the musketry, then it is proper to add to the traverse one or more foot-banks within the defence, for the troops to mount on when they want to fire over the traverse, leaving the upper foot-bank 4½ feet below the crown of the traverse.

61.

Of Palisades.

Palisades are straight wooden stakes of about 9 or 10 feet long and about the thickness of a middling sized leg, but of a triangular or rectangular shape, according as they are of cleft quartering, or sawed stuff; and have one of their ends pointed. Their use is to form a line before those places that are most likely to be taken by surprise.

The best are those made of oak, but where this is wanted, or on emergent occasions, the best are those which can most conveniently be procured.

In planting of palisades, it is usual to sink about three feet of them into the ground the thick end downwards, and the pointed end upwards, and to set them in about four inches from one another; the whole is braced together by pieces nailed across them near the tops, and secured by thick posts at the distance of every four or five yards.

Palisades

Palisades are found to be a very good defence against an enemy not greatly superior, and destitute of cannon; for the defendants may freely use their small arms at the time they are half covered by the palisades; whereas the assailants being quite exposed, attack with the disadvantage of, at least, one to two.

62.

Of Barriers.

A barrier is a kind of fence or gate serving to shut up the roads or passages in works of any kind, to prevent their being carried by surprise, and are of various kinds, as

1st. A strong chain drawn across the passage, at about $3\frac{1}{2}$ or 4 feet above the ground, and fastened at each end to an upright post.

2d. A beam laid horizontally across two upright posts.

3d. A BAR-GATE hanging by hinges to an upright post.

4th. A CHEVAUX DE FRISE, which is a horizontal beam usually cut with 6 sides, each whereof is armed with iron spikes set within five or six inches one of the other: This beam is supported at each end by an upright post.

5th. A TURNSTILE which consists of two or three pieces of timber framed crosswise, making four or six rays like the spokes of a wheel; this frame is put on a post in the middle of a passage to turn horizontally, so that two of its rays always stretch across the passage, and prevent more than one person at a time from passing the same way.

6th. A HERRISON, which is a chevaux de frise, turning horizontally on a post in the middle of a passage like a turnstile.

7th. KLINKETS, are strong doors or gates made like palisades, and are hung and fastened as other gates are.

63.

Of Redoubts.

A REDOUBT is usually a temporary work thrown up to strengthen the lines which shut up, or secure a *pass*: They are usually figures of three, four, five, or six sides, encompassed with a ditch and a bank of earth.

The bank encompassing such works generally consists of two parts, called *rampart* and *parapet*.

64. The RAMPART is a large bank surrounding a work or place, of a height sufficient to cover the habitable parts of the houses (if any) from a direct fire of the enemy; or to raise the defendants so, as to command the neighbourhood; and of a thickness sufficient to resist the efforts of an enemy's cannon for some time.

65. The PARAPET, or breast-work, is a bank raised on the rampart close to its outer edge, to cover and defend the troops posted behind it, on the rampart.

66. On those sides of the work likely to be attacked, the parapet is cut into embrasures and merlons, of the same size and shape as has been shewn at arts. 35, 46.

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Against

Against every merlon, a *foot-bank* is raised on the inside for the musketry to stand on when they are to fire over the crown of the parapet.

To every embasure a platform is to be fitted, either of planks, as shewn in art. 36. or of stone, as described in art. 43.

When these redoubts are intended to defend lines or intrenchments, their faces or fronts should be so placed, that the shot discharged from them may fly directly to the front of those lines; for hereby the enemy may be so galled in flank, as to find it necessary to attack the redoubts, and thereby either weaken his attack on the lines, or cause him to lose time: But when redoubts are built more particularly to defend the spot whereon they stand, their faces must then front the places from whence they may be battered, and also command the ways by which they may be approached.

67. The inner sides of square redoubts are usually between the limits of 12 yards and 32 yards; and when they are to be defended by musketry, the number of men necessary to the defence may be thus determined.

RULE. *Half the side squared gives the number of troops.*

EXAM. In a square redoubt, one of whose sides within is 24 yards: Required the number of troops necessary for its defence?

Half of 24 is 12: Then the square of 12 is 144 for the number of men.

ANB, Twice the square root of a given number of men, shews the length in yards of the side of a square redoubt proper to contain them.

Thus to contain 100 men, whose square root is 10, the side of the square redoubt must be 20 yards.

68. *To construct a Square redoubt.* Plate XV. Fig. 23.

1st. Mark out a square whose side is adapted to the number of troops that may be allotted for the defence; as *AB* for the inside of the rampart.

2d. About this square, at the distance of 10 or 12 feet, describe another square, whose side *CD* is the inner boundary of the parapet.

3d. Make a parapet of about 9 or 10 feet thick, whose outline is the line *EF*; leave a berm of about 3 or 4 feet broad, whose side is *GH*, and dig a ditch of about 16 feet wide, and about 6 or 7 feet deep, which let be rounded before the angles of the redoubt.

4th. Make the rampart from *A* feet to 9 or 10 feet high, according to circumstances; let the parapet be 6 or 7 feet higher, and let the foot-bank be $4\frac{1}{2}$ feet lower than the crown of the parapet.

5th. On that side most secure from the enemy make a bridge across the ditch, and a passage through the rampart: The breadth of this way should be about 4 or 5 feet, if the defence is to be musketry only; but 9 or 10 feet if cannon are to be used; and in either case the passage is to be shut up by a strong gate.

But if the redoubt is to be defended by cannon, both the rampart and parapet should be at least 5 or 6 feet thicker.

69. As

69. As it is customary for troops to prevent their breasts to the parapet, and then fire direct before them, there will be a very considerable space before each angle of the work, that will be in a great measure without any defence: To remedy this defect in these works, it has been proposed to round the inside of the angles of the parapet, and post three or four men there to defend the angle, for more cannot well be applied: But this appearing also defective, Mr. Clairac has shewn an ingenious method to make the fire pretty near equal in all parts; which is done by cutting the inside of the parapet into notches, whose two sides, of a yard each, are at right angles to one another, and make half-right angles with the sides of the work; the manner whereof, and its defence, will be plain from seeing the figure 23, where the lines with dots at the ends represent the fire three different ways from the same side,

There are in many Treatises of Fortification directions to make a kind of temporary works called SCONCES, or FIELD-FORTS; some whereof, which are pretty well contrived for defence, will be here explained; but one species of sconces, called STAR-FORTS, are such *wretched baubles*, that nothing more will be mentioned of them here, than to recommend their plans as pretty things to stick on children's kites.

70. *To construct a Flanked redoubt.* Plate XV. Figs. 24, 25, 26.

Having described the inner figure, where AB is an inner side of the redoubt, suppose of about 18 yards.

Parallel to this figure, at the distance of about four or five yards from it (or more if necessary) describe an outward figure, whose sides are ED, EF, &c.

Divide each side, as ED, EF, &c. into three equal parts, one whereof is EI, EL, &c.; and in these sides prolonged, take DG, EH, &c. each equal to one of those three parts.

From the points G, H, &c. thus found, draw the lines of defence to the angles of the outward figure, as HD, &c.

From the points I, L, &c. draw the flanks, IK, LM, &c. perpendicular to the sides ED, EF, &c. meeting the lines of defence in K, M, &c.

Or, *The flanks may be thus drawn.*

In the line DI continued towards E, take IP equal to twice IE. And from P, as a center with the radius PI, cut the line of defence in K, and draw the flank IK.

Then to each face (HK), flank (KI), and curtain (IG), make a parapet of about 7 feet thick (or more if necessary), which is expressed in the plan by drawing lines at that distance on the inside of the several parts.

Leave a proper berm of about 4 or 5 feet broad, and make a ditch of about 5 or 6 yards wide, whose outer line or counterscarp is to be parallel to the faces and curtains only.

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A redoubt thus constructed is more capable of defence than one without flanks: For the curtain IG is well defended by the flank IK; and the face HK has some defence from the part DG.

71.

Of Redans.

A REDAN is a fortified line, broke into several curtains and flanks; the defences of the flanks being all directed towards one way.

This is a very good work for the defence of any passage; such as a hollow way, a road, a river, or a harbour's entrance.

If the defence is chiefly intended for cannon, the distance of the flanks from one another should not exceed 300 yards, that being about the point blank sight of a musket shot: But if the defence is chiefly intended for musketry, the distance of the flanks may be about 50 yards or less. In general, the distance of the flanks should be regulated according to the number of them intended to be placed within the length of the line. The length of each flank should be about 15 or 18 feet, in order to admit one gun; but if more cannon than one are to be put in a flank, then the flanks must be made proportionally longer.

And if the flanks are made about two or three feet lower than the curtains, there will then be no danger of the troops, posted in the salient angle of one flank, being hit by those in the entering angle of the flank next behind.

72.

To construct a Redan.

In the front line take the equal spaces AB, AB, &c. of the length proposed for the distance of the flanks, as suppose 50 yards; draw the perpendiculars AC, BD, of 18 feet each, and draw the faces BC, AD.

From the salient angles A, B, draw the flanks AE, BF, at right angles to the faces, and the master-line AEBFAE will be constructed.

Make a rampart, parapet, and foot-bank; ramps in the curtains an embrasure in each flank, and as many in each face as convenient.

The thickness of the parapet, if of earth, may be from 9 to 12 feet; but if of masonry, about 4 or 5 feet will do.

SECTION

SECTION IV.

Of the Fortifying of Towns.

Towns are fortified by surrounding them with a rampart, moat and outworks.

73. The **RAMPART** of a town is a large bank of earth surrounding the place, of a height sufficient to cover the habitable parts of the houses from the direct fire of an enemy; and of a thickness sufficient to resist the efforts of his cannon for many days.

The rampart should at certain distances, within the reach of musket shot, have works projecting outwards, either contiguous to the rampart, or detached from it; and these projections are to be so contrived, as to flank or defend one another.

Projecting works contiguous to the ramparts are called *bastions*, and those detached from it are called *outworks*.

74. A **BASTION** is a projecting work consisting of four lines; namely, of two **FACES** forming a salient angle; and of two **FLANKS**, which join one to the end of each face, and have their other ends joined to the rampart.

75. The **CURTIN** is that part of the rampart which lies between two bastions.

76. The **PARAPET** is a breast-work ranging quite round the faces, flanks, and curtains; of a height and strength proper to cover the troops posted on the rampart.

77. The **MOAT** is a large ditch dug on the outside of the rampart and some outworks, quite round the place: The earth that comes out of it serves to make the ramparts, parapets, and other elevated works.

The **SCARP** is that border or wall of the moat next the place, and the **COUNTERSCARP** is the opposite border or wall.

78. The **COVER'D-WAY** is a sort of passage or street running quite round the moat next the counterscarp; and is covered on the side next the country by a bank or parapet.

79. The **GLACIS** is a gentle slope of earth descending from the parapet of the cover'd-way towards the country,

The *head of the glacis* is the highest part thereof, or the top of the parapet next the cover'd-way. This part is sometimes called the counterscarp; but the soldiery usually call the cover'd way and glacis together the counterscarp.

80. The **OUTWORKS**, or those detached from the rampart, are of various forms and uses; some to cover the principal parts of the rampart, and to retard the enemy's approaches towards the town, and others to secure those places, which if neglected, might be of advantage to the besiegers.

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81. *The names of the principal parts of the Plan of a Fortress.* Plate XVI.

The figure HFBMN represents the plan of a bastion. Fig. 4.
 ag the thickness of the rampart.
 ad the breadth of the parapet.
 mm the moat, ditch or foss.
 aaa the cover'd-way.
 bbb the glacis.

82.

Of the principal Lines.

The line AB represents the exterior side of the polygon, and LA its radius.
 IK the interior side and LI its radius.
 JA the capital CD the normal.
 QH the curtain AH the line of defence.
 GI the demi-gorge, the breadth HN the gorge.
The line HF represents the flank } of the bastion.
And BF the face }

83.

Of the principal Angles.

The angle ALB represents the angle of the center.
 $\angle AAB$ the angle of the polygon, or $\angle$ of the gorge.
 $\angle EGH$ the angle of the curtain, or $\angle$ of the flank.
 $\angle GEA$ the angle of the shoulder.
 $\angle FBM$ the angle of the bastion, or flanked angle,
 $\angle BAH$ the diminished angle.
 $\angle ADB$ { the exterior flanking angle; or
 $\angle EHG$ { the angle of the tenail.
 apa { the interior flanking angle.
the angle of the counterescarp.

84.

The names of the principal parts of the Profile of a Fortress.
 Supposed to be taken through the line PR, Fig. 6.

The line A a represents the thickness of the rampart at its base.

AR	the inner slope	} of the rampart.
BR	the inner height	
EC	the walk	
CD	the slope, and DE the top of the foot-bank.	} of the parapet.
EF	the inner face or slope	
FG	the top or crown	
GH	the outer slope or front	} of the scarp.
HI	the face of the wall, called the scarp.	
IK	the breadth of the moat.	
KL	the outer wall of the moat, or counterescarp.	} of the cover'd-way.
LM	the breadth of the cover'd-way.	
MN	the slope, and NO the top of the foot-bank.	
OP	the slope of the parapet, or inner face of the glacis	} of the head of the glacis.
PQ	the gentle slope, and P the head of the glacis.	

Military

Military works of defence should be conformable to the principles which have from time to time been collected and established as fundamental propositions, such are the following.

85. GENERAL PRINCIPLES, *usually called MAXIMS.*

I. The plan of every place that is to be fortified, ought first to be circumscribed with one of the most convenient polygonal figures that can be fitted to that place, and this polygon to be fortified.

II. The more regular the circumscribing polygon is, the better.

III. It is a disadvantage, to be necessitated to extend the works about a fortress.

IV. Every side of the polygon, as much as is possible, should be equally fortified; that the enemy may not have it in his choice to attack a part more weak than the rest.

V. All works ought, as much as is possible, to be easy to build, hard to destroy, and not difficult to be repaired.

VI. The works about a fortress should be so contrived as to cover the defendants, discover the approaches of the enemy, and adapted to the common weapons of defence.

VII. Each work should be defended from as many as may be, and liable to be attacked from as few places as possible.

VIII. These works made to defend one another should be sufficiently large to contain the troops and cannon necessary for that defence.

IX. Works which are to flank one another, ought not to be at a greater distance than the point-blank flight of a musket shot.

X. The works farthest distant from the body of the place should ever be lowest, that they may be commanded by those that are nearer.

XI. None of the works should, if it can be helped, be beyond the point-blank reach of the cannon within the town.

XII. Wet ditches are preferable for small places, dry ones for large places.

XIII. Bastions with large flanks and large gorges are most esteemed.

XIV. The flanked angle of a bastion should not be much less than a right one, but are better when obtuse, not exceeding 120 or 130 degrees.

XV. The faces of bastions should, in length, not differ much from 100 yards.

XVI. A curtain should not be much longer than 180 yards, nor much shorter than 100 yards.

XVII. The angle of the curtain should be between the limits of 90 and 100 degrees.

XVIII. The point-blank flight of a musket-ball is about 300 yards, or within the limits of 240 and 360 yards; and that of a battering cannon is about 1200 yards.

XIX. Battering cannon will drive a ball 14 or 15 feet into a bank of common earth at the distance of 300 or 400 yards.

From the foregoing maxims, constructions have been made for all the regular polygons from four to twelve sides inclusive; and the quantity of the several lines and angles have been obtained from trigonometrical operations; the results of which are contained in the following table.

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86. A TABLE shewing the measures of the principal lines and angles in all regular Fortresses from four to twelve sides inclusive. Calculated from the exterior side, normal and face being taken as known quantities in yards.

Names of the polygons.	The exterior side in yards:	Radius of exterior side in yards.	Interior side in yards.	Radius of interior side in yards.	Capital in yards.	Normal in yards.	Curtin in yards.	Flank in yards.	Face in yards.	Line of defence in yards.	Demitorge in yards.	Angle of the center.	Angle of the polygon.	Angle of the curtain.	Angle of the shoulder.	Angle of bastion, or flanked angle.	Diminished angle.	Exterior flanking angle.	Breadth of the foss, in yards.
Square	360	254,56	231,00	163,34	91,22	45	156,06	40,54	100	265,96	37,48	90°00'	90°00'	97°01'	111°03'	61°56'	14°02'	155°56'	30 yds.
Penta- gon	360	306,24	247,90	210,90	95,34	54	154,22	48,92	100	268,42	46,84	72°00'	108°00'	98°21'	115°03'	74°36'	16°42'	146°36'	30 yds.
Hexa- gon	360	360,00	261,26	261,26	98,68	60	152,80	54,54	100	270,28	54,26	60°00'	120°00'	99°13'	117°39'	83°08'	18°26'	143°08'	36 yds.
Hepta- gon	360	414,82	272,44	313,96	100,96	64	151,80	58,32	100	271,62	60,32	51°26'	128°34'	99°47'	119°21'	89°26'	19°34'	140°52'	40 yds.
Octagon	360	470,36	280,06	365,96	104,40	68	150,54	62,12	100	272,86	64,76	45°00'	135°00'	100°21'	121°03'	93°36'	20°42'	138°36'	40 yds.
Nonagon	360	526,28	285,86	417,86	108,42	72	149,40	65,92	100	274,32	68,24	40°00'	140°00'	100°54'	122°42'	96°24'	21°48'	136°24'	42 yds.
Decagon	360	582,50	288,52	466,90	115,60	78	147,34	71,66	100	276,46	70,58	36°00'	144°00'	101°43'	124°09'	97°08'	23°26'	133°08'	42 yds.
Undecagon	360	638,78	292,60	519,36	119,42	82	142,90	73,96	102	276,30	74,84	32°44'	147°16'	102°15'	126°45'	98°16'	24°30'	131°00'	44 yds.
Dodecagon	360	695,46	296,20	572,14	123,32	86	138,68	76,18	104	276,34	78,76	30°00'	150°00'	102°46'	128°18'	98°56'	25°32'	128°56'	46 yds.

By the help of this table, every thing relative to the master-line of any of these fortresses are readily constructed.

87.

PROBLEM I.

To describe any of the polygons in the table.

FIRST. *To construct a polygon in a circle.* Plate XVI. Fig. 1.

- 1st. With the exterior radius of the given polygon describe a circle.
- 2d. Apply the exterior side equal to 360 yards within the circumference of that circle, from A to B, B to C, C to D, &c. as many times as the proposed polygon has sides.
- 3d. Draw the lines AB, BC, CD, &c. and the polygon will be constructed.

88. SECONDLY. *To construct a polygon upon the exterior side.* Plate XVI. Fig. 2.

- 1st. Draw the exterior side AB equal to 360 yards.
 - 2d. At the ends A, B, make angles equal to the angle of the polygon, and in that position draw the lines AE, BC, each of the length of AB.
 - 3d. At the points E, C, make angles as before, and in those positions draw the lines ED, CD, each of the same length as AB.
- And thus proceed until the polygon is constructed.

Note, The two last lines may be found by describing intersecting arcs D, with the radius AB, from the centers E and C.

89.

PROBLEM II.

To fortify any regular polygon in the table.

Or, To draw the MASTER-LINE of a regular fortress.

FIRST. *From the exterior side.* Plate XVI. Fig. 3.

- 1st. Make the exterior side AB equal to 360 yards, and find its middle C.
- 2d. Draw CD at right angles to AB, and make CD equal to the normal.
- 3d. Through D draw the lines of defence ADH, BDG.
- 4th. Make the faces AE, BF, each of their proper lengths.
- 5th. Transfer the distance FE from F to G, and from E to H.
- 6th. Draw the curtain GH, and the flanks EG, FH.

This work being done for every front or side, the master-line of the fortress will be described.

This method is most fit, when the master-line is to be laid down on a place where there were no works before.

90. SECONDLY. *To fortify from the curtain.* Fig. 3.

- 1st. Make the curtain GH of its proper length, for the given polygon.
- 2d. Make the angles of the curtain HGE, GHF, as per table.
- 3d. In the lines GE, HF, take the flanks GE, HF, of their proper length.
- 4th. Make the angles of the shoulders GEA, HFB, as per table.
- 5th. Make the faces FB, EA, of their proper lengths.

This

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This method is proper to be used when a piece of an old fortification is to serve for a curtain, to save expence.

91. THIRDLY. *To fortify from the interior side.* Fig. 3.

- 1st. Make the interior side IK of the length given in the table for the proposed polygon.
- 2d. From each end set off its proper demigorge KH and QI.
- 3d. On K and I as centers, with the extent of the capital, as per table, describe arcs at B and A.
- 4th. On H and G as centers, with the length of the line of defence, as per table, cut the former arcs in A and B.
- 5th. Draw the lines AH, BG; in which take the faces AE, BF, and draw the flanks EG, FH.

This method is useful when modern ramparts and bastions are to be annexed to simple walls surrounding a place.

In either of the above methods, the work here described for one front is to be applied to every front of the polygon; and it is recommended to the learner, that he applies the precepts given for one front, to a complete polygon; and should a frequent repetition of these operations to a whole polygon become disagreeable, they may be done only on two fronts, which may be thus drawn.

Draw a capital line AL; make the angles LAB, LAA, each equal to half the angle of the polygon; and make the sides AB, AA, each equal to 360 yards.

92.

PROBLEM III.

To construct the plan of a bastion. Plate XVI. Fig. 4.

- 1st. Draw a capital line BK, and make the angles MBK, FBK, each equal to half the flanked angle.
- 2d. Take the lines BF, BM, each of the length of the face.
- 3d. At the points F, M, make the angles of the shoulder BFH, BMN.
- 4th. Make the lines FH, MN, each of the length of the flank.
- 5th. At the points H, N, make the angles FHG, MNa, equal to the angle of the curtain, and draw the pieces of the curtain HG, Na.

This problem may be found useful on many occasions; particularly when the drawings of works in a bastion are to be represented with that bastion only, on a large scale; therefore it was here introduced, more especially for beginners, who are for the most part apt to be somewhat puzzled about constructing part of a work, independent of the whole.

93.

PROBLEM IV.

The master-line of a fortress being given; to draw the plan of the rampart. Plate XVI. Fig. 5.

- 1st. Draw the lines ag perpendicular to each face, flank and curtain.
- 2d. In each of the lines ag take $ag = 24$ yards, $ab = 20$ yards.
 $ac = 8$ yards, $ad = 6$ yards.
- 3d. Through

3d. Through the points *d, c, b, g*, draw lines parallel to the master-line; that is, to each face, flank, and curtain, intersecting one another.

Then the breadth of the rampart is represented by $ag = 24$ yards.
 of the parapet by $ad = 6$ yards.
 of the foot-bank by $dc = 2$ yards.
 of the walk of the rampart by $cb = 12$ yards.
 And of the slope of the rampart by $bg = 4$ yards.

The breadths of these parts of the rampart are sometimes different, according to the quality of the earth, or other materials the work is to be built with.

Sometimes the bastions are made solid; that is, the rampart fills up the whole space contained within the faces, flanks, and neck: The plans of such are represented by continuing the pair of lines *bg*, forming the inner slope of the rampart, against the adjacent curtains, till they meet the capital in the breast of the bastion, as at *R*: But then this pair of lines is to be omitted against the flanks and faces of such bastions.

In plans drawn from a large scale, it is usual to express the outside slope of the rampart by a pair of lines; that is, by drawing a parallel to the master-line at about five or six feet distant from it on the outside.

The master-line is to be drawn the thickest; the inside of the parapet something finer; and the other lines as fine as they can be drawn.

94. Before any of the lines are drawn in ink (for it should be remarked that every thing is first to be drawn in black lead pencil), let the ramps be put in the inner slope of the rampart (52); the embrasures in the parapets of the flanks (46); the barbets in the salient angles of the bastions, fig. 6. with their ramps (56); and the cavaliers, if any are to be, in the solid bastions, fig. 7. with their ramps and embrasures (58).

All these several parts being laid down with black lead pencil, let the lines be neatly drawn with Indian ink, taking care not to draw them beyond their proper terminations; for wherever superfluous ink lines are scraped off, those plans will never look well when they come to be washed with any colour.

When the plan is to be washed, that is, coloured, either with tints of Indian ink or other colours, leave the lower line of the inner slope of the rampart in lead pencil, which will make it appear softer when the drawing is cleaned off.

R E M A R K S.

95. In the middle of bastions, fig. 8. it is usual to build magazines for the stores of powder, bombs, and other fireworks, and also for the provisions for the troops and lodging: For in these places they are not only more out of the way of doing damage to the town on an accident of blowing up, but they are also nearer at hand to be transported to the several works where they may be wanted; and here the troops may securely take their rest when relieved from duty.

These

These magazines are usually built about 20 yards long by 12 broad in the clear, and about 8 feet high in the upright, besides the rise of the semicircular vault: The side walls are about 8 or 9 feet thick at the foundations, and about 8 at top; the end walls about 5 at bottom and 4 at top: And that they may be bomb proof, the work is about three feet thick in the *reins*, or just above the springing of the arch, and eight or nine feet thick in the crown or top; this is effected by building the roof sloping on the outside, like the roof of a house.

These buildings are sometimes strengthened on each side by buttresses 6 feet by 4; and in each side wall, there should be left in the solidity of the wall vent-holes to air the magazine: These vents wind round a kind of shaft of about a foot square. In each gable end, a small window is made to give light.

96. Magazines may be either in hollow or solid bastions; taking care in the latter to leave an area about their walls, if the place is subject to damps: And although there are those who would have the magazines at the foot of the rampart along the curtains, in order (as they say) that the bastion, by being left clear, may be disputed with the enemy inch by inch at last; but as there are very few examples, since the modern practice of war, of such extraordinary resolution in the defendants after a breach is made, therefore the reasons for having the bastions thus unincumbered with buildings, are rather ideal than real, and consequently can have but little weight: However, when the ditch furnishes earth enough, or it can be otherwise easily had, it has been the practice of the most celebrated engineers to make full bastions, especially if they were small and had narrow necks.

97. In the salient angles of bastions, fig. 7. such as the flanked angle, and the angles of the shoulders, it is usual to make little centry-boxes either of wood or stone, to secure the centinels posted there from the inclemency of the weather and random shot: They are furnished in the sides with loop-holes, through which may be seen what is doing in the ditch and covered-way, and also to fire through on occasion.

The plans of these centry-boxes may be either square, round, or pentagonal, and their floors are on a level with the walk of the rampart; their communication is by a passage of about 3 feet wide, brick'd on each side, cut through the parapet.

The centry-boxes built of stone, are usually ornamented with mouldings, and the arms of the prince or slate.

98.

P R O B L E M V.

The master-line of a fortress being given, to draw the plan of the foss and esplanade. Plate XVI. Fig. 4.

1st. On the flanked angle B of each bastion, with the radius BC equal to the breadth of the foss, found in the table, describe an arc CC.

2d. Lay a ruler to touch each arc at C, and the angle E of the nearest shoulder of the next bastion, and in that direction draw the lines CP, meeting one another in P.

3d. From the flanked angle of each bastion, draw lines Bq at right angles to CP.

4th. In

- 4th. In each of the lines $8q$, take $ca=10$ yards (Fig. 4.)
 $co=12$ yards, and $cq=60$ yards.
 5th. Through the points, a, o, q , draw lines parallel to cp , meeting before the middles of the curtains at c and t (Fig. 3.), and before the flanked angle of the bastions, in v, x, y .
 Then will bc represent the breadth of the fofs.

cq of the esplanade.
 co of the cover'd-way.
 ao of the foot-bank.
 oq of the glacis.

The arcs cc represent the roundings of the fofs.

The lines cp the counterescarp.

xc the head

xy the ridges

st the valleys

And yt the foot

} of the glacis.

The head of the glacis is to be drawn strong, and all the other lines are to be fine.

99. When the plan is to be washed, if the foot of the glacis be drawn in lead pencil only, the termination of the glacis, or its concurrence with the adjoining lands, will appear more soft and neat.

In the colouring of the glacis, it is usual first to lay a very light tint of colour over every plane, and then to strengthen the colour on every second plane, beginning at any salient angle; the strongest tint to be next the head of the glacis, and to be washed gradually off; so that the limits of the lightest shade shall lie in a diagonal line from the foot of the glacis at a ridge, to the head of the valley, as from y to s .

Give a light touch of the pencil or brush, with a middling tint of Indian ink, along the inside of the line, denoting the foot-bank.

Wash the borders of the ditch near the scarp and counterescarp, softening the edge of the colour with a brush wet with fair water: In wet ditches the colour used is distilled verdigrise; but in dry ditches use bistre.

The parapet is generally coloured with a middling tint of Indian ink.
 100. By this method of drawing the counterescarp, all the fofs is seen from the flanks, and consequently is defended by them, and must be more advantageous than if the counterescarp was directed to any other part of the flank, the intercepted part next the shoulder being thereby rendered useless for the defence of the ditch; nevertheless, if it was on any account necessary, the direction of the wall of the ditch may point within the shoulder by the thickness of the parapet of the face, and still the fofs would have the whole defence of the flank.

101. Fortifications in general would be very defective without fosses: for the wall or rampart need not be raised so high when there is a ditch, as they must be without one, to secure the place from surprizes by scaling, &c. Neither is a low wall so much exposed as a high one is in the modern methods of attack: Besides, a place is not so liable to the attempts of a surprize with a fofs, as it would be without one; for the enemy, if discovered, may easily retreat while he is on a level with the country, which he would find some difficulty in doing from the bottom of a deep fofs bounded

bounded by walls or banks nearly upright: Add to these, that the earth taken out of the ditch serves to make the ramparts, parapets, glacis, &c.

102. It may appear to some persons, that the broader and deeper a foss is, the better, as it would throw more difficulties in the way of the besiegers when they wanted to pass it: But it must be observed that some advantages are best waved, when, by taking them, greater disadvantages would ensue. For beside the great expence that the digging and removing so much earth would occasion, the enemy might see the foot of the wall at a greater distance in a broad ditch; and in a deep one, the cannon of the flanks could not dip low enough to defend the bottom. And on the whole, if the foss by its magnitude furnished more earth than was wanted, some inconvenience might arise in the disposing of the overplus.

103. These fosses are best which can be filled with water and drained at pleasure by the help of sluices or floodgates; because all the advantages that may be reaped either from a wet or dry foss, are by these means obtained: But as these cannot be had every where, the ditch must be adapted to the situation of the place.

In dry fosses, a row of pallisades is sometimes planted along the middle, either upright, or sloping towards the counter-scarp 20° or 30° from the upright.

In some dry fosses, where water can be had, a trench called *CUVETTE* *nn* (Fig. 12.) is dug along the middle, of about 12 or 15 feet wide and 6 feet deep, for the water to run in.

The breadth of the foss, although unequal in itself, is reckoned from the flanked angles, before which it is always rounded; the depth here may be from 15 to 20 feet, and thence rise gradually to the middle of the curtain, where the depth is 4 or 5 feet less.

Of Gates.

104. A GATE in a fortified place, is a passage through the rampart, which can be shut up or opened by the means of *doors* and a *portcullis*.

105. A PORTCULLIS, is either a HERSE consisting of strong pieces of timber framed like a lattice, the lower ends of the upright pieces being shod with pointed iron, and the whole frame let down by chairs or ropes: Or ORGUES, consisting of several long pieces of timber not framed together; each hanging by its particular rope or chain, whose ends are wound round a windlass which lets them all down together.

The use of a portcullis is to stop the gateway in case of a surprize; and in this, the orgues are by much the best contrivance; for a herse may be broke to pieces by a *petard*, or by cannon shot: But should a beam in the orgues be struck off in the middle, the upper part will descend to supply the place of the part taken away.

Gates are either public or private.

106. PRIVATE or POSTERN GATES, are those passages through the rampart by which the troops can go out of the town unseen by the enemy, when they pass to and from the relief of the duty in the outworks; or on any other occasion which is to be concealed from the besiegers.

167. PUBLIC GATES, are those passages through the middles of such curtains to which the great roads or public ways lead.

They are put in the middles of curtains, as being the best defended place in the outline of the rampart.

The dimensions of a public gateway are usually about 13 or 14 feet high, and 9 or 10 feet wide, continued through the rampart, with proper recesses for the foot-passengers to stand in while the wheel-carriages pass on. It is usual to ornament the front of the gateway on the outside with architecture either of the Tuscan or Doric order, and to cover the passage with a strong vault. Over the vault, close to the town-wall, is erected a building of about 18 or 20 feet square, in which is suspended the portcullis; and on the inside of the rampart there is generally raised another building, of about 100 feet front and 30 deep, and high enough to contain one or two floors of rooms for one of the town-officers, the ground floor serving for guard-rooms for the troops who are on duty at the gates.

108. *To construct the Plan of a gate with the adjoining building.* Pl. XVI. Fig. 9.

Let the line ac represent the thickness of the base of the rampart, and the middle line of the gateway; in which take these measures in feet: $af=4\frac{1}{2}$; $fr=6$; $rb=18$; $ri=30$; and $cm=15$: Through these points lines being drawn parallel to the curtain, take $fd=4\frac{1}{2}$; $fb=14$; $rg=9$; $ck=50$; and $kl=6$ feet, and complete the figure.

Then bg represents the plan of the frontispiece; gb of the portcullis room; ik of the arcade or guard-house; and lk is the breadth of the stairs to ascend the rampart.

The plan to which these directions refer is on the right hand side of the plate.

109. *Of the disposition of the Streets and Buildings in a Fortress.*

When a new town is built and fortified, and all that relates to the rampart is fixed with regard to situation and size; then the plan of the streets are to be considered; and herein, some authors have thought it convenient to adapt the position of the streets to the climate the town is built in.

Thus in hot climates, they would have the streets run NE. and SW.; NW. and SE.; for then the streets will be shaded by the houses from the mid-day sun: In this manner *New Brissac*, part of *Malta*, and some other regular built towns, are laid out. But in cold climates, they advise to place the streets so as to run north and south, east and west, in order to have as much sun-heat as can be.

110. *To design a Plan of the streets and buildings.*

About the middle L of the town, describe a square, whose sides shall be parallel to the position of the streets (already determined); this square is called the great place of arms, whose size should be proportioned to that

that of the town, and to the number of troops designed for its defence. Thus in a square, make the side of the place of arms about 90 yards; in a pentagon, about 100; a hexagon, 120; a heptagon, 140; an octagon, about 160 yards, &c. In this place of arms the troops parade, or are assembled for their daily exercise.

In the continuation of each side of the square, draw streets of about 10 yards wide; likewise through the middle of each side draw a street of about 12 yards broad; cross these streets perpendicularly with others of about 8 yards wide; make the parallel distance of these cross streets about 40 yards, and continue all the lines to the rampart.

It is not necessary that the middle streets going from the great place of arms *L* should terminate directly against the gate *E*, fig. 9. But it is somewhat more convenient, when the circumstances of the place will admit it to be so.

Make streets of about 10 or 12 yards wide, parallel to the curtains, and bounded on one side by the rampart: About 15 yards distant from this street, within, draw another street of about 9 yards wide: In the space *H* between these two streets, the barracks for lodging the officers and soldiers are built; and in the street next the rampart, the piles of shot and bombs are sometimes placed.

The length of the plan of a barrack before a curtain is about 140 or 180 yards, and the plan of a pavilion *I* for the officers may be about 25 yards long by 16 or 18 yards broad.

Against those curtains where the public gates *E* are placed, the barracks are discontinued for about 40 yards on each side the gate, making thereby an area of about 80 yards long, and about 36 broad, serving for a lesser place of arms, where the guards of a gate may draw up, in case of a surprize: Moreover, this area is useful for the carriages that are going out to stop in, until those on the bridge coming in have entered the gate. In figures of 4, 5, or 6 sides, this area is less.

The extremities *I* of the barracks next the gates and bastions are usually assigned for the officers pavilions, being the most airy places.

111. Of the rectangular plats made by the intersections of the streets, the most conveniently situated ones are to be chose for the public buildings; such as the arsenal, church, town-house, market, governor, and for the other chief officers in the service of the state; and on the remaining plats, houses are built for the burghers.

Besides the guard-houses of the gates, there should be others made on the ramparts for the centinels appointed for the night watch: these guard chambers are often placed at the centers of the necks of bastions, when in those places there are no cavaliers or magazines of powder: They are also well disposed, when set on the middles of those curtains where there are no public gates, especially when there is a water-gate through which runs a canal or river.

The arsenal should be erected in a place the least liable to accidents of fire; and if there is a convenience of water-carriage in the town, the arsenal should be placed as near the water as may be.

There are no general rules to be given for fixing the places on which the public buildings are to be erected, because the circumstances by which engineers regulate their plans are various: Therefore in the figure referred

ferred to, which is a regular pentagon fortified in the common way, the streets and buildings are laid out in imitation of their disposition in *New Brissac*; which being in general esteem, is usually proposed as a pattern for beginners.

There are many writers on this subject, who propose to make the great place of arms, or parade, similar to the figure of the town when regular; and from the angles and sides of this parade, draw streets to the middle of the bastions and curtains: But as this disposition spoils most of the corner houses in the town, by making the walls at oblique angles to one another, it was thought sufficient to have barely mentioned such constructions.

Of Bridges.

112. The passage over the moats of fortified places is by bridges, most commonly made of timber, and supported by frames rising from the bottom of the ditch: Such bridges are either *private* or *public*.

PRIVATE BRIDGES serve for the easy communication between the town and the outworks.

PUBLIC BRIDGES, are those by which carriages and passengers go over when they come into or depart from the town.

Bridges consist of two parts, namely, the *fixed* and *moveable*.

The FIXED or DORMANT parts of a bridge, are those parts which are strongly joined to the supporting piers; and are never moved but in the cases of repairs, or during a siege.

The MOVEABLE or DRAW-BRIDGE, is a floor of timber framed together, one end of which serves as an axis, whereby the other end may be drawn up or let down by some easy contrivance: The most common way is by a kind of balance called *plyers*.

The PLYERS are two timber levers about twice the length of the bridge they lift, and joined together by other timbers framed in the form of a St. Andrew's cross, which serves as a counterpoise. The plyers are supported by two upright jaumbs, on which they swing; and the draw-bridge is raised or let down by the means of chains joining the ends of the plyers and bridge.

The breadth of a public bridge is usually about 16 feet, including the rails and footway of about 3 feet on each side: But the breadth of the draw-bridge is only 9 or 10 feet, and its length about 15 feet.

A draw-bridge is generally placed close to the gate, whether it be the gate of the town or that of an outwork; and is then by the means of the plyers drawn up against the gate: But when a bridge is more than 40 yards long, it is usual to put, besides the draw-bridge at the gate, another about the middle of the bridge.

113. *To draw a Bridge in the Plan of a Fortress.*

Before the gates draw four lines across the ditch, making three intervals or spaces, the middle one being 10 feet broad, and the space on each side three feet.

* C 2.

Divide

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Divide the length of the bridge into parts of about 16 feet long each, to express the distance between pier and pier, or the bays of building; and against each division, on the outside of the lines, make a square of a foot, to express the ends of the piers supporting the bridge.

Allow the bay next the gate for a draw-bridge; and if the length of the bridge exceeds 40 yards, allow the middle bay for another draw-bridge: These are to be distinguished by drawing two diagonals across the middle interval of these bays; that of the other bays having lines drawn across them at a small distance from one another, and the space on each side left white for the foot way; which is to be broke off at each draw-bridge. See figs. 13, 15. Plate XVI.

114.

Of the Orillon and Retired flank.

When the besiegers of a fortress have made a breach in the face of a bastion sufficiently perfect, and have every thing ready for the storm, they then cross the ditch to the attack, if the besieged will suffer it to be brought on: In this attack, the fire from the flanks is the most effectual defence; and cannon placed there, loaded with grape or case shot, would then do great execution: But of this the besiegers are well aware, and therefore take care beforehand to dismount all the cannon flanking their intended passage of the foss; and as this cannot be well prevented by the besieged, especially when the flanks are plain ones, several of the most eminent engineers have proposed, and actually built part of the flanks retired behind the ordinary limits, leaving at the shoulder of the bastion a mass of earth, commonly called an Orillon, which is to serve as a screen between the enemy's direct fire and a part of the flank. In this covered part are reserved one or two cannon, to be used only in the defence of the breach in the face of the next bastion.

This notion of covering part of the flank is almost as old as the modern method of fortification.

115. *To construct the Orillon and Retired flank. Plate XVI. Fig. 10.*

1st. At right angles to the face As , draw from the angle of the shoulder the line sb equal to 8 or 9 yards, and make b the center of the arc say forming the *orillon*.

2d. From a point z in the face of the next bastion, fig. 6, about 15 yards from the point A , draw through a a line zm ; in which, and also in the line of defence Ar produced, take the *brasures* or *breaks* am and rn , each 10 yards.

3d. From m and n , with the radius mn , describe arcs cutting in d , which make the center of the arc mn , or *concave flank*.

4th. In dm produced, which serves as the termination at this end, lay the several breadths of the parts of the rampart; as the parapet = 6 yards, foot-bank = 2 yards, walk 12 yards, and the slope = 4 yards; then from the center d , through those points, describe arcs, meeting the parapet, foot-bank and rampart lines of the *brisure*, these being in the continuation of the like lines drawn within the face of the next bastion.

The

The parapet of the orillon is drawn parallel to *sa* at 6 yards distance.

The axis or middle line of each uncovered embrasure in the flank, should tend towards a point about the middle of the foss opposite the flanked angle of the next bastion.

The wall in the upper brisure *am*, as it cannot be battered, need not be above 2 or 3 feet thick to support the earth behind it.

Through this wall there should be a fally-port coming out at the bottom of the foss if a dry one, or a little above the water line in a wet foss; The descent to this port is by a passage under the rampart of the flank; but in straight flanks, the fally-port is in the curtain next the angle of the flank, the passage to it being under the rampart of the curtain.

The entrance into these fally-ports are marked on the inner slope of the rampart against the letters *v*,

116.

Of Double flanks.

As the progress of the works of the besiegers generally depend on their having a greater army than is in the town besieged, and of forming a greater fire on the town than they can return; therefore some engineers have endeavoured to hinder the progress of the besiegers batteries, by furnishing the town with an equivalent, or rather a superior fire; and this has been proposed to be effected, by making second and third flanks, one behind another, gradually more elevated than those before: For as it is found convenient to silence the fire of the flanks, before the besiegers can avail themselves of the breach they make in the face of the bastion; therefore batteries to ruin the flanks are made, and play at the same times as those for the breaches in the faces. Now the batteries for the ruin of the flanks, can be conveniently made only within about 10 yards of the head of the glacis, and their length not greater than about 40 yards, containing at most about 6 or 7 cannon: But if the besieged can by any artifice preserve the cannon in the opposite flank, and have a greater number there than are in the battery opposed, consequently their fire will be superior, and ruin the battery intended to offend them; and this was expected to be accomplished by *double*, or *cazemated flanks*.

In some places two or three tire of cannon were placed one above the other, the upper ones being supported by vaulted work, as in the Tower of London; but such having been found very inconvenient, on account of the great smother contained in the vaults, they have been generally constructed open.

The cazemated flanks have been condemned by many writers, because they may be rendered useless by the enemy's bombs; was this objection just, it would hold against any small work; for the besiegers having once got the length to any place, they may be supposed to render that place also useless to the besieged; and it would thence follow, that it were best not to have any places of defence: But as the utility of such works have been commended by some of the most eminent engineers, it was judged proper here to mention how these flanks may be constructed.

SECTION V.

117. *Of Works for the defence of the Foss.*

Such works are usually called *tenails*, or *ramshorns*, or *caponiers*.

A *TENAIL* is a work raised in the ditch before the curtain upon the lines of defence.

A *RAMSHORN* is a curved tenail raised in the foss before the flanks, and presenting its convexity to the covered-way.

A *CAPONIER* is a kind of covered way or gallery across a dry ditch before the middle of the curtain; and limited between the lines of defence and the counterescarp.

The *tenails* never rise above the common ground level, but are frequently two or three feet lower: They consist of a rampart of about 12 or 15 yards thick, covered by a parapet of cannon proof, with one or two foot-banks. These works are placed at such a distance from the curtains and flanks, that the splinters of stone knocked off them by the besiegers cannon may not hurt the troops posted there to flank the enemy in their passage of the foss; which is the principal use they are made for.

The *caponier* is of like use, but is usually sunk about two or three feet lower than the bottom of the foss: It is about two or three yards broad, and is covered by a parapet about six feet high on each side, which slopes away like a glacis about 15 yards broad. It is frequently covered by a roof made with hurdles, to sustain a floor of earth about a foot thick, and the whole supported with thick planks of musket proof, leaving convenient loop-holes for the defendants to fire through.

118. *To construct Tenails. Plate XVI.*

First sort with flanks, Fig. 11. Parallel to the curtains and flanks, draw the lines *eg* at 4 or 5 yards distance, and *ee*, *gg*, at 10 yards distance: From *f*, and *d*, the middles of *cg*, *ce*, draw the flanks *ff*, *dd*, at right angles to the lines of defence *ag*, *be*; draw the curtain *df*, and put in the rampart, parapet, and foot-bank.

Second sort with faces only. Fig. 12. Take the distances from the curtain and flanks, the same as before; and to the lines of defence annex the rampart and parapet.

Third sort with faces and curtain, Fig. 3. When the lines of defence cross one another in *d*, so near the curtain that there is not room for the thickness of the rampart and its distance from the curtain, make the face *ab*, *cd*, each of $\frac{2}{3}$ of *ad*, and draw the curtain *db*.

119. *To construct a Caponier. Fig. 12.*

From the exterior flanking angle *d*, draw a line *dp* to the angle of the counterescarp *p*; on each side of *dp* draw parallel lines, at the distances of $1\frac{1}{2}$, $2\frac{1}{2}$, and 15 yards from *dp*; these lines being limited by the lines of defence and counterescarp, form the caponier.

This work, which can be used only in a dry ditch, serves, besides its defence of the foss, for a convenient passage between the town and its outworks; and in this case there is usually a circular excavation made in the

the angle of the counterscarp, out of which the troops may defile without being seen by the enemy.

120. *To construct a Ramshorn.* Plate XVI. Fig. 13.

A line Da drawn from D , the angle of the shoulder, at right angles to the line of defence, its intersection a with BI , the other line of defence continued, gives the center a ; from which, with the radius aD , describe the outline Ac of the ramshorn, CD being equal to 10 yards; and annex a parapet, foot-bank, and walk, as in the other tenails.

This work seems preferable to either of the other tenails, both on account of its simplicity, and the defence for which it is constructed.

SECTION VI.

Of Outworks.

121. OUTWORKS are all those works advanced beyond the foss, serving to augment the defence of the town, to cover the gates, bridges, and other weak parts; to inclose those eminences, not too far distant, which command the town; to surround and defend the suburbs, if any; and, in fine, to throw so many difficulties in the way of the besiegers, that by the length of the siege, the enemy may waste both his time and troops; besides, such protraction frequently affords an opportunity of obliging him to raise the siege, either by the approach of a relieving army, or by the setting in of a season unfit for military business.

The outworks recorded by engineers are various, such as *ravelins*, *half-moons*, *lunets*, *counterguards*, *horn-works*, *crown-works*, *tenails*, and several others: But many of them having been found defective, are now disused in all new works and alterations; and therefore those only which are now in most esteem will be here treated of, referring the more inquisitive readers to the writings of *Mallet*, *Dedier*, *Le Blond*, *Muller*, and others.

122. The constructions of outworks depend on much the same principles as those of the place itself; to which may be added,

1st. Every part of an outwork ought to be flanked or defended either from some neighbouring work, or from the town itself; whereby the enemy may not find any place of real shelter among the outworks, without being at the labour of throwing up a covering for himself.

2d. As the chief defence depends on the musket, therefore the parts mutually flanking one another, ought to be within the ordinary reach of that weapon; that is, not exceeding about 300 yards.

3d. Every outwork ought to be commanded from the town; and therefore the more advanced works are ever to be the lowest. The gradation usually observed from work to work is about three feet; that is, the ramparts of all works advanced before a place gradually diminish three feet in height from the rampart of that place: Thus if the town-ramparts are 18 feet high, the rampart of the next outwork is 15 feet high, that of the work next advanced is 12 feet, and so on; whereby the more advanced outworks are commanded by those next within.

*C 4

4th. The

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4th. The flanked angle of an outwork ought not to be less than 60 degrees.

5th. Each outwork should be surrounded with a ditch communicating with the foss of the town : When all the ditches can be filled with water, they ought to be equally deep ; but when that cannot be obtained, the ditches of outworks should be shallower than that of the town ; whereby the fronts of the outworks will be better defended from the parts which flank them. The breadths of these ditches are usually from about 20 to 24 yards, and rounded at that breadth before the salient angles, with the counterescarps parallel to the faces or outlines of the work they encompass.

6th. The parapets of outworks are to be equally strong and high with those of the town, as they are to be cannon proof, and to cover the troops posted on the ramparts of those works ; So that it is usual to make the parapets of outworks of about 18 feet thick, 6 or $6\frac{1}{2}$ feet high, with a foot-bank $4\frac{1}{2}$ feet lower than the crown or crest of the parapet ; and the walk of the rampart about seven or eight yards broad, with a slope within of about once and a half its height.

When the plan of a fortification is to be drawn, in which are outworks, the counterescarp of the great foss is to remain in black-lead pencil, until the outworks are annexed in pencil also, with their fosses ; then the covered way and glacis are to be drawn surrounding the fosses of all the works.

123. Of the Ravelin. Plate XVI. Figs. 14, 15, 16, 17.

A RAVELIN is a work made before a curtain, serving to cover it and the adjoining flanks from the direct fire of an enemy. It is composed of two faces, AB, AC (fig. 14), and the two demigorges *p* B, *p* C, taken on the counterescarp.

To construct a ravelin. In a line drawn at right angles to the curtain through its middle, take the capital *p* A equal to about 100 yards, and draw the faces AB, AC, directed to points in the faces of the next bastions, about 10 yards distance from M, the angle of the shoulder.

In this work put a rampart of about 16 or 20 yards, and a parapet of about six yards ; and before the faces put a foss of about 24 yards wide, rounded before the salient angle A ; also annex ramps in the slope of the rampart, and if proper, construct a barbet in the salient angle.

The gorge of the ravelin, or the part next the town, is left open without any rampart ; because the enemy cannot attack it there ; and also, that it may be entirely commanded from the ramparts of the town.

R E M A R K S.

124. It is proper to make a ravelin before every curtain, as the flanks are thereby covered from the enemy, and cannot well be ruined before he is master of the ravelin ; without which, he would have it in his power to batter the flanks from the counterescarp, both from before the curtain and the face of the bastion ; the latter of these being the only place he can do it from, when a ravelin is interposed, of which he is not master :

master : But the difficulty of erecting a battery on the head of the glacis against the face of a bastion, is rather too great to be undertaken, while the place is defended by the whole fire of the adjacent face of the ravelin ; and therefore the besiegers can scarcely hurt the flanks before they have taken the ravelin ; consequently such works greatly augment the defence of a town.

125. As the face and fofs of a ravelin is defended by part of the face of the opposite bastion ; therefore should the faces of the ravelin be directed somewhat within the angle of the shoulder, at least as much as is the breadth of the parapet and foot-bank of the flank ; besides, by this construction they more effectually cover the flanks, which is one of the principal things wanted, as the flanks are of most importance to be preserved the longest.

126. In wet ditches, where the troops pass from the town to the ravelin in boats, it is proper to make, in the gorge of the ravelin, a kind of harbour, where the boats may lie covered from the fire of the enemy ; and this may be done various ways : One is, by making an excavation limited by an arc described from A (fig. 15), the angle of the counter-scarp, with a radius of about 20 yards. In dry fosses, it is proper to have either ramps or stairs in the gorge of the ravelin, to preserve a free communication, should the bridges be broken by the enemy's shot.

127. In some ravelins, (fig. 16) there is constructed a kind of *redoubt*, or *Keep*, which is a small work similar to the ravelin, with a ditch of about 10 or 12 yards wide : The capital may be about 30 or 40 yards ; and the work covered in the faces by a wall of a foot or two thick, furnished with loop-holes for the musketry to fire through.

This *keep* serves to secure a retreat for the troops who defend the ravelin, and on this account the defence of the breach in the ravelin may be more obstinately maintained ; and even when the defendants are obliged to give way to a superior force, they may, when retired into the *keep*, prevent the enemy from making a lodgment in the outward part of the ravelin, or at least greatly obstruct it ; and cannot be drove from this place until the enemy has erected a battery, and brought cannon on the ravelin to batter the *keep*.

The construction may be otherwise done, by taking on the gorge of the ravelin the distance BA equal to about 20 yards ; and out of the rest form the *keep* with its foss ; the scarp and counter-scarp being drawn parallel to the faces of the ravelin, whose two fronts form a work covering the *keep*, and is usually called the countergard.

128. The ravelins, called by some authors *half-moons*, have sometimes their corners at B and C, fig. 14, cut off, either parallel to the capital P A, or at right angles to the foss ; thereby making a kind of flank of about 12 yards long, with its rampart and parapet, and serves to give a more direct fire along the covered-way, which they are proposed more particularly to defend : But then this partly lays open the flanks, which the ravelins were originally intended to cover.

129. The celebrated General *Coeborn*, in the ravelins which he built at *Bergen op-Zoom*, contrived a very good defence for the covered-way before the faces of the bastions, by making retired flanks in the breasts of the ravelin, where one or two cannon might be placed as secure from the

the enemy's fire, as those behind the orillon of a bastion. These covered cannon in the ravelin seem to have been intended for the defence of the covered-way on the last emergency, like as those in the bastion were for the defence of the breach when the besiegers brought on the storm: But the covered cannon in the ravelin seem to have this preference; that as towns are not usually given up until the enemy is master of the covered-way, the secreted guns in the ravelin have an opportunity of being employed in the use for which they were placed there; whereas the guns covered by the orillons are very rarely of any use, as there are few garrisons which stand the storm of the breach in a bastion.

130. *Coeborn's* ravelins may be imitated by the following construction. In the gorge of the ravelin take ab , fig. 17, equal to about 20 yards, draw the flank bc at right angles to ab , make bc equal to about 12 or 14 yards, and draw the cover cd parallel to ab : To the flank bc annex a rampart, parapet, embrasures, and ramp; and to prevent these flanks being raked from the enemy's batteries, particularly by the *ricochet*, or bounding shot, there may be made a traverse of about 10 feet thick across the top of the flank at c , with a small mine under it, to blow up the traverse when the besieged are obliged to quit the ravelin, lest it should serve to cover the assailants from the fire of the town.

Coeborn has also made flanks at the corners d of the ravelin, which may be constructed by taking an equal to 10 yards, and drawing ne parallel to bc , meeting cd in d ; then de is the flank required.

It seems proper, when such ravelins are used, that the faces should spread on those of the adjoining bastions 10 or 12 yards farther from the angle of the shoulder, than in ravelins which have no flanks; and this is done at *Bergen-op-Zoom*, where the faces of the ravelins would fall between 20 and 30 yards from the angle of the shoulder on the faces of the bastions.

131. *Of the Countergard.*

A COUNTERGARD is a work generally serving to cover a bastion: It is composed of two faces, forming a salient angle before the flanked angle of a bastion.

To construct it. Plate XVI. Fig. 18.

Having described the ravelins, and drawn their fosses; then in the counterescarp of the ravelin take ab equal to about 24 yards, and draw the face bc parallel to the counterescarp of the foss before the bastion; and the outline of the countergard will be determined; the inner boundary being the counterescarp of the grand foss.

In this work put a rampart of about 16 or 18 yards broad, with a parapet of six yards, and annex the ramps and a barbet if necessary: Also make a foss of 24 yards broad, the counterescarp being parallel to the faces.

The countergard seems to be, next to the ravelin, one of the most useful outworks; for it occupies but little ground, is of no great expense, covers the faces of the bastion so effectually, that they cannot be battered in breach until the enemy has made himself master of this work; and

and when he has it, he will meet with difficulties enough in finding earth to cover himself and erect his batteries; and must therefore be at a considerable trouble in bringing it from beyond the foss, while he is exposed to the fire of the faces of the neighbouring ravelins; which, with the flanks of the bastions, are the defences of the countergard.

132.

Of the Tenaillon.

A TENAILLON is a work composed of a ravelin and two detached works called *lunettes*, which either partly or wholly cover the faces of the ravelin from an enemy's direct fire.

When the detached works cover only the lower part of the faces of the ravelin, they are called *small lunettes*: But when they cover the whole faces, they are called *great lunettes*, and this is the work which is here meant by the word *tenaillon*.

133.

To construct a Tenaillon. Plate XVI. Fig. 19.

Having described the ravelin and its foss, whose counterescarp is *rt*; then in the face *ef* continued, take *tv* equal to about 60 yards, and on the counterescarp of the grand foss take *rs* equal to 25 or 50 yards; then drawing *sv*, the figure *rsvt* is the *great lunette*; to which a rampart, parapet, and ramps are to be annexed against its faces *tv*, and branches *vs*, of the same kind with those in the ravelin.

134. A great lunette may be considerably strengthened, by making across its middle, at A, a retrenchment, consisting of a rampart, parapet, and a foss of about six or eight yards broad, and seven or eight feet deep; the parapet being drawn at right angles to the counterescarp of the ravelin against the middle of its face; and the ditch of the retrenchment communicating with that of the ravelin.

A *tenaillon* is a work capable of affording great defence to the besiegers, as was evident at the siege of *Lisle* in 1708, where the besiegers were twice or thrice drove out of a *tenaillon* they had taken and retaken.

135.

Of a Horn-work.

A HORN-WORK is a fortification composed of two long sides, called branches, tending towards the town, and of a front formed by a curtain between two half-bastions.

This work, when used, is sometimes put before a bastion, but oftener against a curtain.

136. To construct a Horn-work before a curtain. Plate XVI. Fig. 20.

In a line drawn from O, the angle of the counterescarp, at right angles to the curtain, take OC equal to about 250 or 260 yards; through C draw AA at right angles to OC, and make CA, CA, each equal to half OC, so AA is equal OC.

Take

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Take the *normal* CD equal to $\frac{1}{2}$ of AA , through D draw the lines of defence AI ; make the faces AB equal to $\frac{1}{2} AD$; transfer the distance BA from B to I the line of defence, draw the flanks BI and curtain II , and draw the branches AQ towards some point in the face of the bastion, about 8 or 10 yards from the angle of the shoulder.

Against the faces, flanks, curtain, and branches, put a rampart and parapet as in the ravelins; make a foss of 24 yards broad before the flanked angles A , the counterescarp of the foss in the front being directed towards B , the angle of the shoulder, and the rest drawn parallel to the branches.

A small ravelin is usually put before the curtain of a horn-work, for the same uses as that before the curtain of the town: Make the capital CP about 60 yards, draw the faces as in the large ravelins, and make a foss of about 15 or 16 yards wide, whose counterescarp is parallel to the faces of the ravelin, which are to be furnished with a rampart and parapet.

137. Retrenchments are sometimes made in the horn-work, as in the great lunette, and for the same reason, namely, to increase the difficulties of an enemy in a siege.

Also a ravelin with its foss is made before the curtain of the town, independent of the horn-work, which it commands.

138. There is another work, called a Crown-work, whose front consists of a whole bastion, two half-bastions, and two curtains, and is bounded on the sides by two long branches directed towards the town. This work is like two horn-works connected together, by their branches being put side to side.

Horn-works and *Crown-works* have been chiefly used to inclose some rising or hollow ground, or a suburb; but are now in general disesteem, on account that their defence is very small in proportion to their cost, a circumstance by which all works should be partly rated.

There are several other works called outworks, which may be found in the treatises wrote on Fortification, but are here omitted on account of the intended brevity of this work.

139. Of the Covered-way and Glacis.

When all the intended outworks are annexed to the plan, and the several fosses are drawn, communicating one with another, then the covered way and glacis are to be drawn parallel to the counterescarps of the fosses; and a foot-bank is to range quite round the covered way, at the foot of its parapet. The top of this foot-bank should be at least four feet broad, that there may be convenient room left for the troops to stand on after the palisades are planted, a row of them being always to run quite round the covered way. The manner of setting them, as recommended by the most celebrated engineers, and particularly by the great marshal *Vauban*, one of the most experienced men in military works that ever lived, is to put the palisades on the upper foot-bank at about a foot distant from the foot of the parapet, sunk in the ground at least three feet, and their top on a level with the head or crest of the glacis.

Palisades

Palisades thus placed are not exposed to the cannon of the besiegers; the most they can do will be only to brush their points: Neither can they be easily jumped over by the troops commanded to the attack of the covered way, nor readily cut down; and their distance from the parapet leaves a space sufficient for men to go along to repair any disorder.

140.

Of Places of Arms.

PLACES OF ARMS in the covered-way, are spaces made at the entering and salient angles, considerably larger than the street of the covered-way, and formed by turning the head of the glacis into two faces projecting towards the country: Those at the salient angles are formed by the roundings of the counterescarp; but those at the entering angles are thus constructed. Plate XVI. fig. 3.

In the head of the glacis take the gorges co , co , from 20 to 30 yards; draw the faces $o\ t$, $o\ t$, so that the angles $t\ o\ a$ be between 90 and 100 degrees; and parallel to those faces draw a glacis of about 50 yards.

The faces $o\ t$ of the *entering places of arms* should not make angles greater than 100 degrees with the *branches of the covered-way* $o\ a$, because those branches could not then be defended so well by the musketry, as when their shot was delivered directly along the glacis, nearly parallel to the branches, which can only be done when the defences are nearly at right angles to the places defended.

Some writers have advised to round all the salient angles of the places of arms, because those angles could then be defended by more men: But perhaps there might be some advantage in notching the faces somewhat like a redan, as mentioned at article 69.

141. The great *Coebern* was so conscious of the advantage of places of arms at the entering angles for the defence of the covered-way, that he was very particular in their description, and took such care in the construction of those at *Bergen-op-Zoom*, that they very well answered his design, as was evident in the famous siege of this place in the year 1747.

Coebern's Places of Arms may be thus imitated.

Plate XVI. fig. 21. where the work is on a larger scale, to be plainer.

Take the gorges ab , ab , about 45 yards, and the faces bc , bc , about 50 or 60 yards: At 10 yards distant from the faces, draw the *first lunette* A , with a ditch before its faces, both together being about ten or twelve yards broad, its ends being terminated by the line ab : Within this lunette, at about 10 yards distance, draw a second B of about five yards broad, its ends stretching to about half way the breadth of the covered-way: Behind the ends of each lunette, at the distance of two or three yards, put a traverse of about five yards thick, reaching from the counterescarp to about two or three yards farther than the ends of the lunette: And about the angle of the counterescarp make a skreen of about six yards in each face and gorge; which skreen is of brickwork with loop-holes for the musketry, to favour the retreat of the defendants when drove from their works in this well-contrived place of arms.

142. *Of*

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142.

Of Traverses in the Covered-way.

When the besiegers have pushed on their approaches so far, as to be within 30 or 40 yards of the head of the glacis, it will be in their power to rake the covered-way from some of their batteries; to prevent which, and also to shelter the troops in some measure from the ricochet or rebounding shot, traverses are thrown across the covered-way, particularly on each side of every place of arms, and in every branch of the covered-way where the length of that branch exceeds 50 or 60 yards.

Each traverse is furnished with a foot-bank, and a row of palisades planted on the foot-bank; and the passage round its end should be furnished with klinkets or doors, to shut them up when necessary.

143.

Of Sally-ways.

In the middle of some faces of the entring places of arms, is made a sally-way, which is a passage cut through the parapet, or head of the glacis, for to facilitate the sallies of the besieged. To make sally-ways in the places of arms salient, would be improper; because as those places are the most convenient to be first attacked, they should not by any means be weakened.

These passages are cut oblique to the faces of the places of arms, and wind towards the salient angle in such a manner, as not to be easily entered or raked; they rise gradually from the foot of the parapet, until they meet the slope of the glacis, at the distance of seven or eight yards from its head. The roads to the town commonly pass through these sally-ways; every one of which should be furnished with a good barrier, to shut up against surprizes and sudden attacks.

144.

Of Communication.

Every detached work in a fortification should have a ready and free communication preserved, otherwise they might be disadvantageous to the place.

Communications are preserved various ways, otherwise than by bridges, which are liable to be destroyed by the enemy's cannon. In dry fosses it is thus:

From the town to the tenail, by a subterraneous way under the rampart in the middle of the curtain into the foss, and rise thence by ramps or stairs into the tenail: From the tenail by a subterraneous way into the caponier, and thence to the ravelin, either by ramps, stairs, or a timber stage: From the ravelin to the works beyond, the communication is kept in like manner; or sometimes a way is sunk across a ditch, and covered with a parapet next the enemy; and the way from the foss to the places of arms is also by stairs, or by a timber stage.

Over wet ditches the troops are conveyed by boats, or rafts, or by little bridges about three feet wide lying nearly level with the water's edge; but the latter are inconvenient in the night, and much more so when the defendants are obliged to quit the work precipitately.

In the gorges of ravelins and other outworks, a space or passage of about five or six feet wide is left between the counterscarp and the end of the rampart, which serves for the communication between that work and the covered-way.

When

When wet fosses are to be represented in plans, it is done by giving them a tint with distilled verdegise along the borders of their scarps and counterscarps; but in printed plans, it is expressed as in the ditch of the ravelin at fig. 15; and dry ditches are shewn in coloured plans, by washing the said borders with a tint of brown bistre; but in prints, dry fosses are expressed as in the ditch of the ravelin at fig. 14: And a *cuvet* is represented by a double line drawn along the middle of the foss, as against the front BB of the polygon.

145. *Of Works beyond the Glacis.*

Sometimes, when earth is wanting to raise the several works to their proper height, or when the defence is to be increased, a foss of about 20 or 25 yards broad is sunk at the foot of the glacis, encompassing either the whole or some convenient part thereof: But then it is proper that the glacis should slope quite to the bottom of this foss, if it is a dry one, or a wet one which can be drained; for otherwise, when the enemy comes to be possessed of it, he would find a trench ready dug to skreen him from the fire of the covered-way, and other places.

To this advanced foss is usually annexed, when it can be, a covered-way and glacis; and there are not wanting fortifications which have three covered-ways and glacises one before the other.

146. Before some of the salient angles of the glacis there are sometimes constructed a kind of work, called a *lunet*, or by some an *arrou*. It consists only of a parapet and foot-bank forming two faces, making a salient angle, whose legs are parallel to the head of the glacis, through whose ridge runs a narrow passage to the lunet.

147. Sometimes works called *redoubts* are constructed before the salient angles; some of these works have fosses, covered-way and glacis; others are made to depend on their own defence; But to all these detached works, a proper communication must be preserved with the covered-way, and in such a manner as not easily to be cut off. When this passage is two or three times as long as the glacis is broad, the passage should have one or more traverses across it, to prevent its being raked; also, on either side, it should be covered by a parapet and small glacis.

148. *Of Profiles.*

The **PROFILE** of a work is a figure expressing the several heights, depths and breadths of that work.

The profiles of works are regulated according to their use, and also according to the quality of the materials with which those works are to be made.

Thus the ramparts of the town are ever thicker than those of out-works; the breadth of the parapet only remains nearly the same in most works: Besides, a rampart made of good firm earth, need not be so thick as one made of loose and sandy materials: From 20 to 24 yards will do in the former case; but less than 25 or 30 yards will not suffice with poor earth.

Profiles

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Profiles are also regulated with regard to the expence intended to be bestowed on the works; that is, according to the outside coating or walling of the rampart and parapet, namely, whether it be done with earth or intire masonry, or half one and half the other.

149. When a work of earth is raised, and the surface is covered or plastered with a tenacious substance made of well tempered earth or loam; this coating or plastering is called *placcage* or *crusting*.

And when the surface is covered with grass turf, or, as it is most common, with clods of fat mould cut in the form of wedges, and called *gazons* or *sods*: This kind of coating is called *gazonage* or *sodding*.

150. When the whole front of a work, from the bottom of the foss to the crown of the parapet, is covered or fronted with a wall of stone or brick: This kind of work is called *inire masonry*.

151. When the front of the rampart, from the bottom of the foss to the horizontal line, or ground level, or a little higher, is fronted with a wall of brick or stone, and the remaining part of the front is coated with earth; then this work is called *half masonry*.

Construction of Profiles.

152. When the works are coated with earth. Plate XVI. Fig. 22.

Let *AHLQ* represent the horizontal, or ground line, taken through the face of a bastion, and across the foss, and through the covered-way and glacis: In this line take the the following measures; namely,

$AR = 3$ or 4 yards; $RC = 10$ yards; $cd = 6$ feet; $de = 1\frac{1}{2}$ feet; $eg = 18$ or 20 feet; $ga = 12$ or 14 feet; $ah = 9$ feet; $hl = 12$ feet; $lm =$ breadth of foss; $ml = 12$ feet; $LM = 5$ yards; $mp = 7\frac{1}{2}$ feet; $pQ = 50$ yards.

Through the points *R, c, d, e, g, l, m, M, p*, draw lines perpendicular to *AQ*.

In these lines take $RB = 3$ or 4 yards; $cc = RB + 2$ feet; $de = cc + 1\frac{1}{2}$ feet; $ef = de + 4\frac{1}{2}$ feet; $gG = cf + 2\frac{1}{2}$ feet; $li = 18$ feet; $mk = 18$ feet; $mo = 2$ feet; $pp = 8$ feet.

Draw the lines *AR, BC, EF, FG, Ga, HI, KL, Lo, PQ*. Then draw *ED* parallel to *AH*, and equal to 3 feet, and draw *DC*; also dispose of the space between *o* and *p*, in like manner as between *c* and *f*.

Then will the several parts of the profile be completed, and represented; namely, *BA* the inner, and *Ga* the outside, slopes of the rampart, whose walk is *BC*; *FE* the inside, and *FG* the crown of the parapet; *CD* the slope, and *DE* the top of the foot-bank; *aH* the berm or foreland; *HI* the scarp, *KL* the counterescarp, and *IK* the bottom, of the ditch; *LM* the covered-way; *oN* the slope, and *NO* the top, of the foot-bank; or the inside of the parapet, and *pQ* the slope of the glacis.

The palisades *k* are put on the top *NO* of the foot-bank.

153. In ramparts coated with earth, it is usual to set a row of palisades in the front of the rampart, as at *b*, about five or 6 feet below the crown of the parapet. These are not planted upright, but nearly horizontal, the point dripping a little below the level; and this work, which is called *sraising* or *staccage*, is done to prevent surprizes by scaling, and also to hinder desertions.

154. In most ramparts coated with earth, whether of the town or out-works, it is usual to plant on the berm, either a row of upright pallisades, as at *f*, or a quick-set hedge of thorn, which should be kept constantly cut, to make it grow thick: Either of these methods serve to cover the troops in going their rounds. Sometimes both pallisades and hedge are used, the hedge standing foremost.

The salient angles of all earth works should be rounded off; otherwise the weather would soon destroy the angular edges, and made them look worse than the other parts.

155. *When the coating of the work is entire masonry.* Fig. 23.

Having determined all the parts through the rampart to the line *Gg*, as at art. 152; in *Gg* take $GI = 3$ or 4 feet, for the upright face of the parapet; on *I*, with the radius of one foot, describe a semicircle representing the *cordon*, which is a large semicircular moulding that ranges along the whole front; take *gr* equal to 18 feet, and draw *rest* parallel to the ground line, and this represents the bottom of the foss: Take *rc* and its parallel *1a*, each equal to 5 feet; draw the scarp *1c*, and draw *ab* parallel to *Gg*, which continue to about 6 feet below *rc*, and this line represents the back of the wall; through *b* draw a line *b d m n* parallel to the ground line, and this represents the bottom line of the foundations, which is usually about 6 feet below the bottoms of the fosses; though in some cases it must be more; and let the foundation line *b d* project about 2 feet before the scarp line *1c*: Draw a line about 3 feet behind the front line of the parapet *GI*, parallel to it, and the thickness of the front wall of the parapet will be thereby represented; to which must be annexed, at *G*, a little projection, of about 6 inches in fall and height, to express the coving stone, and the outlines of the profile of the rampart will be completed.

156. From the directions already given, it will be easy to conceive how to regulate the profiles of outworks, whether they are to be fronted with turf or masonry; and according to art. 155, were the other profiles of the masonry annexed in the figures 24 and 25; that of 24 being a tenail before the curtain, whereof no more than the parapet rises above the ground level; and consequently the walls to support this work need not be so thick as those are which sustain the lateral thrusts of higher banks; and for the same reason, the counter-scarp walls are of a less strength. In the ravelin, fig. 25; the front is half masonry, or the walling is carried up to the ground level, and the front of the rampart and parapet is supposed to be coated with earth; but the walling in this case should project about 2 or 3 feet below the foot of the rampart, in order to preserve a proper berm.

157. The walks of all ramparts, and also the covered-way, should slope backwards, in order to carry off the rain; this sloping may be about half an inch, or an inch in a foot.

158. The scarping usually given to earth facings, is about 8 inches on 12, or the base of the slope is $\frac{2}{3}$ of the height; and the scarpings generally allowed to fronts of masonry is about $\frac{1}{3}$ or $\frac{1}{6}$ of its height; and with regard to the thickness of the walls, Mr *Vauban* usually allowed about 5 feet at top, and gradually thickened towards the bottom by their

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scarping of 1 foot in 5 of height; and this profile he has experimented in the works of 150 towns, wherein he has directed the building of more than 400000 of cubic yards of walling: But in order to add a greater strength to his walls, he puts, at distances of 18 feet on the inside, buttresses built of rough stone, whose length was about the thickness of the wall at bottom, and their breadth, on a mean, about half the length; and these buttresses being carried up as high as the cordon, greatly contributed to take off the thrust of the bank against the wall.

159. At the flanked angles of bastions, and other works, the parapet should rise about a foot or two higher for about 12 or 14 yards on either side the angle, than along the rest of the faces; and the same should be observed for about 4 or 5 yards on each side of the more exposed salient angles of the covered way: For by these means, the troops ranged along those faces will be somewhat better defended from the ricochet, or bounding shot.

160.

Of Irregular Fortifications.

When a town or other place is to be fortified, it generally happens that the polygon which best circumscribes its plan will be very irregular, and consequently the fortifying of that polygon will not fall under the rules already given for regular fortification: It is therefore proper to point out to a learner how he may proceed on such occasions.

161. Let the plan of the place be circumscribed, if possible, by a polygon, whose angles are all salient, none less than 60 degrees, nor much greater than 120; and neither of the sides less than 200 yards, nor greater than 350: This polygon, which is to be the interior one, should have its sides not less distant from the houses than 50 yards; and the fortification is to be made from the interior side.

Through every angle of the polygon draw a line bisecting it; and on these lines, continued without the polygon, lay from the angular points, the capitals, (in the table, art. 86) of those polygons, whose angles are nearest to the angles in the given irregular polygon: Also on each side of the capital, from the said angles, lay on the sides of the polygon the demigorges corresponding to the capitals used.

Then the lines of defence being drawn, and the corresponding faces laid thereon, the *master-line* of the fortification to the given irregular polygon may be readily completed (as in art. 91); to which the ramparts, fosses, and proper outworks, may be annexed, as before shewn.

162. In the circumscribing of the given place with a polygon proper to it, should it so happen, that the sides, when drawn at the distance of 50 yards from the houses, make a polygon very different from that above directed; then should the positions and lengths of all those lines, or of some of them, be changed, and tried various ways, when the situation will admit of those various trials, until one is fitted to the place that will come near the foresaid limits; notwithstanding the sides should, in some places, run many more yards from the houses than 50, and a space of ground greater than the first proposed spot should be included within the polygon: For although by these means the fortification will be more extended, and thereby incur a greater expence both in building and defending; yet as it will, by its nearer approach to regularity, be stronger

in itself, and thereby more effectually answer the end of fortifying, the latter considerations may on some accounts be preferred to the former.

163. It should be observed, that regularity is not strictly confined to such polygons only as can be circumscribed by a circle, or nearly so; for a polygon inscribed in an oval kind of figure will have a great regularity, and may be made pretty nearly equally strong, by putting the stronger kind of outworks at the narrow ends: For as these ends, on account of their small breadth, in proportion to the longer sides, cannot flank the approaches of an enemy so well as the longer sides, and the enemy thereby not necessitated to extend his works; consequently the smaller ends may be more advantageously attacked, and therefore should have their outworks of the stronger sort.

164. If the situation is such, that one or more of the sides of the circumscribing polygon cannot by any easy means be made less than 400 yards or more; then should such sides be divided into parts of some number of yards, between 200 and 350, as many as its length will admit of, which may then be considered as composed of so many polygonal sides: At every division, make such a bastion as the limits will admit, taking care that the faces of each are commanded by the adjacent flanks of the next bastions.

165. When any of the angles become too acute to make in that place a whole bastion, the angle may be diminished only on one side, making there a half bastion, and let the other side be either left as one line, or make in it two or three small flanks, in the nature of a redan (71); observing that the whole front is commanded by the flank of the next bastion: Or should the angle be too small even for a half bastion, then both sides may be fortified in the manner of redans, and commanded from the flanks of the next bastions: But in the case of these acute angles, the angular point should be made very solid, and the sides about such angle shorter than the others.

166. Should the polygon have one or more entering angles, which cannot be avoided; when that angle is near 90 degrees, and the adjacent sides do not each exceed 250 yards; those sides, with the assistance of the flanks of the next bastions, will very well defend one another, and also the entering angle: But should the angle exceed 120 degrees, then put against that angle a work like a bastion, if it can be; or if there is not room for the projection of the flanked angle, make but one face, observing that it be raked from one of the adjacent flanks.

167. Where a town has been fortified in the ancient manner, and has already a good rampart and foss, such a town may be put into a good state of defence against the modern methods of attack, by adding outworks so disposed as to cover the town wall, be mutually defended by one another, and commanded from the ramparts of the town; such a place, with the outworks judiciously disposed, may hold out as many days as a place formed on a regular plan; which has been evinced from the good defence made by several towns fortified in this manner.

168. There are a multitude of other things that might be said on the business of irregular fortification, which, on account of the designed brevity of this tract, must be omitted: But after all, the best notions are acquired in these matters from studying the plans of the fortified towns in several parts of the world. Collections of these plans may be met with at most print shops.

PART II.
Of Marine Fortifications.

SECTION I.

Of the kinds and properties of Harbours.

169. A HARBOUR or PORT is a hollow cavity in the land, into which the sea runs with a depth of water sufficient to float ships or other vessels capable of performing voyages on the ocean, and where such vessels may lie secured from the tempests of the sea.

As harbours may be of very different forms, so there may be a multitude of ways proposed for their defence ; But herein the business will be confined to a few of the more common situations.

Harbours in general may be distinguished into two kinds, namely, artificial and natural.

170. *Artificial harbours* are such inlets of the coast, as are rendered safe for shipping to ride there, by building about its entrance certain works called MOLES or PIERS, which defend the ships within from the surges of the sea without : And when the moles or piers can be brought so close, as to be shut up by sluices or gates, the harbour is then called a BASIN : But the inner part of a harbour, where the ships ride in still water, is usually called the *basin*.

All artificial ports of these kinds, are easily fortified by building on the moles or piers either batteries, redoubts, or forts, which command the entrance into the harbour : But as these works are usually done at the time when the moles are made, there need be no more said of them in this work.

171. *Natural harbours* are those where the natural figure of the land forming them, is such as contributes to the safety of the shipping therein, by sheltering the vessels from the fury of the sea ; and this happens in various manners, such as a shoal lying off the harbour's entrance, which breaks the waves, and thereby keeps the water still within ; or by the points of land forming the harbour's mouth, stretching themselves so far into the sea, and coming so near to one another, that the surges are broke at their entrance ; or by a narrow inlet that runs into the land for a considerable way, and where the shores near the entrance receiving the shock of the waves, render the upper ports smooth enough to anchor in, of which kind are navigable rivers ; or by a large deep bay, wherein islands, or other obstructions, contribute to render a part secure for the riding of ships at anchor, &c. &c.

172. The properties of a good harbour are, a sufficient depth of water for large ships to enter at any time, whether the tide be in or out ; good offing, and an easy access out of all danger from the winds ; free entrance, without rock or sand-bank ; the entrance not too wide to be easily barred and defended upon occasion ; not subject to overflow, and where ships may lie close to the keys ; where the vessels are sheltered from all winds by the high mountains surrounding the harbour, and from whence there is an extended view of the sea ; lastly, that the shipping therein be out of all danger, either from being set on fire, bombarded, or cannonaded from sea by the ships of an enemy.

173. There are many places in which ships anchor that lie open to the sea, and are only safe to ride in when the weather is calm, or when particular winds blow; such anchoring places are called *roads*.

SECTION II.

Of the fortifying of Roads and Harbours.

174. FIRST SORT. *When a town lies open to the sea upon a curved or straight bold shore, and has before it a sufficient depth of water and good anchorage, within gun-shot of the shore:* As such places can afford shelter for shipping, only at certain times of the year, when they are generally sure to have calm weather, or gentle gales, in which vessels may safely ride at anchor, they cannot so well be secured from the attacks of an enemy, as in places more inclosed: However, the shipping lying there during the season of trade, may be well defended by forts built near the water's edge on each side of the anchoring place; such forts being so contrived as to have two or three batteries, one higher than the other, and these furnished with many cannon, carrying shot from 24 to 48 pounds, will awe the ships of an enemy in general from undertaking any thing against the vessels that are lying in that road.

175. But in order to defend the town itself, there should be a rampart or wall, well flanked, built along the shore, beside the fortifications on the land side. The works along the shore should be carried so near to the water's edge, that were any troops to land under the cannon of a fleet, they might not find any ground to intrench themselves on.

176. Forts built on a bold shore, to which ships of war can come within musket-shot, are in general liable to be easily taken, unless some precautions are used: For as the batteries on shore are seldom raised much above the water's level, in order to be more certain of their mark; the troops posted at the guns are liable to be commanded by the marines quartered in the round-tops, and other elevated parts of the ships: However, this may be prevented, either by driving several rows of piles (which are long pieces of timber, or the bodies of trees, pointed, and shod with iron at one end when necessary) in the water before the fort, whereby the shipping cannot approach it; or by fixing along the battery a kind of *shed* made with planks of musket-proof, and covered with raw hides or earth, that they cannot be set on fire by combustibles thrown from the ships: These sheds are not only convenient to preserve the troops posted at the guns from musket-shot, during the time of an attack, but they would also preserve the cannon and carriages from the weather at all times; for the expence of a few rough planks, used now and then to repair such sheds, added to the first cost, might perhaps, in a length of time, be found of less expence than the gun-carriages they would preserve; and should it turn out otherwise, the framing and planks might be prepared in time of peace, and set up only when there are apprehensions of an attack; but it should be observed to fix them so, as to be the least exposed to the cannon from the ships. In such sheds, as they would be open behind, the smoke would be soon dissipated, and might be sooner thrown off.

off, by making holes in the shed over every gun ; which holes may be covered by a short plank raised before them.

177. SECOND SORT. *When a harbour, being a bay, has a shoal or small island lying before its entrance:* Such a place is fortified by building a strong fort on the island, in a place where it can command the entrance, on both sides, when the size of this island is not too large for this purpose ; But when the mouth of the harbour, and the extent of the island, is too large for one fort on the island to guard the entrance, then two or more forts should be raised thereon, in the most commodious places that can command the avenues to the bay.

178. Beside the forts raised on such an island or shoal before a harbour's entrance, there should be others raised on the most convenient points of land forming the mouth of the bay ; whereby the defence of the passage is made more secure, should the ships of an enemy pass the advanced defences.

179. When forts are thus built, it is not enough to contrive them in such a manner, that their batteries may annoy the shipping which attempt a forcible passage ; but they should be also carefully fortified, to resist attempts from an enemy ashore ; for should this be neglected, a body of marines, or seamen, landing on the island or place where the battery or fort is, would frustrate the intention of such fortifications or defences ; of which there are many instances to be met with in history.

180. As there may be several spots about the points judged proper to erect the defences on, that will command in the manner as above, it is of considerable importance to know which will have most advantages, and fewest disadvantages ; for both circumstances are to be brought into the account, and that only chosen which has the most favourable balance. Thus, the soil should be the firmest, and most capable to bear the cannon and building to be erected.—Of a height sufficient to be out of the reach of tides and floods, and not much higher than is barely necessary to avoid those inconveniences ; for the cannon commands best when near the water's level.—A place difficult for an enemy to land in its neighbourhood ; and if landed, where they may be incommoded in their march to the fort.—A place where succours may easily approach.—Where the troops on duty may without much trouble obtain the conveniences of life.—And where there are the fewest noxious exhalations.

When the most convenient place for a fort or battery is chosen, the next thing to consider, is whether the work is to be temporary or perpetual ; that is, whether the work is only to serve for a present emergency, or to be kept in repair under a probability of its being frequently attacked ; and whether the work be temporary or perpetual, it is necessary that it be fitted to the materials at hand, or that can be procured with the least trouble and expence.

181. As it may frequently happen, that in such situations here referred to, the passage between the island and main is rather too wide for cannon to command across ; therefore, if the place is of sufficient importance, there may be many ways contrived to remedy this defect, when it is possible to be avoided. The best are those which oblige shipping to keep in one track or channel ; and when this can be accomplished, either by sinking of old shipping, running out of moles, or contriving to form

form sand banks or shoals, or by any other means that the situation of the place can furnish; then may forts or batteries be advanced so far into the water, as to command that channel: Or if the passage cannot be reduced to less than two channels, then a fort should be erected, if possible, between them.

182. In the building of these works, great care should be taken that the foundations are laid sufficiently firm to support the superstructure; and as these places are generally low, either in marshy or sandy situations, a great number of piles should be drove into the soil, to the depth of 10, 20, 30, or more feet, according as they drive with more or less difficulty: If a quantity of such piles be driven, at the distance of 12 or 18 inches from one another over the place intended for the foundation, and the tops of them to be cut level some feet below the surface, then may a floor of timber be laid over the tops of the piles, and the building raised thereon.

183. THIRD SORT. *When the harbour is in a bay, whose points forming the entrance stretch into the sea, and approach one another within cannon-shot:* Such a harbour is fortified by building on both sides its entrance one or more forts; and if it is possible, let a fort be also built within the harbour's mouth, in such a manner, that its cannon can rake the shipping fore and aft as they come in; for this would be a good reserve, should an enemy's fleet force their way between the forts which command the entrance: But should there be no proper spot for such a fort, then let others be erected on the most convenient points that can command the turning to the right and left within the mouth of the bay; for by these means an enemy will be obliged to endure a kind of running fire, which, if the forts be well constructed, he will scarcely be able to sustain.

184. FOURTH SORT. *When a harbour is formed by a cluster of islands:* Such a harbour is not difficult to fortify, when the channel between the islands is not too wide for the command of cannon from one or both shores; the directions how to place the forts in the foregoing cases being equally applicable to this. And in places where the channel is more than cannon-shot from either shore, the shipping that can ride there, must depend upon being defended from the batteries ashore, as in the first case, when the shore is bold enough for the ships to run under the cannon.

185. FIFTH SORT. *When the harbour lies in an inlet or river some miles above its mouth.* If the passage to the port lies straight, and can be commanded from side to side, then a fort built at each point of the entrance, and two others between them and the harbour, but not opposite to one another, unless the width of the channel requires it, will in most cases be a proper security for the shipping in such a harbour: And when the channel or river is winding, the forts should be built where they can command a reach at least; or be so placed at the bends, as to command two adjacent reaches; for as vessels must tack near the forts so placed, they will at those times be more under the command of the batteries, than they would be when sailing by them on a direct course.

In the directions for the placing of forts in this, and the other cases, there has been no mention of the figure which it would be most proper

to construct them of; for the engineer, who has the charge of such works, generally does, or should take care, to fit the spot with a figure properly adapted to it: But for the sake of the mariner, it was judged proper to point out the methods usually taken on such occasions.

186.

Of the figure and size of Forts.

Beside the forts described as redoubts (art. 68, 70), there are many others, such as squares, pentagons, hexagons, or the halves of these figures, which are fortified with intire bastions, by much the same kinds of constructions, as are the polygons in the table, art. 89: only the measures of the *exterior side*, *normal*, and *face*, are much less than those for a town; and are usually about the following numbers.

Make the side or front about 180 yards; the normal one seventh, or one sixth of the side; and the face about twice the normal, or a little less; with a ditch, if necessary, of about 20 yards broad; a rampart broad enough for a platform, and a parapet about 12 feet thick.

The exterior sides of forts may be considerably less or greater than 180 yards, even between the limits of 60 yards to 200; according to the size of the spot on which it is to be built, joined to the importance of the place it is to defend: For it would be a great absurdity, either to build a large expensive fort to secure a harbour or port of little more consequence than to give shelter to a few fishing-boats; or, on the contrary, to build a small trifling work to guard a haven where many ships of the utmost importance are laid up, or riding at anchor.

It is not always necessary, or even convenient, to make these forts on figures perfectly regular; for the situation may require some of its sides to be shorter than others: Neither is it necessary that the figures should be fortified with bastions, or half-bastions, at the angles; for there are many cases where it will answer the purpose equally well, or even better, to make a bastion on the side of a figure towards the middle; but then the faces of the bastion should be so drawn, that they may be raked from the extremities of that side, by a defence of at least 10 or 12 yards.

In many cases there need no other than a direct defence from one, two, or three contiguous batteries, making one, two, or three sides of the figure given to the fort, suitable to the shore.

187.

Of proper Forts, and their disposition.

If the place on which the fort is to be built is a spot surrounded with water, forming a channel on both sides; then the figure is to be disposed so as to have either one or two of its fronts turned towards each channel, whereby the batteries in the flanks will scour both up and down the stream, while those in the faces and curtains command the passage directly before them. That part to seaward, may either have a front extended along it, or have one of the points of the figure directed that way, according as it may be judged most convenient; and if before these fronts there is any place at which an enemy can land, then one or more batteries should be so formed, that their guns may be brought to bear on that landing place; and if this be not thought sufficient, a small outwork should

should be built near that place, to make the landing more dangerous; and on account of the convenience of landing, the fort should be furnished with a ditch, covered-way, and glacis: The part next the harbour will be sufficiently secured by a line thrown up, properly flanked. Should the situation be on a rock, where there is no danger of an enemy's landing; then a good line, capable of covering the troops and cannon, will suffice. *If there is only a channel on one side fit for shipping to pass through;* then that side should be fortified as in the former case; and least the enemy's boats should get through on the other side, it should be also put in a state of defence: But here a slight redan, furnished with a few pieces of small cannon, may be found a sufficient defence against the boats.

188. When a fort is built on a point of the main, the same kind of disposition used in the foregoing case, next to the channel, will also obtain in this: but it is necessary to take proper care of the land side, which in this case is more liable to be attacked than in the former. In the fortifying of the land side, the works should be proportioned to the strength of the enemy that may probably appear before it; For there are many places, where there is no likelihood of ever seeing an enemy with a stronger force than can be exerted by a fleet of three or four ships of war; and at these cannot be furnished sufficiently for undertaking a regular land-siege, therefore a well flanked line, with a ditch, if properly defended, will secure the place against the attacks of sailors or marines unprovided with artillery: Or should a few pieces be brought against the place, their passage to it will be very dangerous, unless they can be brought near enough to act under cover; and to leave such a cover, would be an unpardonable oversight in those who had the care of constructing that fort. But when the importance of a place may bring before it a large fleet, and a land army properly appointed, then should the fortifications on the land side be put in a state capable of sustaining a vigorous siege.

There are many low points on which batteries may be raised for the defence of a channel, over which a lofty ship may have a considerable command: In such ports proper traverses should be placed, that the line of the battery be not raked by the ship's guns.

189. In the constructions of all forts, it should be remembered, that the figure of fewest sides and bastions, that can probably answer the proposed defence, is ever to be preferred; as the works on such plans are sooner executed, and with less expence; besides, fewer troops will serve, and they are more readily brought together in cases of necessity: And with regard to the executive part of such works, enough has been already said, observing that where stone and brick are scarce, and there is plenty of wood, then timber works are to be most used; and, on the contrary, stone or brick earth being plenty, and wood scarce, masonry is to be preferred; taking care never to use sea-water in the lime, nor to make the parapet entirely of stone; for the splinters drove about by an enemy's cannon, will make it dangerous for the defendants to continue their posts.

SECTION III.

190. *Of the Fortifying Harbours by Booms.*

Notwithstanding the forts built for the security of a harbour, or passage, there are many instances that might be produced to shew that such forts alone are not sufficient. The great Sir Walter Rawleigh says, "In this age, a valiant and judicious *man of war* will not fear to pass by the best-appointed fort of Europe, with the help of a good tide and a leading gale of wind; no, though 40 pieces of great artillery open their mouths against him, and threaten to tear him in pieces.

"In the beginning of Queen Elizabeth's reign, when Denmark and Sweden were at war, our Eastland fleet, bound for Liefeland, was forbidden by the king of Denmark to trade with the subjects of his enemies, and he threatened to sink their ships if they came through the freights of Elfsneur. Notwithstanding this, our merchants, having a ship of her majesty's called the *Minion* to defend them, made the adventure, and sustaining some volleys of shot, kept on their course. The king made all the provision he could to stop them on their return: But the *Minion*, commanded (as I take it) by William Burrough, leading the way, did not only pass with little loss, but did beat down with artillery a great part of the fort of Elfsneur, which at that time was not so well rampard, as perhaps now it is; and the fleet of merchants that followed him, went through without any wound received. Neither was it long since, that the duke of *Parma*, besieging *Antwerp*, and finding no possibility to master it otherwise than by famine, laid his cannon on the bank of the river, so well to purpose, that he thought it impossible for the least boat to pass by. Yet the Hollanders and Zealanders, not blown up by any wind of glory, but coming to find a good market for their butter and cheese, even the poor men attending their profit, when all things were extreme dear at *Antwerp*, passed in boats of ten or twelve tun, by the mouth of the duke's canon, in despite of them, when a strong westerly wind and tide of flood favoured them; as also with a contrary wind, and ebbing tide, they returned back again: So as he was forced in the end to build his *stockado* overthwart the river, to his marvellous trouble and charge. The fort of *St. Philip* terrified us not in the year 1596, when we entered the port at *Cádiz*; neither did the fort at *Puntal*, when we were entered, beat us from our anchoring by it, though it played upon us with four demi-cannon, within point-blank, from six in the morning till twelve at noon. The siege of *Osband*, and of many other places, may be given for proof, how hard a matter it is to stop the passage of a good ship, without another to encounter it. Yet it is true, that where a fort is so set as that of *Angra* in *Tercera*, that there is no passing along beside it, or that ships are driven to turn upon a bow-line towards it, wanting all help of wind and tide; there, and in such places, it is of great use and fearful: Otherwise not."

But since Sir Walter's time, the Sound has been better fortified, yet in 1700 Sir George Rook passed it, notwithstanding the Danish fleet lay to impede his passage. The actions at Londonderry, Vigo, Porto-bello, Carthagea, and at many other places, are all sufficient proofs, that nei-

"ther

ther forts nor castles can stop the enterprises of the brave; and therefore it has been judged proper on many occasions, besides forts and batteries, to lay booms and other obstructions, in a stream, to secure the shipping there, from the attacks of a powerful enemy.

191.

To lay a Boom.

Provide a great number of wooden battlings or spars of about 20 30, or 40 feet long, more or less, and between 5 and 10 inches diameter: Then moor two boats, having a sheet-anchor in each, in the place near one side of the river where it is intended the boom shall begin; bend two cables to these anchors, and round them place the spars or poles, frapping on each with rattling stuff, or better with four inch rope, until the boom is 7, 8, 9, 10, or more feet diameter (according to the hands employed), the cables being in the middle; next, with iron hoops rivetted together, worm the boom, and drive through the hoop a nail into almost every spar: After having wrought a good birth for the anchors, drop them, and continue the work till it is brought near the other edge of the river, and there drop two anchors more, with the cables bent to them: Then over all lash the spare yards and top-masts, with the top-chains, so far as the channel goes.

To that part of the cable within the boom, over the channel, let two or more cables be fastened, and these bent to anchors laid down the stream; over the clinch of these cables let battlings or spars be wrought for near ten fathoms down the stream, that the enemy may not cut these cables: These will be serviceable when the enemy's ships come *stemlings* against the boom; for if he force it in one place, the whole will not be opened by that fracture.

In places where wood is scarce, and proper spars are not to be readily had; or in cases where there is not time sufficient to prepare them; it may be found sufficient to woold two cables together with old ropes, lashing to them the oars, top-masts, and yards, and worming the whole about with iron hoops; and let every part be well *payed* with pitch, and small gravel strewed on it while the pitch is warm. A boom so prepared cannot be easily cut.

It is in most cases convenient to have the boom so contrived, as to open at one end for the passage of vessels; and this may be done several ways, one is. Let one end of the cable be clinched to an end of a large mooring-chain, having the other end floated with a buoy, and this end fixed to the ring of the anchor by a shackle: Now the chain being loosed from the anchor at the ebb with a slack hawser fixed to it, the boom will swing down the stream during the ebb, and upon the flood the boom may be relaid, if the enemy appear in fight, which he must do the first of the tide, for upon the ebb there is no danger of his coming; because, if the wind be right in, a prudent enemy will not adventure against the tide, a ship then making such wild steerage; and should she ground, she must lie there till flood, which may prove fatal to her from the batteries ashore; and against both wind and tide the enemy cannot come in. On the contrary, should the boom be carried up the flood, and the enemy appear at the beginning of the next flood, the boom cannot be relaid till the ebb, and before that time the enemy may have accomplished his design.

192. If

192. If there be good store of timber at hand, then a *stockade* may be made by driving several rows of piles in the channel before the boom; or should the depth of water be too great for this work, the trees may be usefully applied, by making a raft to ride before the boom by good anchors, observing that the cables are so made fast, that the enemy cannot come to cut them: These rafts will be of singular use in putting the enemy into disorder, by laying rubbish on them to make fires of when the enemy appears, which will put him into some consternation, and perhaps may cause him to chop to anchor, and lose time or his tide; and this may be done each flood, observing not to have the smoke drive into the works, which might give the enemy too great an advantage.

193.

To lay a Boom in a straight Channel.

Suppose the wind for the most part to blow obliquely across the river or channel on the starboard quarter going up the river: Now if the boom be laid directly thwart the channel, the ships coming with the tide of flood and a leading gale will run stemlings against it, and so hazard its breaking, by striking with a force that comes direct; whereas, could the boom be laid obliquely thwart the river, nearly in a line with the wind, so as the ship must take it with her bow, the blow would be diverted by the ship's casting: because in this position the enemy cannot run stemlings against it; for in sailing up the river, the ship must be near before the helm; and to bring her head laid to a boom obliquely, the helm must be put down, and then it is ten to one whether she come to so nicely as to take the boom: Not to mention the mistakes in a confusion committed by the man at the *helm*, and he upon the *cond*, nor the smoke; for exclusive of all these, it may be taken for granted, that the ship would cast along-side the boom: Then the batteries at the end of the boom on the larboard side, would rake him fore and aft, while the opposite batteries on the starboard side playing on his broadside with *double-round* and *partridge*, must make a great slaughter among his men cutting at the boom. And should he not swing along-side the boom, but lie stemling against it, then will the batteries on the starboard side of the river, which are to be made above the weather-end of the boom, rake him fore and aft, while those on the larboard side, against the leeward end, play on his broadside.

194. The ships within the boom, which it is to protect, should be moored in a kind of half-moon, with their broadsides flanking the boom. And several old ships, or those the least useful, may be sunk as soon as a signal, signifying that the boom will be cut, is made at one of the forts which defend it; for this purpose, those ships should have large scuttles ready cut: And for a farther security, it would be very proper to have a small boom to divert the enemy, that the ships may be sunk in the channel before he boards them.

As to the rest of the ships, they are left to their commanders judgment: and if there is no other remedy, the people may get ashore to windward if they can, and there make the best defence in their power against the enemy's boats and fireships. From duly weighing the whole of these circumstances, it will be found that the chief strength is in the boom;

boom; therefore if a double, triple, or fourfold boom was laid, if the materials could be procured, and the value of the shipping and cargoes were of sufficient importance, it would make the place so much the stronger, and the enterprise of the enemy more hazardous. In stretching these booms, the trouble of many anchors may be spared, by making all the cables fast to the first, and so let them float in a bight, and by a small anchor ride upon the ebb, to keep clear of one another.

195.

To set up a boom in the bend of a river.

From the point formed by a bend, stretch two booms across the channel, one towards the middle of the opposite bight, and the other so much higher, as to lie directly thwart the channel, leaving a kind of angular space between them.

Next the point from whence the two booms stretch, erect a proper fort or battery to command the channel both below and above the bend.

On the other side of the river, erect another fort or battery against the bight a little above the end of the lower boom, and so disposed, that its cannon may rake the channel coming up, as well as command both the booms.

From such a disposition, it is a great chance if a ship answer her helm so timely in *bearing* or *loosing* about the point, as to take the boom stemlings, and if she smite it with her bow, she casts; and in either case she will be raked fore and aft by one fort, and have her broadside battered by the other.

Let some old vessels be fitted up for fireships, and placed between the two booms; from each ship let two hawfers be carried ashore, one on each side, and fixed to crabs or capstans set up; so that as soon as the enemy has passed the first boom, these ships being set on fire, and heaved in their way, nothing can hinder the enemy's destruction: As soon as fire is set to the train, the boat may pull ashore, under covert of the ship, without any apparent danger; for such will be the enemy's consternation, that they'll soon leave firing. The hawfers may be fastened to clamps below the water-line, that they be not burnt, nor in sight of the enemy to cut them. If the fireship is clapt aboard the weathermost ship, they may be both heaved together aboard another to leeward.

The ships to be defended, may be moored in a half-moon, with their broadsides so laid, as to batter the enemy when he attempts the boom: And if other works than what are here directed be judged necessary they must be adapted to the situation of the place.

196. When no strong attacks by land are to be feared, the mooring of ships behind a point is best, on account of laying the boom: Now should the boom be forced, which must be upon the flood, a fireship, instead of falling on board a ship thus moored, will by the tide be hurried beyond her; and if the place is any ways favourable to the ships moored there, it will be found impracticable to board a ship thus moored, with such a wind and tide as the enemy must have to break the boom, unless he expose his boats in carrying out an anchor to *warps* over, which will be a very dangerous attempt; or some unaccountable accident intervene.

197. It

197. It may be objected, that ships thus moored cannot bring their broadsides to bear upon the enemy when coming to force the boom; and that they may be battered by the enemy over the point. To this it may be answered, first: It is no matter whether the broadsides do or not; for perhaps it might be the best way to get the guns ashore, if there is time, to be there used; and to put the smaller fort only on the lower deck, and turned to flank the boom. Secondly, The enemy will have but an uncomfortable *birth* in lying to batter the ships over the point, as they will have to deal with the fire from the fortifications ashore. What is here said, is only to be understood when five or six ships are to be defended; for the wake of a point would not hold any considerable fleet.

198. *Maxims to be observed in fortifying a harbour and mooring the shipping therein.*

I. Let all the land-marks be removed that may direct the enemy's steerage into, or up the river or harbour, when there is a probability of an attack.

II. Choose such a place if possible for erecting the works on, that cannot be laid under water, either by hasty rains, or by any art of the enemy.

III. Let the eminences which command the works within cannon shot, be secured by erecting small forts on them, furnished with small cannon.

IV. All woods within cannon-shot of the fortifications are to be cut down, and the timber employed about the works.

V. Secure a spring of fresh water by a fort and proper guard.

VI. Take all the buoys from the anchors that ride the booms, that the enemy may not trip them.

VII. Let the powder and other munitions be separated and kept in different places, that if an accident happens, the whole may not be destroyed.

VIII. The ships should not be posted near any town or village, unless they are intended to cover that place, or the thing be unavoidable; lest the shot and bombs which miss the ships destroy the town.

IX. The ships should never be moored before the fortifications.

X. Moor the ships so as to rake the enemy fore and aft when he batteries the works.

XI. Let not the ships when moored touch one another, that the enemy may be obliged to burn each ship singly.

XII. Unrig the ships, and strip the throwds from the mast-head, that the grapplings of fireships may have no hold.

XIII. Let the sails be carried ashore for tents, and most of the gunner's stores; that if the worst happen to the ships, there may be store of ammunition for the forts.

XIV. Unless the cargo be ashore, let not the chests nor clothes of any person, from the captain to the swabber, be carried out of the ship; where by every man may have some inducement to exert himself in the defence.

XV. Let outguards and centinels be placed upon and near the banks of the river or harbour; and let some hands in nimble boats, armed, pass down the stream each night to watch the motions of the enemy.

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SECTION IV.

199. *Of the mooring of ships in fortified harbours.*

Before the boom is laid, it will be necessary to appoint the place where to moor the vessels, that they may be best secured from the enemy's fire-ships, and yet annoy their men of war; and as this can be only done either in the stream or out of it, the advantages and disadvantages resulting in both cases will be here considered.

200. *Considerations in mooring with regard to the stream.*

FIRST. *The advantages an enemy has of a ship moored in the stream.* A ship moored in the stream will find some difficulty in bringing her broadside to gall an enemy in his approach, who always comes with the stream; and if the tide runs too swift for one to lie thwart, it certainly does so for the other; both having ground tackle equally good.

2d. In this situation there is no hindering a fireship, that has passed the boom, from being aboard you, as none but the chase-guns can be brought to bear on him; and should you heave thwart, he will be the surer of grappling you, unless by bringing your whole broadside to bear, you sink him; for should the officer, through want of courage, forsake the fireship, yet the tide will bring him aboard: Moreover, in such a position the enemy's ships of war may board you, and then your batteries ashore are useless.

3d. In riding in the stream, the cables are exposed to the enemy: which if hit by a chance shot, the ship will swing upon the tide, and so the enemy have the opportunity of raking you fore and aft; and if the enemy has any conduct, he may lay his anchors so, as his cables shall be secured from your shot.

4th. Should you moor thwart the stream, the enemy may drop a small anchor out astern, veer thwart your hawse, and so constrain you to alter your position, or rake you fore and aft.

5th. In riding thwart the stream, the cables by bearing a more than ordinary strain, are apt to break, or start an anchor; whereby the ship swinging on the tide, it is a great chance but the other anchor starts, and she swings to leeward. But these advantages are only on the enemy's side while the tide sets in; for the defendants have them when the tide sets out: Therefore, in such places where the stream runs continually out, the best way is to moor in it, if the wind does not blow always in.

201. SECONDLY. *The advantages a ship moored in a stream has over an enemy.*

1st. A ship moored thwart the tide lies convenient to rake an enemy fore and aft in his approach, and so may do him a considerable damage before he is thwart the stream to batter.

2d. In riding thus in the stream, according to the common situation of rivers, the enemy cannot batter you under covert.

202. THIRDLY. *A ship moored out of the stream has these disadvantages.*

1st. That she cannot rake the enemy fore and aft so well as while lying in the stream; unless by some particular situation of the place.

2d. The

2d. The enemy's men are not much exposed in mooring; for it is but dropping his anchor, and he may veer along your side; or, if you lie thwart the river, across your hawse: But this may be prevented, by riding in the wake of a point; for, should the enemy drop anchor before he be about the point, he cannot veer along-side; and if he let go his anchor after he is about the point, he may be astern before he is brought up.

203. *FOURTHLY. A ship moored out of the stream has these advantages.* 1st. There is no danger of the fireships driving aboard her, unless directed by a strong wind: And for the same reason, the enemy cannot be thwart your hawse, unless he tows there with his boats, and then they and their crews are exposed to your cannon.

2d. A ship thus moored is in no danger of driving, should her cables be shot, if it is calm, as it commonly happens in engagements.

Considerations in mooring with regard to the wind.

204. *FIRST. With regard to windward.*

Beside the advantage of being free from the smoke, you may cause the enemy to drive to leeward should you hit his moorings; and if he tries to heave off, by carrying an anchor to windward, his boats will be exposed to your cannon; and if the boats *lay to pass*, his ship while he heaves will be raked fore and aft, which must do great execution among his men while they stand thick at the capstan: But it is highly probable, a ship so grounded could not be got off.

When to windward, the cables are covered from the enemy by the ship, and may be *seized to clamps* and *cleats* nailed to the side, and so the mooring secured from random-shot: And this ought to be carefully done for preventing the only disadvantage that attends ships moored to windward, which is, that if their cables are shot, they drive out upon the enemy, or swing head at wind; if the former, they may be boarded by the enemy's fireships, or ships of war, and then your precautions are rendered fruitless; if the latter, they may be raked fore and aft by the enemy; and if neither of these happen, the ship may drive to leeward and be grounded.

205. *SECONDLY. With regard to leeward.*

A ship so moored cannot prevent the enemy's being sent on board, unless a shoal or stockado lie between them.

In this position, should you shoot the enemy's cables through his carelessness in covering them with his hull; yet as he drives ashore, or upon you, his boats are covered with his hull while they carry an anchor to windward to heave off.

Besides, your moorings lying to windward are exposed to the enemy; which should they be cut, you will swing head at wind, and in that position be raked fore and aft; and if you attempt to carry out an anchor to heave your broadside against the enemy, your boats will be too much under his command; indeed, instead of carrying out an anchor, it may be best to clap a spring upon the cable, but this will fail if the ship is too near the shore.

Upon the whole, it appears best to moor to the windward in the wake of a low point, when such a situation can be obtained; but the circumstances being so various, it must be left to the judgment of the commander, who, if he rightly considers what has been said, will chuse for the best.

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three mortars: Captain Latham was sent to receive the keys and take possession of the fort; and Colonel Clive, with the king's troops, marched in about 5 o'clock.

215. *Of the siege of Louisburg in 1758.*

Louisburg is a strong town in the island of Cape Breton; it stands at the S. W. end of a very fine bay, whose length extends NE. and SW. above two miles, and is about three quarters of a mile in breadth: the entrance, of about one quarter of a mile broad, is near the middle of the bay, its southern side is formed by a cluster of islands, in one of which stood a large battery, called the island battery, of 36 guns; and on the coast opposite the entrance, stood the royal battery of 42 guns, with a covered way and glacis before it: In the harbour were 13 sail of ships, most of them lying to the NW. of the town, whereby they lay ready to defend the town on the harbour side, and to impede the approaches on the land side: the garrison was about 4000 strong, had about 220 cannon, 18 mortars, and well-stored with provision and ammunition.

On the 28th of May, the fleet under Admiral *Boscawen*, and the army under General *Amburst*, assembled before Halifax, and on the 2d of June, they anchored in Gabarus bay, about two miles to the SW. of Louisburg: but on account of the blowing weather, nothing towards landing could be done till the 8th, when seven frigates drew near shore, and for about a quarter of an hour cannonaded the French defences thrown up along shore, and then the boats landed the troops amidst a very heavy fire, both of the French musketry and cannon, and a most dreadful surf, which overset and flaved several of the boats, and drowned many soldiers; and upon the whole there were above 100 boats lost in landing the troops and provisions.

On the 12th, Brigadier Wolfe was sent round the harbour to the north side of the entrance, to construct a battery near the light-house, and thence to ruin the island battery: he drove the enemy from all their defences on that side, took several cannon, and by the 20th had completed the light-house battery, which, with the fire from the ships, silenced the island battery by the 23th: the light-house battery being considerably strengthened by ship-guns, and intrenched, Brigadier Wolfe was ordered round with his detachment and cannon, and to advance towards the west gate, and try to destroy the shipping. He took post on the hills for that purpose on the 30th.

The 29th, the French sunk four ships in the harbour's mouth, with their masts cut down; and on the 21st of July, one of their ships was set on fire by a shot from the marine battery, this fire communicating to two others, the three were burnt. On the night of July 25, Admiral Boscawen detached the boats of the fleet in two squadrons under the command of the Captains Laforey and Balfour to take or burn the only two French ships, then in the harbour: the boats going silently along the middle of the harbour, the noise of the oars was not heard at land, nor at the ships till the boats were nearly along side; and then the courage and activity of the seamen rendered all resistance useless:

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useless: one of the ships, the *Prudente* of 74 guns, being aground, they set her on fire; and the other, the *Bienfaisant* of 64 guns, they cut from her anchors, and towed into the NE. harbour amid the fire of the musketry and cannon from the town and other places.

In fine, the batteries of the English being opened, well served, and well directed; and the enemies ships, and many of their defences being ruined; they though proper to capitulate on the 26th of July, and the English took possession of the town on the 27th.

216. *Of the taking of Quebec in the year 1759.*

QUEBEC, the capital of the French dominions in North-America, stands on the NE. bank of the great river St. Lawrence, about 360 miles from its mouth, and on the south side of the river St. Charles; which falling into the river St. Lawrence on the north of the town, forms a fine basin of about three miles in length and breadth; the east and NE. parts are terminated by the isle of Orleans, which is a most delightful and well-cultivated spot of about twenty miles long and seven broad, formed by the river St. Lawrence: the greatest part of Quebec stands on a rocky hill, in some places near fifty feet above the river; and on a small strand at the foot of the hill is built the part called the lower town: the high land stretches up the river St. Charles, and also, with a steep front, for many leagues up the river St. Lawrence: the town was fortified and well-defended by many batteries, besides an army of about 10000 men, under the brave and cautious General *Montcalm*, occupied posts well intrenched, at every accessible spot for an extent of near five miles on the side of the river St. Charles, imagined to be the only part by which the town could be approached by an enemy: these posts had in front a sand-bank between them and the river, and thick woods in their rear.

The English army, of about 7000, commanded by General Wolfe, were landed by Admiral Saunders, who commanded the fleet, on the 27th of June, on the Isle of Orleans; and on the 29th of June a detachment drove the enemy from point Levi, a high promontory on the south shore of St. Lawrence, which commanded the basin, and even the town of Quebec, and from whence the English batteries considerably damaged the upper town, and intirely destroyed the lower one: and about the same time a detachment took possession of the westernmost point of the isle of Orleans, which also commanded the basin. Admiral Saunders, with his division of the fleet, was posted below the town in the north channel, and Admiral Holmes, with his division, was stationed above the town, (to divide the enemies attention, and guard the batteries playing on the town and works.

On the 30th of June, about mid-night, six fireships, well prepared for burning, their sails being covered with a composition that burnt a considerable time, were almost in an instant lighted up, and made a most terrible appearance; they came down with a gentle breeze and a strong tide towards the middle of the fleet, then riding in the north channel, pretty close to one another; but this being expected by the Admiral, the boats were ordered out, and these dreadful machines towed ashore

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206. *Of some remarkable actions concerning the attack of ships in fortified harbours.*

Having shewn the methods that have been directed by experienced men, to be used in the defence of harbours, we shall now think of closing this work with the accounts of some of the most remarkable actions recorded within the last 100 years, wherein the English have had any share; and this was thought useful on two accounts. First, as examples to the preceding precepts; and secondly, to point out to the young mariner the means by which many have acquired great reputation and honour.

207. *The battle of Santa Cruz in the year 1657.*

Santa Cruz is a town and bay on the west side of the island of Teneriff, one of the Canary islands: The bay, which lies rather open than inclosed, has deep water near in shore; but the best anchoring is about half a mile from shore, in 30, 40, or 50 fathoms water, black slimy ground: If there be many ships, they must ride close one by another. The shore is in general high land, and in most places is steep to the water.

Admiral Blake lying with some ships near Cadiz, to watch for the return of the Spanish plate fleet, had intelligence they were in the bay of Santa Cruz, in the island of Teneriff; hereupon he broke ground the 13th of April, and on the 20th arrived at Santa Cruz, where he found the Spanish fleet, to the number of sixteen, moored in the bay in a half-moon: Near the mouth of the bay was a castle well furnished with near 40 heavy cannon; and besides that, there were seven forts round about the bay, with 6, 4, or 3 great guns in each, all united by a line of communication from fort to fort, and well lined with musketeers. *Don Diego Diagues*, the Spanish general of the fleet, upon sight of the English fleet, caused the ten smaller ships to be moored close to the shore, and set six great galleons, well manned, further out at anchor, with their broadsides to sea; in this posture the Spanish admiral vainly thought himself so secure, that a Dutch merchant ship going out of the harbour, he sent a message by him to *Blake*, that he might now come if he durst.

The English admiral, having well viewed their posture; saw it would be impossible to bring off the galleons; however, he resolved to burn them, and to that end sent in Captain *Stayner*, commander of the *Speaker frigate*, with a squadron to attack them; He soon forced his passage into the bay, whilst other frigates entertained the *forts* and *lines* with continual broadsides: These were presently supported by *Blake* himself with the whole fleet, who placing some of his ships to batter the castles and other forts, he with *Stayner* continued to engage the galleons; over whom, in six hours, he gained (notwithstanding they had reinforcements of men several times from the shore) a complete victory. The Vice-admiral and Admiral being blown up in the engagement, and the rest drove ashore; where, notwithstanding the fire from the forts, they were all set on fire by the English, and every one of them was burnt to the water's edge. They had no sooner done this, than the wind luckily turned, and carried the fleet, without the loss of one ship, out of the bay, and put them safe to sea again.

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Londonderry is a small but pretty strong town in the north of Ireland: it stands on the west side of the river *Loughfoyle*, about three miles from a lake of the same name; which is about ten miles broad and fifteen miles long, communicating with the sea by a streight of about a mile broad, so that the town stands about eighteen miles from the sea: The river next the town is about $\frac{1}{4}$ of a mile over, and keeps this breadth nearly, between the town and lake, where its mouth is defended by Culmore fort, which is a large square with four bastions.

This town was besieged by the Irish and French in the year 1689, and the garrison brought into great streights for want of provisions, the besiegers having strongly fortified the river. There was a boom framed of a chain and cables, covered with timbers, stretched across the river about two miles below the town, from a point on the east side: Each end of the boom was defended by a fort. Between the boom and Culmore fort were two others, one on each side of the river, and another fort above the boom opposite the town. The sides of that narrow river were intrenched and lined with musketeers, and several boats were sunk, and stockadoes drove, armed with large iron spikes.

Notwithstanding all these preparations, the boom was forced, the town relieved, and the siege raised, by the assistance brought by three ships only, in the following manner: The *Montjoy of Derry*, commanded by Captain *Browning*, and the *Phoenix of Colrain*, by Captain *Douglas*, being both laden with provisions, were sent towards the town, under the convoy of the *Dartmouth* frigate, commanded by Captain *John Leake*. They were forced to stand a furious fire of the enemy from Culmore fort, and from the forts and intrenchments on both sides of the river, which they received and returned with the greatest bravery imaginable. The *Montjoy* made a little stop at the boom, occasioned by her rebound, after striking and breaking it, so that she was run a-ground. Upon this the enemy gave a loud and joyful shout (though a dreadful one to the besieged), fired all their guns upon her: and were preparing their boats to board her. The trouble and concern of the besieged, to see their last hopes disappointed, is not to be expressed; but by great providence, firing a broadside, the shock loosened the ship so, that she got clear and passed the boom. Captain *Douglas* was engaged all this while, and the *Dartmouth* gave the enemy very warm entertainment, till at length the three ships, having overcome the great difficulty of breaking and passing the boom, got up to the city, and brought unspeakable joy and transport to a garrison, which reckoned only upon two days life, having nothing left but nine lean horses, and a pint of meal to each man. This brave and successful undertaking so discouraged the enemy, that they raised the siege the next day, and retreated by night in the utmost confusion; but made a most miserable havock in the country, by robbing and burning all before them for several miles.

The boom appears to have been laid directly thwart the stream, and so was easier to force, than if it had been laid obliquely across.

The English fleet under the command of Sir George Rooke and Sir Thomas Hobson were conveying of a very large fleet of merchant ships towards the Mediterranean; but falling in with the French fleet under the command of Admiral Tourville, the English admiral gave the signal for the merchantmen to shift for themselves; and among others, four heavy-laden Turkymen; namely, the *Shandois*, the *Italian merchant*, the *Asia*, and the *Loyalty*, stood away for the Spanish ports, and put into the bay of Gibraltar; where the commanders of the Turkey ships, being hindered from prosecuting their voyage by some dissension on board, and the fear of the French fleet, thought proper to fortify themselves.

For this purpose, the commanders posted their four ships in a line within the new mole, which stretches to the NW., with their broadsides flanking towards the north, or entrance into the mole; the ships were unrigged, and consequently having no fails to manage, they turned all hands to the cannon, bringing as many as they conveniently could to bear on one side. On the SE. end of the mole, or land-side, was a castle, in which was mounted ten small cannon, four pointing into the bay towards the west, three commanding the mole, and the rest flanking the NE. shore. Opposite to the mole-head was a kind of redent, very advantageously built to flank the entrance into the mole; but neither on this, nor the mole-head, had the Spaniards any cannon. From the castle there ran northward a line of communication, or a strong stone wall, to the south end of the town, where was a battery of cannon, but too far from the mole to be of any service in defending it.

The English stretched a cable from the mole-head to the redent, the ends being bent to two sheet anchors, and to this cable they lashed their *spare yards* and *top-masts*. Upon the mole-head was planted six of the Shandois's guns to flank the boom, and this battery was committed to the care of some English seamen, who behaved themselves very well. The commanders intended to plant some guns at the other end of the boom, upon the redent, but being obliged to go through the castle, the draw-bridge broke, and down went the first gun that came upon it into a ditch near forty feet deep, with some of the Shandois's men; by this it may be judged in what condition the Spanish works and garrison were.

This was the posture things were in, when the French came into the bay with fourteen or sixteen men of war, and two bomb-ketches; and standing within shot of the castle, the Spaniards shewed their dexterity in managing their ordnance, by firing about one shot in half an hour. The commanders perceiving their awkwardness, sent a quartermaster from on board the Shandois, and the gunner's mate of the Loyalty, to their assistance, and some dozens of cartouches; but through want of judgment in the Spaniards, they were both blown up. All this time not a ship could bring a gun to bear, and so without any damage the enemy passed by into the bay, where they anchored.

In the afternoon they sent down two *men of war* to view in what posture the ships lay; as soon as they came within shot, the English fired briskly at them, but before many broadsides were returned, they stood away to their

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their squadron, and continued quiet that day: Next morning a ship of upwards of fifty guns came towards the boom, and it proving calm when she came within shot, there was pretty brisk firing on both sides, on which she endeavoured to tow off, but her boats were sunk from the battery on the mole-head, and had not boats come from the squadron to her assistance, she had most certainly been sunk, having above 100 cannon playing upon her from the English. The enemy finding such entertainment provided for them before the boom, sent off four *men of war* and the two *bomb-ketches*, who lying to the southward of the mole, and firing over it, raked the English fore and aft; who on account of their former disposition could not bring one gun to bear upon the enemy, neither from the four ships, nor from the mole; under these circumstances, the commanders judged it best to sink the ships. In the evening the French stood out of the bay, and next day stood in again; and having posted some *men of war* and their two bomb-ketches against the castle and mole, they sent in a store-ship they had turned into a fire-ship, and burnt the upper works of three sinking ships; for the *Italian merchant's* forecalle being under water, saved the Shandois.

210.

Disposition of the English fleet at Cadiz.

The 16th of October Sir George Rooke arrived in the bay of Cadiz with the fleet under his command, and on the 21st it was resolved in a council of war, that as there were but 30 ships of the line, which was not a force sufficient to oppose the enemy, who were expected to be above double that number, all that could be proposed till the fleet should be reinforced from England, was to protect the trade: But lest the French should appear at Cadiz before this reinforcement arrived, it was agreed that the ships should be removed within *Puntal* castle, and formed in three lines; which was all that could be done, the ships being in general but half manned, and the Dutch ships very foul:

These three lines were to be as follow: The first, to consist of the largest English ships, to lie from *Puntal* thwart the channel, to the creek's mouth called *Trucadero*, next within the north castle: The second, to be composed of the smallest English and Dutch ships, along the shoal on the south side of the harbour: And the third, to be of the largest Dutch ships, to begin from the upper end of the second line, and to trench away thwart the channel, to the mouth of the upper creek, which goes to *Port Real*; and the small frigates, bomb vessels, and fireships, were to be posted to the best advantage, as the wind and other circumstances might permit. However, the fleet was not attacked, and about the middle of March Sir George put to sea on his return to England, and arrived in the Channel the 22d of April.

211. *Fortifications in the harbour of St. John in Newfoundland, 1697.*

Commodore Norris being sent with a fleet to recover Hudson's bay, put into the harbour of St. John in Newfoundland, upon hearing a large French fleet were upon the coast of North America; and not knowing the strength of the enemy, he judged it prudent to secure his force, that it

it might be employed in the service he was sent on. He therefore put his squadron into a line of battle, which was composed of four fourth rates, two fifth, and two sixth rates, with two fireships, two bomb-vessels, and a hag-boat : Two booms were laid across the harbour, and Colonel Gibson's regiment, that was carried by the squadron from England, was embarked, and several guns mounted on the batteries ashore.

When the French squadron came in sight, the foregoing disposition was altered : All the *men of war* lay in a half-moon to the harbour's mouth ; and each ship's broadside commanded the two booms : Colonel Gibson's regiment was posted at the two batteries, and about the harbour's mouth, and Captain Richards (who was the engineer for fortifying the harbour) was making such works as he judged necessary on this occasion ; and that the squadron might be better able to do service, all the men were taken from the merchant ships and put on board them.

The French fleet made no attempt on the harbour ; but, on the contrary, were glad to find that the English had been misinformed of their strength, and so shaped their course for Europe.

• 212. *The battle of Vigo, in the year 1702.*

Monsieur *Chateau-Renault* arriving early in September at *Vigo* in Galicia, with the Spanish galleons under his convoy, received information of the English and Dutch fleets being before Cadiz ; of which timely intelligence he made all the advantage he could in preparing for their reception.

In the upper end of *Vigo bay* is a river running into the bay ; up this river is *Redondella* harbour, which is surrounded with hills in such a manner, as to be capable of being made very strong, the harbour's mouth not being above a musket-shot over. On the north side the entrance was a battery of eight brass, and twelve iron guns, and on the south side a platform with twenty stately brass guns, and twenty good iron guns, besides a stone fort, with a trench about it, mounted with ten guns, and defended by 500 men. A little above the battery on the right, from two points in a narrow gut, was stretched a cable, to which was lashed masts, yards, casks, &c. with their top-chains, and frapped about with four-inch rope very thick : The bight of the boom was rid by three anchors down the stream, that should the English ships come stemlings against it, they might not bring home the anchors upon the shelf at the end of the boom ; or if they cut it in one place the whole might not be open. To flank the boom, there was moored the *L'Esperance*, a ship of seventy guns, on the south side, and the *Le Bourbon*, a ship of seventy-six guns, on the north side ; also five men of war of sixty and seventy guns each, lay moored with their broadsides flanking the gut. The French admiral, to give him his due commendation, had taken all human precautions to secure his fleet.

When the confederate fleet came before the place, Sir George Rooke called a council of war, and it was resolved to attempt the forcing of the harbour the next morning. As soon as the land-forces were got on shore, the 12th of October in the morning, the admiral gave the signal to weigh, the line was formed, and the squadron was briskly bearing up

to the boom ; but when the van was got within shot of the batteries, it fell calm, so that they were necessitated to come to anchor : However, a fresh gale springing up not long after, Vice-admiral *Hopson*, in the *Tor-bay*, being next the enemy, immediately cut his cables, clapt on all his sails, and bearing up directly against the boom, amidst all the enemy's fire, broke through it at once, cast anchor between the L'Esperance and Bourbon, and with unparalleled resolution received several broadsides from them. The rest of Vice-admiral *Hopson's* division, and Vice-admiral *Vandergot's*, with his detachment, having weighed at the same time, sailed abreast towards the boom, to add the greater weight and force to the shock, but being becalmed they all stuck, and were obliged to hack and cut their way through ; but a fresh gale blowing again, the Dutch Vice-admiral made such good use of it, that having hit the passage which the valiant *Hopson* had made, he went boldly in, and made himself master of the *Bourbon*. All this while Admiral *Hopson* was in great danger, for being clapt on board by a French fireship, whereby his rigging was presently set on fire, he expected every moment to be burnt ; but it very fortunately fell out, that the French ship, which indeed was a merchant-man laden with snuff, and fitted up in haste for a fireship, being blown up, the snuff in some measure extinguished the fire, and preserved *Hopson's* ship from being consumed, which however received very considerable damage in this action : At the same time Captain *Bokenham*, in the *Association* of ninety guns, laid his broadside against a battery of seventeen guns on the left side the harbour, while Captain *Wyvell* in the *Barfleur*, a ship of like force, was sent to batter the fort on the other side. Thus for a long while there was a considerable firing of great and small shot on both sides, till the French admiral seeing the platform and fort in the hands of the English, his fireship spent in vain, the *Bourbon* taken, the boom broke, and the confederate fleet pouring in upon him, he set fire to his own ship, and ordered the rest of the captains under his command to follow his example, which was done in great confusion ; yet could he not be so punctually obeyed, but that several men of war and galleons were taken by the *English* and *Dutch*.—In this action it should be observed, that the success of the fleet was in a great measure owing to the land-forces under the command of the Duke of *Ormond*, who not only drove away several bodies of troops destined for the defence of the shore, but also took the castle and platform on the south side, which might otherwise have done great damage to the fleet.

213. *The action of Carthagena, in the year 1741.*

Carthagena, one of the best towns the Spaniards have in South-America, lies on the Caribbean Sea, almost due south of the east end of Jamaica. An irregular shaped peninsula, running in the direction of the sea-coast, nearly north and south, forms a fine harbour of about ten miles long, and in some places above two miles broad : near the north-end of this harbour stands the city, its NW. side being washed by the sea, which being very shoal and rocky to some distance from the shore, together with a great surf, hinders the town from being molested to sea-ward. Now the south end of the harbour is an entrance called Boca-chica, of about
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one third of a mile broad, with sufficient water for the largest ships. The north side of this entrance is near a mile long, turning east and west, and the south side not above half that length; just within this passage, on the south side, lies a small island, between which and the east end of the north side is about one fourth of a mile.

In the spring of the year 1741, England being at war with Spain, an expedition against Carthagea was undertaken by the English; the fleet was under the command of Admiral *Vernon*, and the land-force was conducted by General *Wentworth*: the Spaniards, apprized of the design, made an excellent disposition for the reception of their enemies, who found the following impediments to their sailing into the harbour.

Near the east end of the north side of the entrance, stood a well-built regular square fort, called Boca-chica castle, two of its bastions commanding the passage, the walls of stone, and mounted with 82 cannon and three mortars; and at the west end were the forts of St. Philip and St. Jago, serving as redoubts to the castle, and guarding a small bay where there was tolerable landing. On the western end of the south shore, called the Barradera, was a fascine battery of 15 heavy cannon, about five of which commanded to sea-ward, and the rest flanked the west shore of the other side, and even the castle itself. On the island stood St. Joseph's fort with 21 guns; between this fort and the castle was stretched a boom obliquely across the passage; and above this boom were moored, in a line commanding the boom, four men of war, each carrying from 60 to 70 guns.

Three English 80 gun ships, in about three hours cannonading, obliged the enemy to retire from the forts of St. Philip and St. Jago; and the army was landed, encamped, and formed the siege of the castle: eight days afterwards, the camp having been much galled by the Barradera battery, it was attacked, burnt, and most of the guns spiked, by the boats of the fleet, under the command of Captain *Boscawen*: but the Spaniards, knowing the value of that battery, repaired it; and by its fire so much interrupted the progress of the army, that seven days after its first destruction, three ships were ordered to lie against it, and the boats of the fleet were again manned under the command of Captain *Watson*, when they effectually destroyed the whole battery. Two days after, on the 25th of March, while Boca-chica castle was storming by the army through the breach they had made, the Admiral sent the ships boats, under the command of Captain *Knowles*, to attack St. Joseph's fort, which he took possession of about ten o'clock at night, and also of the Galicia, the Spanish admiral Don Blas's ship; the Spaniards having, upon hearing the English huzzas at the castle, abandoned the castle, St. Joseph's fort, and the four ships, which were already scuttled for sinking to barr up the passage; but the enemy, in the utmost panic, had not time to follow their orders, so only two of their ships were sunk, and a third they set on fire. Captain Knowles immediately ordered the boom to be cut, and thereby opened the passage, through which Admiral Vernon with the fleet failed the next morning: and now the English having possession of the harbour, the necessary dispositions were made for gaining the town: but herein they failed; and the officers thought it better to retire with the fleet and army, than to stay longer in a country whose unwholesome

wholesome climate was infinitely more to be dreaded than the utmost efforts of its timid inhabitants.

214. *The proceedings of the English in the river Hughley, in Bengal, in India, 1757.*

In the year 1756, the government of Bengal devolving on a nabob in the French interest; he, on a frivolous quarrel dispossessed the English of most of their settlements on the river Hughley: however, the governor of Calcutta, and many of the factory, secured themselves and part of their effects on ship-board, falling down the river out of the nabob's reach. To regain these places, and also to punish the nabob and the French for their perfidious dealings, Admiral Watson, commanding the fleet, and Colonel Clive the land-forces, entered the river Hughley: on the 15th of December the army was landed, and, in conjunction with the fleet, attacked and took every place of defence on the river up to Calcutta, which they also retook on the 2d of January, 1757; and before the 11th they also took the town of *Hughley*, which they plundered, and afterwards destroyed.

On the 5th of February, Captain *Warwick*, at the head of a party of seamen, by the admiral's order, joined Colonel Clive, and they on the same day gained a victory over the nabob's army, which obliged him to make a peace, very advantageous to the English.

Afterwards, Colonel Clive having taken all the French posts in the neighbourhood of Chandenagore, except one between the fort and the river, Admiral Watson, who was proceeding up the river to second the land operations, found himself obliged to come to anchor about two miles below the place; for the French had done every thing in their power to obstruct the passage of the English: by sinking two ships, a ketch, a hulk, a snow, and a vessel without any masts, all directly in the channel, within gun-shot of the fort, and laying two bomks, moored with chains, across the river.

The Admiral, conceiving that a passage might be found between those sunken vessels without weighing any of them, sent on nights some expert persons, under the direction of Mr. Delamot, master of the Kent, who, without being perceived by the French, found out a sufficient channel, and cut the booms; which being effected, and the boats on their return, Delamot * hung up a light at the mast-head of one of the sunken vessels, which the enemy amused themselves with trying to strike with a constant cannonade during the night: however, on the 24th the Admiral, with three ships, passed through this channel, and coming abreast the fort, he anchored; and in less than three hours, after a very brisk firing, the enemy hung out a white flag, and capitulated, notwithstanding they were 1200 strong, and had 183 cannon and

* Perhaps Delamot, to cover his last operation, hung up the light at a distance from the place where he was at work, to draw the enemies fire clear of the people employed.

on the Isle of Orleans, where they quietly burnt without doing any mischief. The enemy's next attack was by a raft of fire-ships composed of fir logs, each stage about the size of a long-boat; these were lashed together, filled with combustibles, loaded pistols, and gun-barrels, and turned adrift towards the ships, but with no better success than the other: the whole was about 200 feet long, and made a fine bonfire.

General Wolfe having encamped on the 9th of July, at Montmorenci, on the continent below the town, and tried, during almost two months, several ways to bring the enemy to a battle, which were all frustrated, either by the difficulties of the place, or the vigilance of *Montcalm*, he determined to try what could be done above the town: he first removed the camp to point Levi on the opposite shore, and Admiral Holmes's squadron made for several days some movements up the river, the English army moving at the same time along the southern side. The French general thought it proper to send M. Bougainville with a body of 1500 men along the northern side to watch these motions. On the 12th of September, about mid-night, the English were embarked in the flat-bottomed boats, and fell silently down the stream, while Admiral Holmes made some movements up the river to amuse the enemy: the boats landed on the north side, not exactly where intended, and the troops were obliged to scramble up a woody precipice, which however they effected, dispersing a captain's guard posted there, and the army was formed by day-light.

While this was doing, and in order to draw Montcalm's attention another way, as had been concerted, Admiral Saunders ordered all the boats of the men of war and transports, well filled with men to make a formidable noise, to feign an attempt to attack the French intrenchments below the town: the number of boats, and the manner in which this false attack was conducted, kept the French general too much alarmed to think on what might be doing above the town, and which he could scarcely believe when informed of; but being soon convinced, he assembled his troops and marched to attack the English: both armies drew up with great circumspection; and the French advancing, began the engagement with a galling, though irregular fire; which was received and borne with great patience, till they were come to the distance of 40 yards; and then the English poured in such a well-aimed and dreadful fire, that the French army immediately staggered; and being pushed with a true British spirit, they broke, and yielded up that victory, which they were unable to seize in the presence of such superiors; this decisive battle was followed by the surrender of Quebec, by capitulation, five days after, and the English troops took possession of that city and its neighbouring country.

217. The late wars, particularly the last, have furnished many examples of sea engagements, and of the attack of strong places on the sea-coasts; wherein both the sea and land officers have shewn themselves possessed of all the military knowledge, which good sense and experience could produce; beside that intrepidity so natural to Britons: the few examples here selected, chiefly to illustrate the laying of booms, are neither the most interesting, nor wherein British valour has

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Has shone in its greatest splendor: there are many actions of equal importance, and accompanied with as striking circumstances; which may be found among the accounts published on those occasions. This subject cannot be better closed, than with a remark of Admiral Vernon's; That where men of war can be brought to act at a small distance, such as about 100 yards, or less, no fort can withstand their fire: but at a greater distance, suppose half a cannon-shot, a fort can receive very little damage from the fire of ship-guns.

F I N I S.



*Collated & Perfect
J. R.
1783*

*the omission of the Pages
between 144 and 193 (in Vol. 2)
being only an error of the Press.*

